

# Sequential Estimation of Bending Stiffness and Damping Parameters of Transmission Line Conductors

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**Identificación de Propiedades de Materiales por  
Métodos Inversos – Mar del Plata**

# OUTLINE

- ❑ INTRODUCTION
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- ❑ MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM
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- ❑ FINAL REMARKS
- ❑ FUTURE WORKS

# INTRODUCTION



## □ GENERAL OVERVIEW

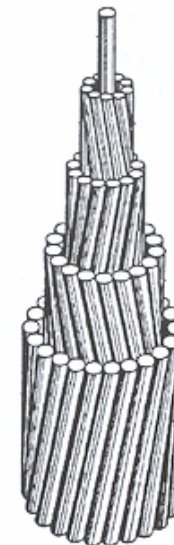
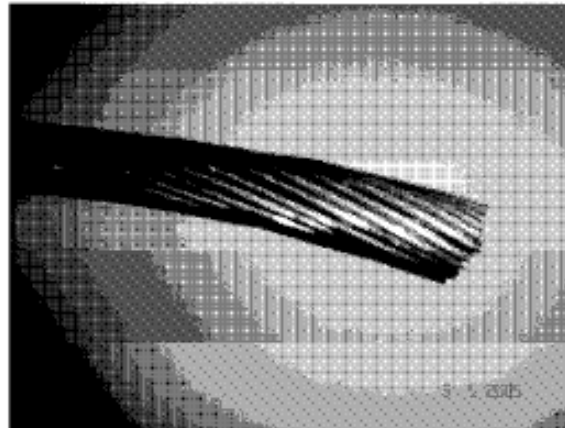
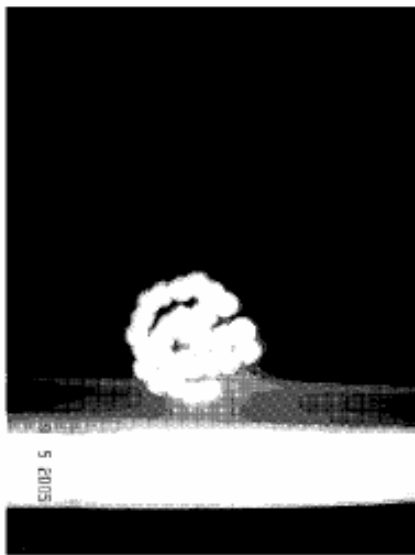
- Wind-induced vibrations on transmission line conductors (TLC) caused by vortex-shedding (Rawlins, 1979; Hagedorn, 1982; Meynen et al., 2005)
- Main features: frequency range 5 Hz – 60 Hz and amplitude range  $0.01 D$  –  $1D$  ( $D$  denotes de conductor diameter 15 mm – 30 mm)
- Well-known galloping vibrations of very low frequencies (below 1 Hz), caused by aerodynamic instabilities, are not addressed here
- TLC are composed of wires helically wrapped around a central core

# INTRODUCTION

## □ GENERAL OVERVIEW (cont.)

### Aluminum Conductor Steel Reinforced (ACSR Conductors)

Layers of aluminum wires helically wrapped around a central steel core



photograph and sketch of a typical TLC



# INTRODUCTION

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## □ GENERAL OVERVIEW (cont.)

- TLC are subjected to very high tensile loads (20 kN – 40 kN) and clamped at the suspension towers
- Frequency spectrum almost continuous; two natural frequencies separated by approximately 0.1 Hz – 0.3 Hz
- Vortex-shedding frequency almost always close to one of the natural frequencies of TLC
- Field and laboratory measurements indicate that TLC have low internal damping, mainly in the frequency range 0 – 30 Hz
- Main damping mechanisms are: (i) interstrand friction among the wires (structural damping); (ii) aerodynamic damping and (iii) material damping

# INTRODUCTION



## □ GENERAL OVERVIEW (cont.)

- Wind-induced vibrations on TLC occur for wind speeds in the range 1 m/s to 10 m/s
- Reynolds number lies in the sub-critical range ( $10^3$  to  $10^4$ )
- Vortex-shedding across stationary bluff bodies in this Reynolds range has a well defined frequency, expressed in terms of a nondimensional parameter called Strouhal number  $St$
- For smooth and circular cylinders  $St = 0.2$
- For TLC, field measurements indicate that  $0.185 < St < 0.22$  (Kraus and Hagedorn, 1991; Rao, 1995)

# INTRODUCTION



## □ GENERAL OVERVIEW (cont.)

- Vortex-shedding across a stationary cylinder is not yet completely understood [Williamson and Govardhan (2004)]
- Concerning wind-induced vibrations on TLC, other complicating factors come into picture
  - (i) the dynamic interaction between wind flow and TLC vibrations;
  - (ii) the turbulent nature of wind flow;
  - (iii) TLC structural vibrations due to lack of information regarding the bending stiffness and damping parameters of TLC

## □ MOTIVATION

- Wind-induced vibrations are a critical problem for safety and reliability of transmission lines
- Bending strains and stresses caused by such vibrations may cause fatigue damages of conductor wires

# INTRODUCTION



## ❑ MOTIVATION (cont.)

- Fatigue damages may lead to complete rupture of the conductor and, consequently, to the interruption on the supply of electric energy
- Therefore, the understanding of wind-induced vibrations on TLC is a relevant issue
- Accurate predictions of such vibrations depend, of course, on the knowledge of stiffness and damping properties of TLC

## ❑ OBJECTIVE

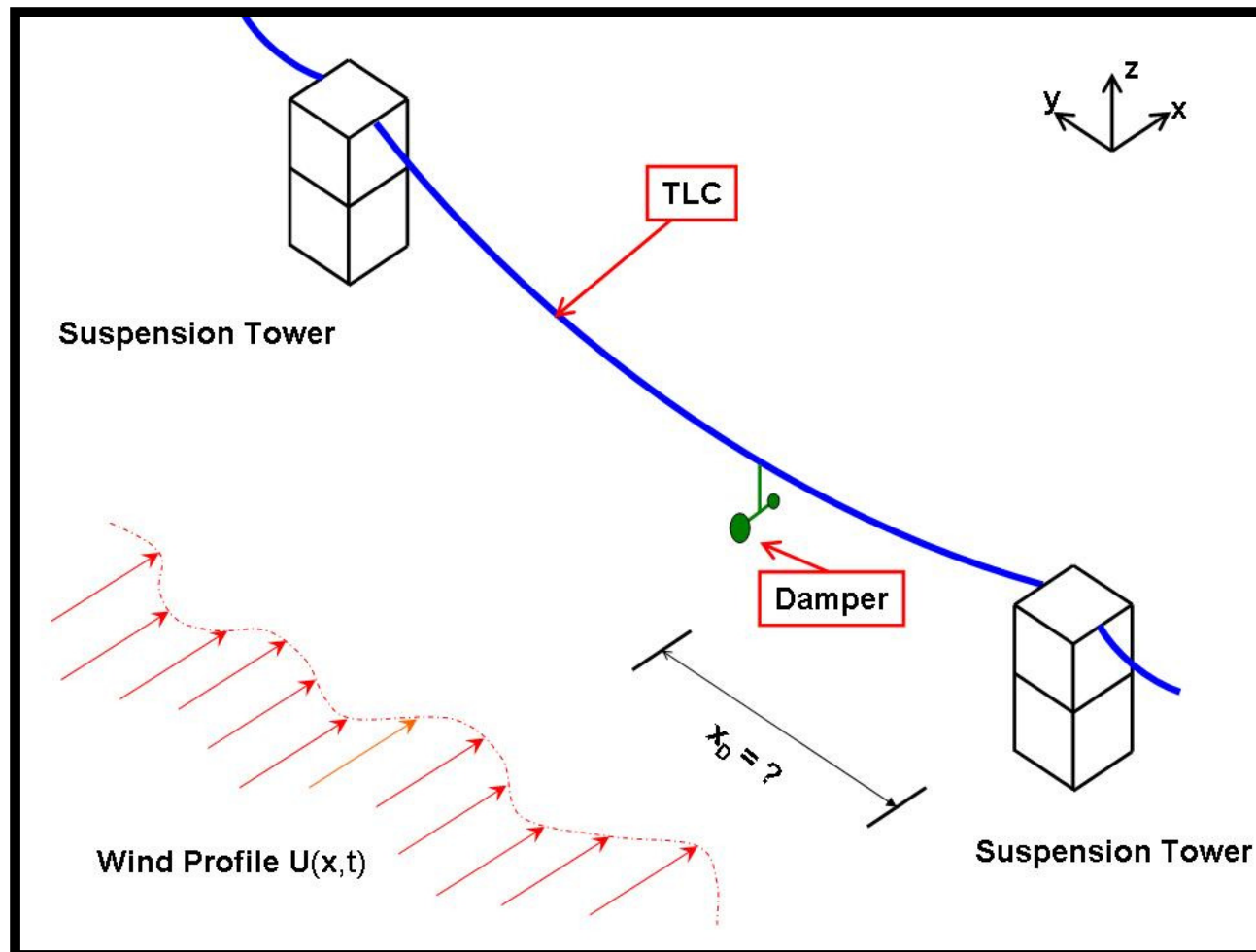
- Estimate the bending stiffness and damping parameters of a typical TLC based on inverse analysis

# INTRODUCTION

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□ ALL IN ALL, WHAT IS THE SYSTEM UNDER ANALYSIS ?



# INTRODUCTION



❑ WHAT IS THE OBJECTIVE OF THIS PROJECT ?

- ❑ To reduce the vibration levels of TLC.

❑ WHAT ARE THE TARGETS ?

- ❑ Target 1 : Determine the bending stiffness and the damping parameters of transmission line cables.
- ❑ Target 2: Determine a suitable mathematical model for the StockBridge Damper.

# INTRODUCTION



## □ WHAT ARE THE TARGETS ? (Cont.)

- Target 3 : Analyze the fluid Structure Interaction.
  
- Target 4 : Determine the optimum number of StockBridge Dampers as well as their optimal positions to reduce the amplitude of vibrations of TLC when the system is excited by wind.

# INTRODUCTION



□ WHAT IS THIS PRESENTATION ABOUT ?

- Target 1: Estimation of the bending stiffness and damping parameters of transmission line cables

□ WHAT ARE THE MAIN ISSUES CONCERNING THIS TARGET ?

- Which model should we use ?
- Which damping model should we use ?
- How do we estimate the model parameters ?



# BRIEF LITERATURE REVIEW



- The majority of theoretical models proposed to predict wind-induced vibrations idealize TLC structure as a continuous (Claren and Diana, 1969; Dhotarad et al., 1978; Hagedorn et al., 1987; Diana et al., 2000; Vecchiarelli et al., 2000; Barbieri et al., 2004; Meynen et al., 2005)
- The simplest models idealize TLC as homogeneous taut strings without bending stiffness; more complex ones idealize TLC as homogeneous elastic beams with structural damping being represented as of hysteretic kind
- Authors rarely report the values adopted for the bending stiffness and damping parameters; there is a current lack in the literature about mechanical properties of typical TLC
- Most of data available refers to the power dissipated by TLC during standard self-damping tests on a laboratory span. Discrepancy among measurements performed by different authors may reach 100%!
- Authors rarely compare their theoretical predictions against experimental measurements (Claren and Diana, 1969; Diana et al., 2000; Barbieri et al., 2004)

# BRIEF LITERATURE REVIEW



- Few works have attempted to account for the helicoidal structure of TLC and all damping mechanisms, most authors adopts a constant value for the bending stiffness
- Recommendation is to choose such the bending stiffness as a constant value between the minimum and maximum values [CIGRÉ (1989)]
  - minimum value ( $EI_{\min}$ ) is obtained by considering TLC as a bundle of individual wires free to move relative to each other
  - maximum value ( $EI_{\max}$ ) is obtained by considering TLC a bundle of individual wires unable to move relative to each other due to contact pressure
- Nevertheless, Papailiou (1997) presented a more sophisticated model which accounts for helicoidal geometry of the wires, interlayer friction and slipping during bending (non-linear model)
- Such a model leads to a bending stiffness which changes with amplitude and mechanical load applied to TLC
- Papailiou (1997) compared his theoretical predictions against experimental measurements performed on laboratory and the agreement was satisfactory

# BRIEF LITERATURE REVIEW

## MINIMUM AND MAXIMUM VALUES OF THE BENDING STIFFNESS [CIGRÉ (1989)]

$$EI_{min} = \frac{\pi}{64} (E_s d_s^4 N_s + E_a d_a^4 N_a)$$

$$EI_{max} = E_s \sum_{i=1}^{n_s} I_{max,i} + E_a \sum_{j=1}^{n_a} I_{max,j}$$

$$I_{max,i} = \frac{N_i \pi d^2}{8} \left( \frac{d^2}{8} + R_i^2 \right)$$

$E_s$  Young's modulus of elasticity for steel

$E_a$  Young's modulus of elasticity for steel

$d_s$  Steel wire diameter

$d_a$  Aluminum wire diameter

$N_s$  Number of steel wires

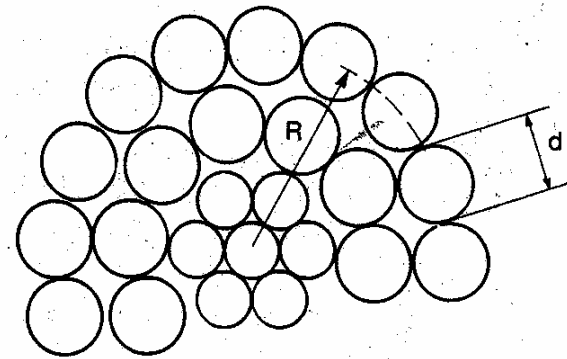
$N_a$  Number of aluminum wires

$n_s$  Number of layers of steel wires

$n_a$  Number of layers of aluminum wires

$N_i$  Number of wires in the  $i^{th}$  layer

$R_i$  Radius of  $i^{th}$  layer



ACSR conductor Grosbeak

26 aluminum wires  $d_a = 3.973$  mm

7 steel wires  $d_s = 3.089$  mm

$EI_{min} = 28.4$  N.m<sup>2</sup>

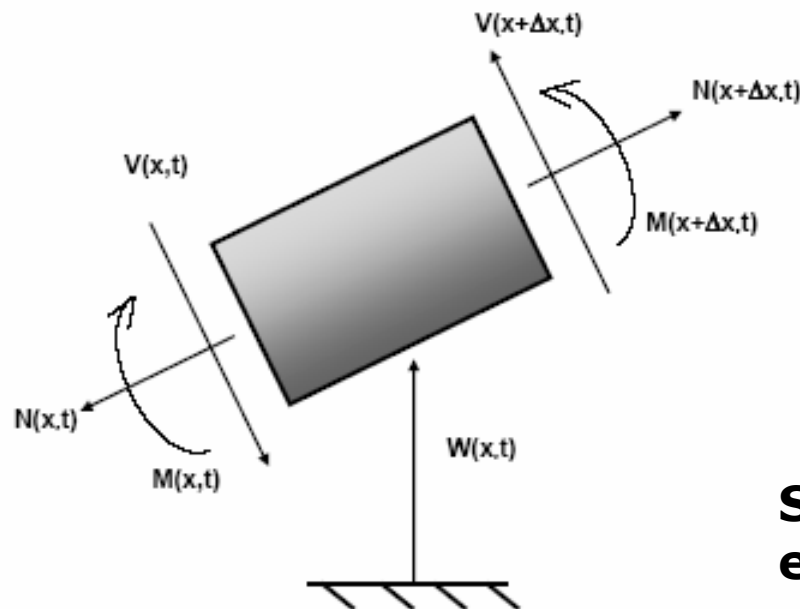
$EI_{max} = 1027$  N.m<sup>2</sup>

What value of  $EI$  should we use?

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

## □ HYPOTHESIS

- TLC modelled as an Euler-Bernoulli beam with constant bending stiffness and subjected to a constant axial load
- Small displacements
- Aerodynamic damping of viscous type
- Material damping modelled from constitutive equation



**Sketch of a differential element of the TLC**

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

## □ EQUILIBRIUM EQUATIONS

$$H(x,t) + \frac{\partial}{\partial x} \left( T \frac{\partial w}{\partial x} \right) - \frac{\partial^2 M}{\partial x^2} - \alpha \frac{\partial w}{\partial t} = \mu \frac{\partial^2 w}{\partial t^2}$$

$$M(x,t) = \int_A -y \sigma(x,t) dA$$

## □ CONSTITUTIVE EQUATION ?

□ Linear or nonlinear ?

□ Candidate models ?

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



- CONSTITUTIVE EQUATION – LINEAR ?
  - STICK-SLIP BETWEEN THE STRANDS (PAPAILIOU, 1997)
  - FOR TLC THE AXIAL TENSION IS EXTREMELY HIGH
  - EXPERIMENTS AIMED AT ASSESSING WHETHER TLC POSSES LINEAR BEHAVIOUR WERE PERFORMED.
  - BASED ON EXPERIMENTAL DATA WE DECIDED TO ADOPT A LINEAR MODEL FOR THE TLC USED (HORIZONTAL, HIGH AXIAL TENSION AND LOW SAG)

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ CONSTITUTIVE EQUATION

- Generalized damping model (time domain)

$$\sigma(x, t) = E \varepsilon(x, t) + \int_0^t K(x, \tau - t) \frac{\partial \varepsilon}{\partial \tau} d\tau.$$

- Kelvin-Voigt model (time domain)

$$\sigma = E\varepsilon + \xi \frac{\partial \varepsilon}{\partial t}$$

- Hysteretic damping model (frequency domain)

$$\hat{\sigma}(x, j\omega) = (EI + j\bar{\eta}I) \hat{\varepsilon}(x, j\omega)$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ KELVIN-VOIGT MODEL

- Constitutive equation defined in time domain what enables one to obtain the governing partial differential equations also in time domain.

- Governing equation

$$EI \frac{\partial^4 w}{\partial x^4} - T \frac{\partial^2 w}{\partial x^2} + \xi I \frac{\partial}{\partial t} \left( \frac{\partial^4 w}{\partial x^4} \right) + \alpha \frac{\partial w}{\partial t} + \mu \frac{\partial^2 w}{\partial t^2} = H(x, t)$$

- + Boundary conditions



# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ HYSTERETIC DAMPING MODEL

- Constitutive equation defined in frequency domain.  
Corresponds to the dissipation model mostly adopted for the analysis of TLC.
- Characterized by a constant loss factor.
- Non-causal. Therefore, we cannot obtain the governing equations in time domain.

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ DIRECT PROBLEM

- Comprises the solution of the above equations subjected to appropriate boundary and initial conditions, assuming that parameters  $EI$ ,  $\xi I$ ,  $\bar{\eta} I$  and  $a$ , and the excitation  $H(x,t)$  are known
- Two techniques were used to solve the direct problem:  
(i) the finite-element method FEM (Hughes, 2000; Reddy, 1993) and (ii) the generalized integral transform GITT (Cotta, 1993; Özişik, 1993)

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

## □ DIRECT PROBLEM BY FEM : KELVIN-VOIGT DAMPING MODEL

### ■ Weak Form

$$\int_{x_e}^{x_{e+1}} \left( -T \frac{\partial w}{\partial x} \frac{\partial u}{\partial x} - EI \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 u}{\partial x^2} - \xi I \frac{\partial^3 w}{\partial x^2 \partial t} \frac{\partial^2 u}{\partial x^2} - \alpha \frac{\partial w}{\partial t} u - \mu \frac{\partial^2 w}{\partial t^2} u + Hu \right) dx = 0$$

### ■ Galerkin Discretization

$$w(x, t) = \mathbf{N}(x) \mathbf{w}_h(t)$$

### ■ System of Ordinary Differential Equations

$$\mathbf{M}^e \ddot{\mathbf{w}}_h + \mathbf{D}^e \dot{\mathbf{w}}_h + \mathbf{K}^e \mathbf{w}_h = \mathbf{f}^e$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

## □ DIRECT PROBLEM BY FEM : KELVIN-VOIGT DAMPING MODEL

### ■ Mass, Stiffness and Damping Matrices

$$\mathbf{M}^e = \int_{x_e}^{x_{e+1}} \mu \mathbf{N}^T \mathbf{N} dx$$

$$\mathbf{K}^e = \int_{x_e}^{x_{e+1}} \left( EI \frac{\partial^2 \mathbf{N}^T}{\partial x^2} \frac{\partial^2 \mathbf{N}}{\partial x^2} + T \frac{\partial \mathbf{N}^T}{\partial x} \frac{\partial \mathbf{N}}{\partial x} \right) dx$$

$$\mathbf{D}^e = \frac{\alpha}{\mu} \mathbf{M}^e + \int_{x_e}^{x_{e+1}} \xi I \frac{\partial^2 \mathbf{N}^T}{\partial x^2} \frac{\partial^2 \mathbf{N}}{\partial x^2} dx = \frac{\alpha}{\mu} \mathbf{M}^e + \frac{\xi}{E} \mathbf{K}^e - \frac{\xi T}{E} \int_{x_e}^{x_{e+1}} \frac{\partial \mathbf{N}^T}{\partial x} \frac{\partial \mathbf{N}}{\partial x} dx.$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ DIRECT PROBLEM BY FEM : HYSTERETIC DAMPING MODEL

- The relationship between the excitation and the response of the system can be written only in the frequency domain

$$\left[ (j\omega)^2 \mathbf{M} + (j\omega) \mathbf{D}_\alpha + \left\{ \mathbf{K} + j \mathbf{D}_{\frac{-}{\eta}} \right\} \right] \mathbf{w}(j\omega) = \mathbf{F}(j\omega)$$

- Elemental matrices

$$\mathbf{D}_\alpha^e = \frac{\alpha}{\mu} \mathbf{M}^e$$

$$\mathbf{D}_{\frac{-}{\eta}}^e = \int_{x_e}^{x_{e+1}} \eta I \frac{\partial^2 \mathbf{N}^T}{\partial x^2} \frac{\partial^2 \mathbf{N}}{\partial x^2} dx$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

## □ DIRECT PROBLEM: HYBRID SOLUTION BY GITT – KELVIN-VOIGT

### □ Auxiliary Eigenvalue Problem

+ homogeneous B.C.'s

$$EI \frac{d^4 \Psi_m}{dx^4} - T \frac{d^2 \Psi_m}{dx^2} - \mu \lambda_m^2 \Psi_m = 0$$

■ Inverse-Transform Pair

$$\left\{ \begin{array}{l} \bar{w}_m(t) = \int_0^L \Psi_m(\lambda_m, x) w(x, t) dx \quad (\text{transform}) \\ w(x, t) = \sum_{m=1}^{\infty} \frac{\Psi_m(\lambda_m, x)}{N(\lambda_m)} \bar{w}_m(t) \quad (\text{inverse}) \end{array} \right.$$

### ■ Transformation of Original Problem - System of Coupled Ordinary Differential Equations

$$\zeta_m \equiv \frac{\alpha}{2\mu\lambda_m}, \quad \zeta_m^* \equiv \zeta_m + \frac{1}{2} \frac{\xi I}{EI} \lambda_m$$

$$\frac{d^2 \bar{w}_m}{dt^2} + 2\zeta_m^* \lambda_m \frac{d\bar{w}_m}{dt} + \frac{\xi I}{EI} \frac{T}{\mu} \sum_{n=1}^{\infty} \Lambda_{mn} \frac{d\bar{w}_n}{dt} + \lambda_m^2 \bar{w}_m = \frac{H_0}{\mu} \Psi_m(\lambda_m, x_s) e^{i\Omega t}$$

$$\Lambda_{mn} \equiv \frac{1}{N(\lambda_n)} \int_0^L \Psi_n(\lambda_n, x) \frac{d^2 \Psi_m}{dx^2} dx = -\frac{1}{N(\lambda_n)} \int_0^L \frac{d\Psi_n}{dx} \frac{d\Psi_m}{dx} dx.$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM

- The unknown parameters are the bending stiffness  $EI$ , the viscous damping coefficient  $\alpha$ , and the constitutive damping parameters  $\eta I$  and  $\xi I$ .
- Additional information used to estimate these two parameters are the complex frequency response functions 'measured' at prescribed locations  $x_p$ ,  $p = 1, 2, \dots, N_s$ , and circular frequencies  $\omega_q$ ,  $q = 1, 2, \dots, N_f$  ( $N_s$  and  $N_f$  denote, respectively, the number of sensors and frequency data)

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM

- Solution of inverse problem comprises the minimization of a suitable error function  $S(\mathbf{p})$ , viz.

$$S(\mathbf{p}) = f(\mathbf{p}, \mathbf{H}^{Exp}, \mathbf{H}^{Est}(\mathbf{p}))$$

- What is a suitable choice for  $S(p)$  ?
- What are the characteristics of our experimental data and what is our interest ?



# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM - FIRST STAGE

- Use the experimental data concerning the lowest frequency bands
  
- Estimate EI and one of the damping parameters by the the classical *Levenberg-Marquardt* iterative procedure [Beck and Arnold, 1977; Özişik and Orlande (2000)].

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM - FIRST STAGE

### □ Error function

$$S(\mathbf{p}) = [\mathbf{H}^{exp}(\mathbf{p}) - \mathbf{H}^{est}(\mathbf{p})]^T [\mathbf{H}^{exp}(\mathbf{p}) - \mathbf{H}^{est}(\mathbf{p})]$$

### □ Iterative procedure

$$\Delta \mathbf{p}^k = [\mathbf{J}^{kT} \mathbf{J}^k + \lambda^k \mathbf{\Lambda}^k]^{-1} \mathbf{J}^k [\mathbf{H}^{exp} - \mathbf{H}^{est}(\mathbf{p}^k)]$$

$$\Delta \mathbf{p}^k = \mathbf{p}^{k+1} - \mathbf{p}^k$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM - SECOND STAGE

□ Use the experimental data containing information of the higher frequency bands

□ Use the estimated parameters obtained in the first stage as *a priori* information for the estimation of the unknown parameters.

□ Error function

$$S_{MAP}(\mathbf{p}) = [\mathbf{H}^{exp} - \mathbf{H}^{est}(\mathbf{p})]^T \mathbf{W} [\mathbf{H}^{exp} - \mathbf{H}^{est}(\mathbf{p})] \\ + [\mathbf{p}_\mu - \mathbf{p}]^T \mathbf{V}^{-1} [\mathbf{p}_\mu - \mathbf{p}]$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM - SECOND STAGE

### □ Error function

$$S_{MAP}(\mathbf{p}) = [\mathbf{H}^{exp} - \mathbf{H}^{est}(\mathbf{p})]^T \mathbf{W} [\mathbf{H}^{exp} - \mathbf{H}^{est}(\mathbf{p})] \\ + [\mathbf{p}_\mu - \mathbf{p}]^T \mathbf{V}^{-1} [\mathbf{p}_\mu - \mathbf{p}]$$

### □ Iterative procedure

$$\Delta \mathbf{p}^k = [\mathbf{J}^{kT} \mathbf{W} \mathbf{J}^k + \mathbf{V}^{-1}]^{-1} \{ \mathbf{J}^k \mathbf{W} [\mathbf{H}^{exp}(\mathbf{p}) - \mathbf{H}^{est}(\mathbf{p}^k)] + \mathbf{V}^{-1} (\mathbf{p}_\mu - \mathbf{p}^k) \}$$

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM - SECOND STAGE

- In the present work the iterative procedure for the second stage was implemented in a convenient form for computational purposes, which avoids matrix inversions. For this we have employed the sequential estimation technique [Beck and Arnold, 1977, Orlande, 2002 , Beck, 2003].

# MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM



## □ INVERSE PROBLEM - SECOND STAGE - ALGORITHM

- Step 1: Initialize the iterative procedure by setting the iteration index  $k$  to 0 and  $\mathbf{p}^{(0)} = \mathbf{p}_\mu$ .
- Compute the estimate for the vector of unknown parameters sequentially, by using

$$\mathbf{A} = \mathbf{V}_n \mathbf{J}_{n+1}^T$$

$$\Delta = \mathbf{J}_{n+1} \mathbf{A} + \mathbf{W}_{n+1}^{-1}$$

$$\Gamma = \Delta^{-1} \mathbf{A}$$

$$E_{n+1} = H_{n+1}^{exp} - H_{n+1}^{est}(\mathbf{p}^k)$$

$$\mathbf{p}_{n+1}^{k+1} = \mathbf{p}_n^{k+1} + \Gamma [E_{n+1} - \mathbf{J}_{n+1}(\mathbf{p}_n^{k+1} - \mathbf{p}^k)]$$

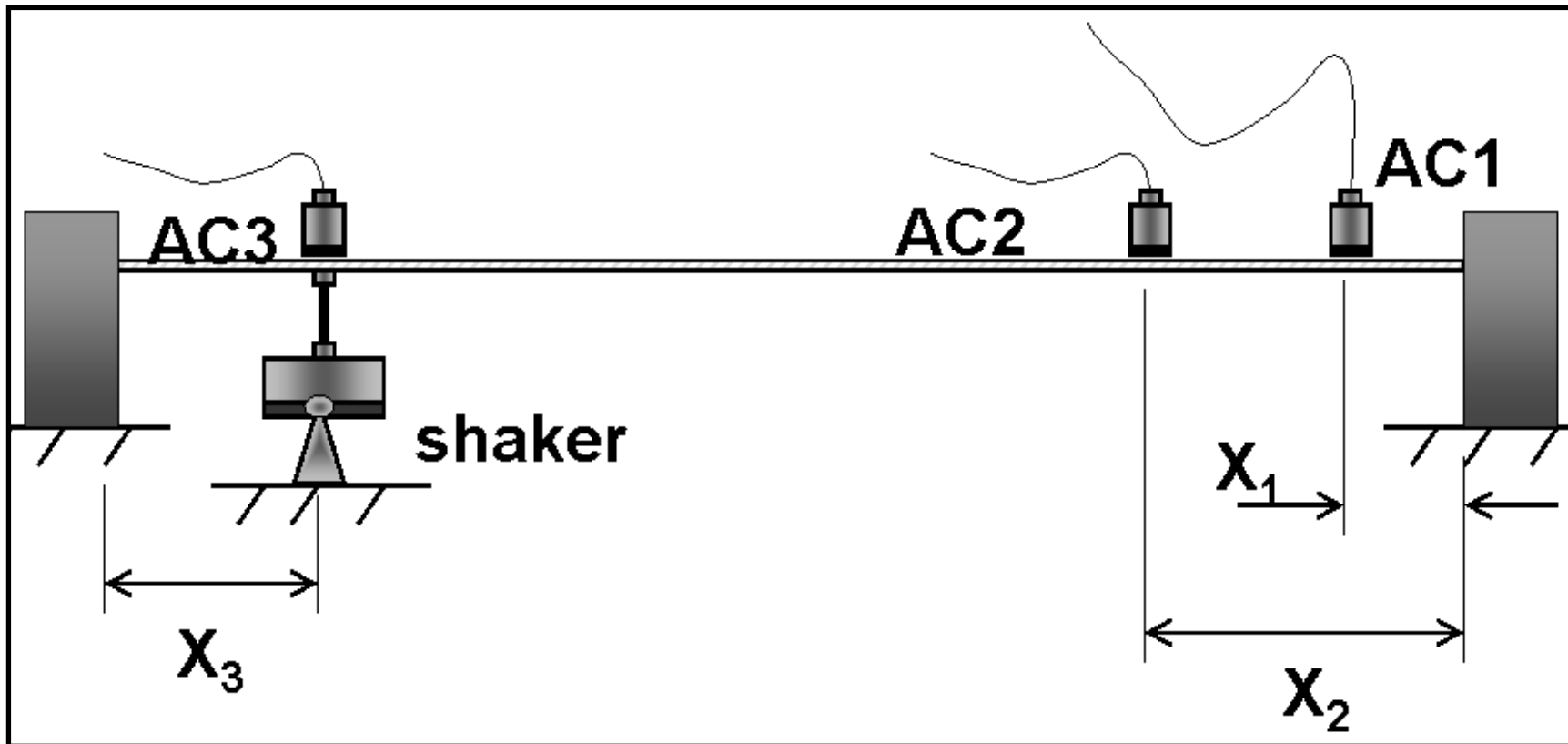
$$\mathbf{V}_{n+1} = \mathbf{V}_n - \Gamma \mathbf{J}_{n+1} \mathbf{V}_n$$

$$\mathbf{V}_0 = \mathbf{V}, \mathbf{p}_0^k = \mathbf{p}_\mu, \mathbf{W}_{n+1} = \sigma_{n+1}^{-2}$$

$$\mathbf{J}_n = \left. \frac{\partial H_n^{est}}{\partial \mathbf{p}} \right|_{\mathbf{p}=\mathbf{p}^k}$$

# RESULTS AND CONCLUSIONS

## □ EXPERIMENTAL SET-UP (Sketch)





# RESULTS AND CONCLUSIONS

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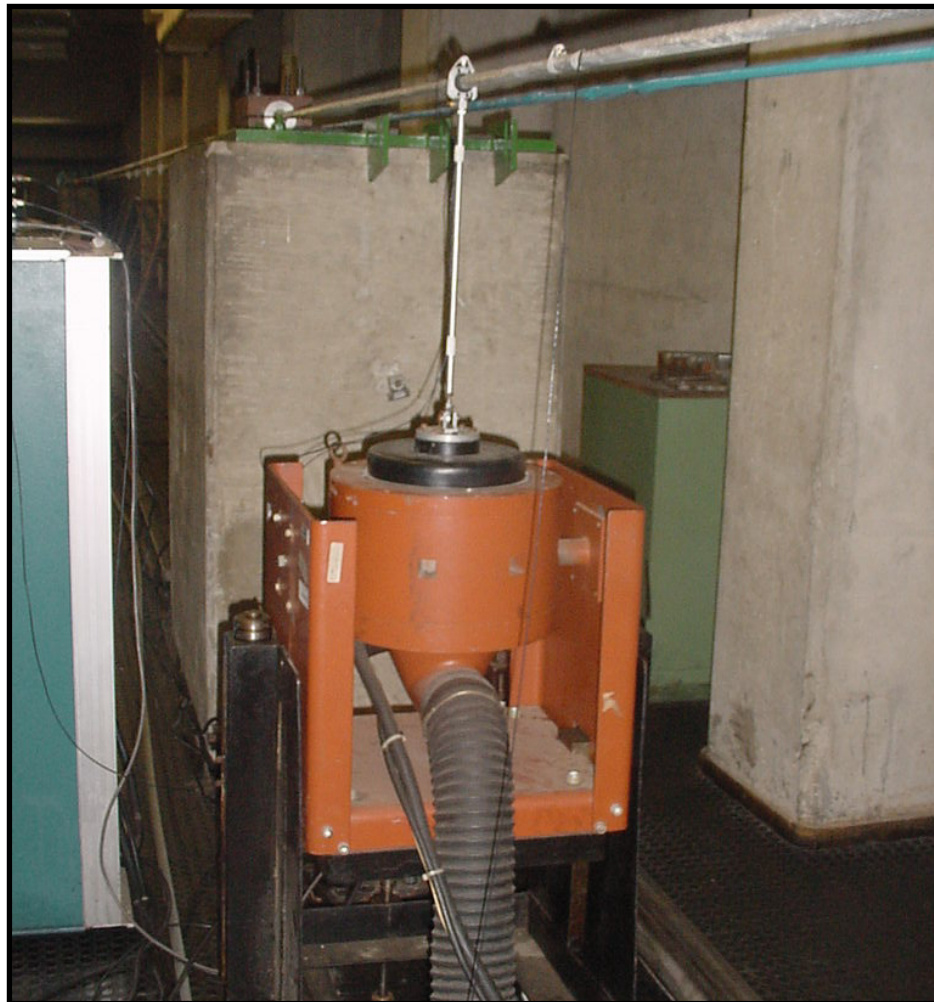
## □ EXPERIMENTAL SET-UP – CEPEL'S LABORATORY SPAN





# RESULTS AND CONCLUSIONS

## □ EXPERIMENTAL SET-UP – SHAKER



# RESULTS AND CONCLUSIONS

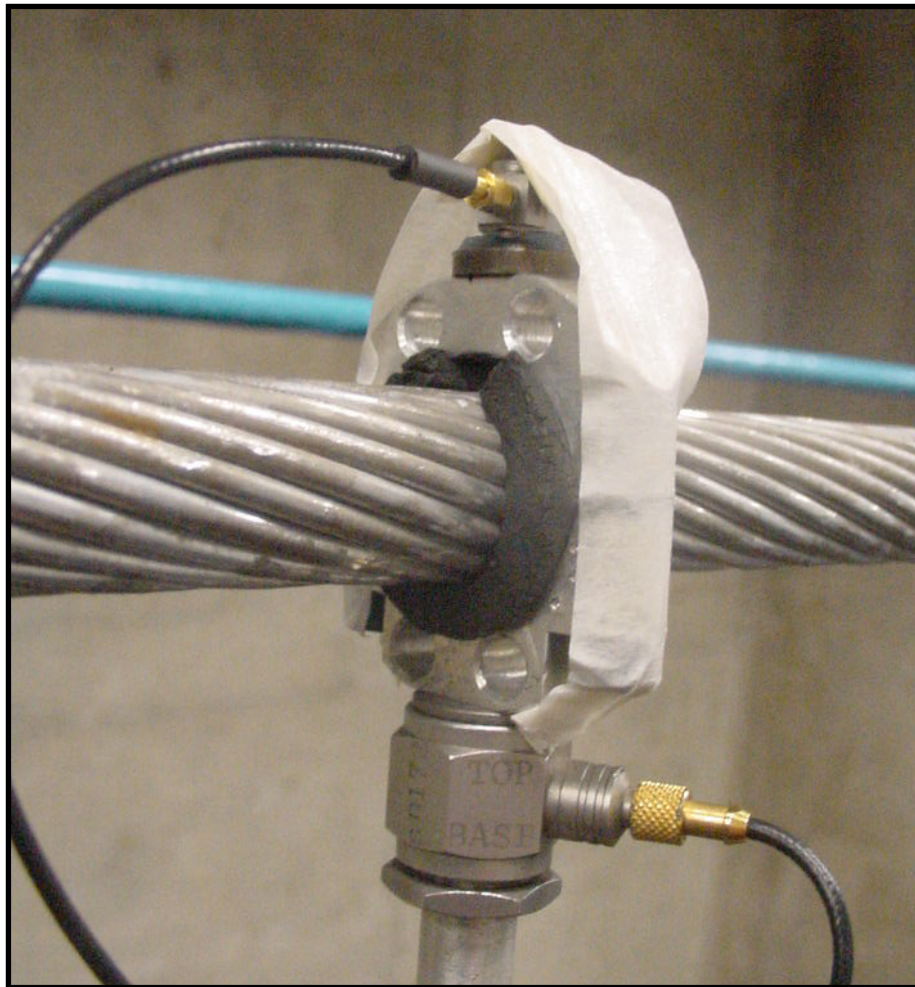
## □ EXPERIMENTAL SET-UP – LOAD CELL





# RESULTS AND CONCLUSIONS

## □ EXPERIMENTAL SET-UP – ACCELEROMETER AND LOAD CELL



# RESULTS AND CONCLUSIONS

## □ EXPERIMENTAL SET-UP

- Grosbeak ACSR conductor
- $\mu = 1.30271 \text{ kg/m}$
- $T = 21778.2 \text{ N}$  and  $T = 27468.0 \text{ N}$
- $L = 51.905 \text{ m}$
- $X_1 = 1.39 \text{ m}$
- $X_2 = 0.70 \text{ m}$
- $X_3 = 1.61 \text{ m}$

# RESULTS AND CONCLUSIONS



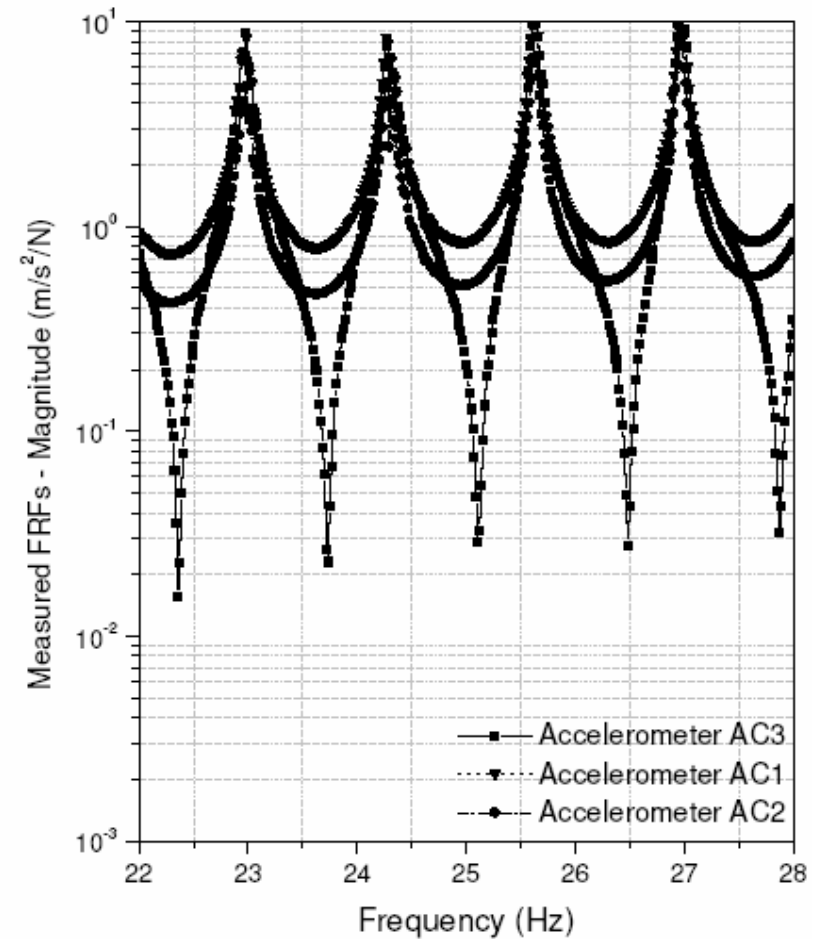
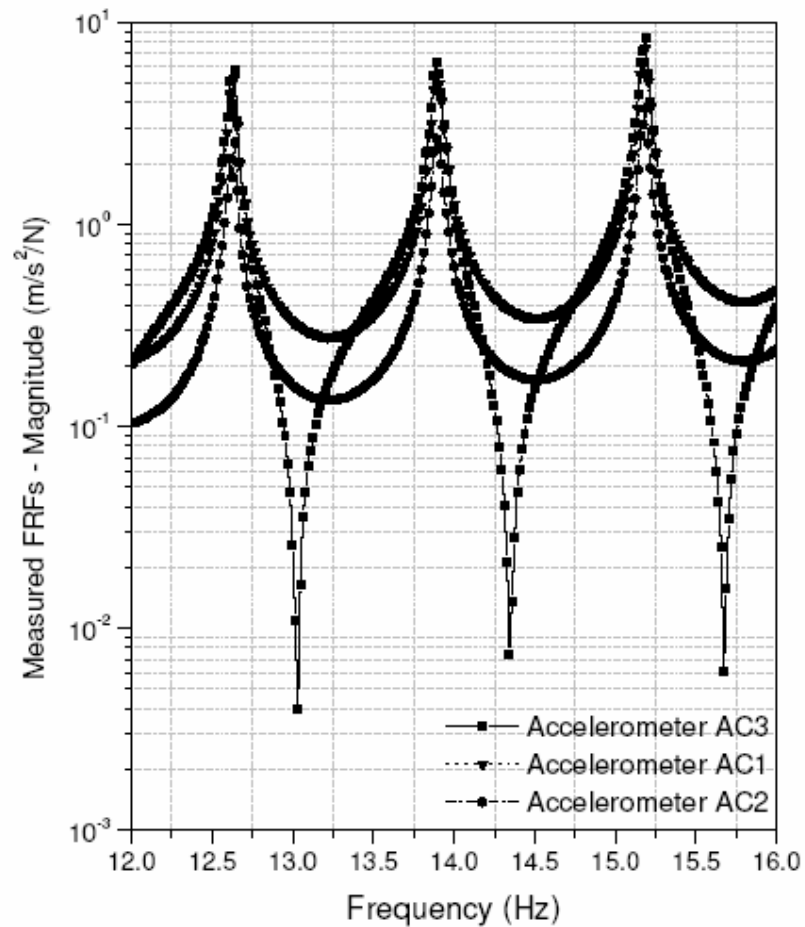
## □ EXPERIMENTAL SET-UP : FRFs

Band (Hz)	$n_d$ (Number of averages)	Accelerometers
[5, 17.5]	30	AC1, AC2 and AC3
[17.5, 30]	30	AC1, AC2 and AC3

- All the FRFs are measured with 801 equally spaced frequency points

# RESULTS AND CONCLUSIONS

## □ EXPERIMENTAL SET-UP : FRFs



# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C1

□  $T = 21778.2 \text{ N}$

□  $[5, 17.5] \text{ Hz}$

□ Kelvin-Voigt model

□ Unknown parameters  $\alpha$  and  $EI$  and for case C1 it is considered that  $\xi I = 10^{-4} \text{ N.m}^2\text{s}^{-1}$ .

□ Initial Guess for  $EI$  and  $\alpha$  ?

# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C1

- We do not have any information about a suitable initial guess for  $\alpha$ , therefore we have considered  $\alpha^{(0)} = 10^{-2} \text{N.s.m}^{-2}$ .
- Concerning the parameter EI, the reports by CIGRE provide the maximum and minimum values:  $EI^{\min} = 28 \text{ N.m}^2$  and  $EI^{\max} = 1027 \text{ N.m}^2$ .
- Parameterization  $EI = p_1 \times 10^3$  and  $\alpha = p_2$ .



# RESULTS AND CONCLUSIONS

## RESULTS: CASE C1

Estimated bending stiffness and aerodynamic damping coefficient (Case C1).

GITT					FEM				
$EI^{(0)}$	$EI$	$\alpha$	$\frac{\sigma EI}{\sigma}$	$\frac{\sigma \alpha}{\sigma}$	$EI^{(0)}$	$EI$	$\alpha$	$\frac{\sigma EI}{\sigma}$	$\frac{\sigma \alpha}{\sigma}$
28	748.9	0.355	2.8	0.012	28	742.2	0.3494	2.8	0.0114
527.5	748.9	0.355	2.8	0.012	527.5	741.9	0.3495	2.8	0.0114
1027	748.9	0.355	2.8	0.012	1027	741.9	0.3495	2.8	0.0114

## DEFINITION: Normalized norm

$$|\mathbf{q}^{est}|_N = \frac{(\mathbf{q}^{exp} - \mathbf{q}^{est})^H (\mathbf{q}^{exp} - \mathbf{q}^{est})}{(\mathbf{q}^{exp})^H \mathbf{q}^{exp}}$$

# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C1

- Definition of a normalized norm of the FRFs

$$|\mathbf{q}^{est}|_N = \frac{(\mathbf{q}^{exp} - \mathbf{q}^{est})^H (\mathbf{q}^{exp} - \mathbf{q}^{est})}{(\mathbf{q}^{exp})^H \mathbf{q}^{exp}}$$

- Norms for case C1

Band ( <i>Hz</i> )	<i>AC1</i>	<i>AC2</i>	<i>AC3</i>
[5,17.5]	0.6761	0.6665	0.6894

# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C2

- [17.5, 30] Hz
- Use the results provided by case C1 as *a priori* information.
- Unknown parameters  $\alpha$  and EI and  $\xi I$ .
- Parameterization:  $EI = p_1 \times 10^3$ ,  $\alpha = p_2$  and  $\xi I = p_3 \times 10^{-2}$
- The components of the covariance matrix  $\mathbf{V}$  and vector  $\mathbf{p}_\mu$  associated to EI and  $\alpha$  are obtained from case C1.

# RESULTS AND CONCLUSIONS



## □ RESULTS: CASE C2

- What about the components of the covariance matrix  $\mathbf{V}$  and vector  $\mathbf{p}_\mu$  for parameter  $\xi_I$  ?
- As we do not have any information concerning  $I$  we simply consider that its mean is a small number and its standard deviation is a large number; therefore we have chosen  $10^{-5}$  and  $10^{10}$ , respectively.

$$\mathbf{p}_\mu = \{0.7419, 0.3495, 10^{-5}\}^T$$

$$\mathbf{V} = \sigma^2 \times \begin{pmatrix} 7.91 \times 10^{-6} & -2.78 \times 10^{-7} & 0 \\ -2.78 \times 10^{-7} & 1.20 \times 10^{-4} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix}$$

# RESULTS AND CONCLUSIONS

## RESULTS: CASE C2

FEM					
$EI$	$\alpha$	$\xi I$	$\frac{\sigma_{EI}}{\sigma}$	$\frac{\sigma_{\alpha}}{\sigma}$	$\frac{\sigma_{\xi I}}{\sigma}$
678.9	0.393	0.029	0.347	0.008	0.0037

GITT					
$EI$	$\alpha$	$\xi I$	$\frac{\sigma_{EI}}{\sigma}$	$\frac{\sigma_{\alpha}}{\sigma}$	$\frac{\sigma_{\xi I}}{\sigma}$
682.4	0.393	0.026	0.336	0.013	0.0048

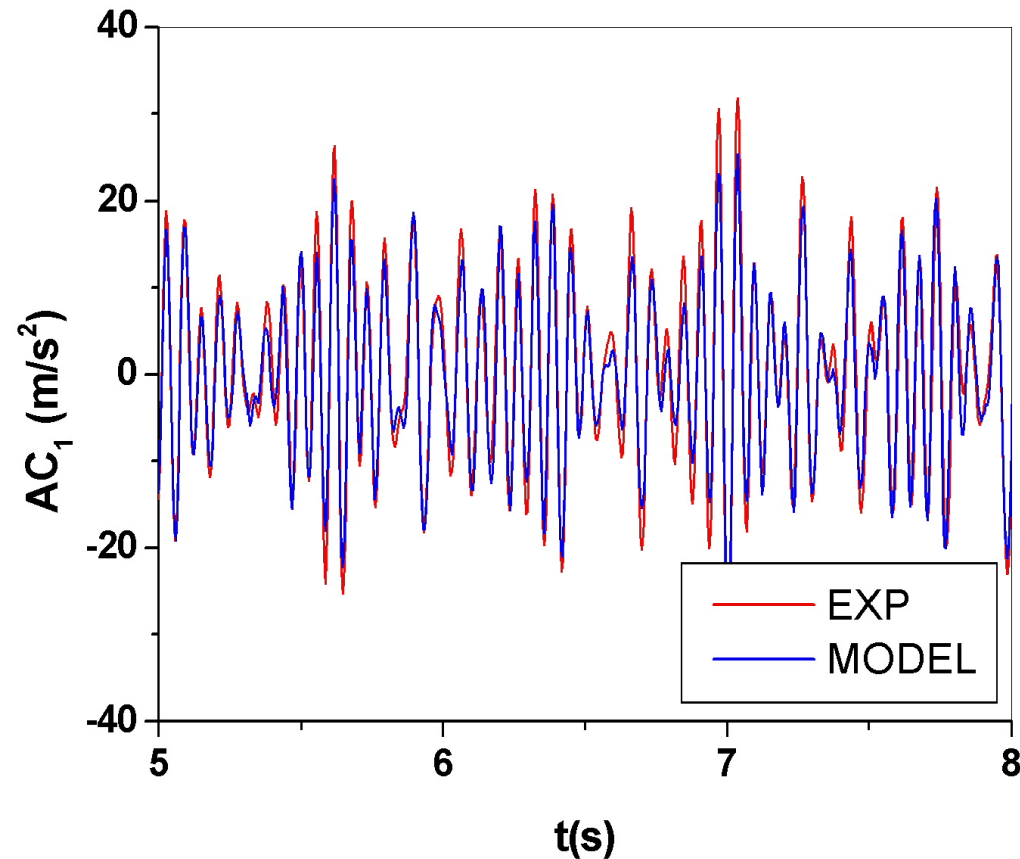
## Norms for case C2

Band (Hz)	AC1	AC2	AC3
[5,17.5]	0.8201	0.8084	0.8320
[17.5,30]	0.4663	0.4409	0.5032

# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C2

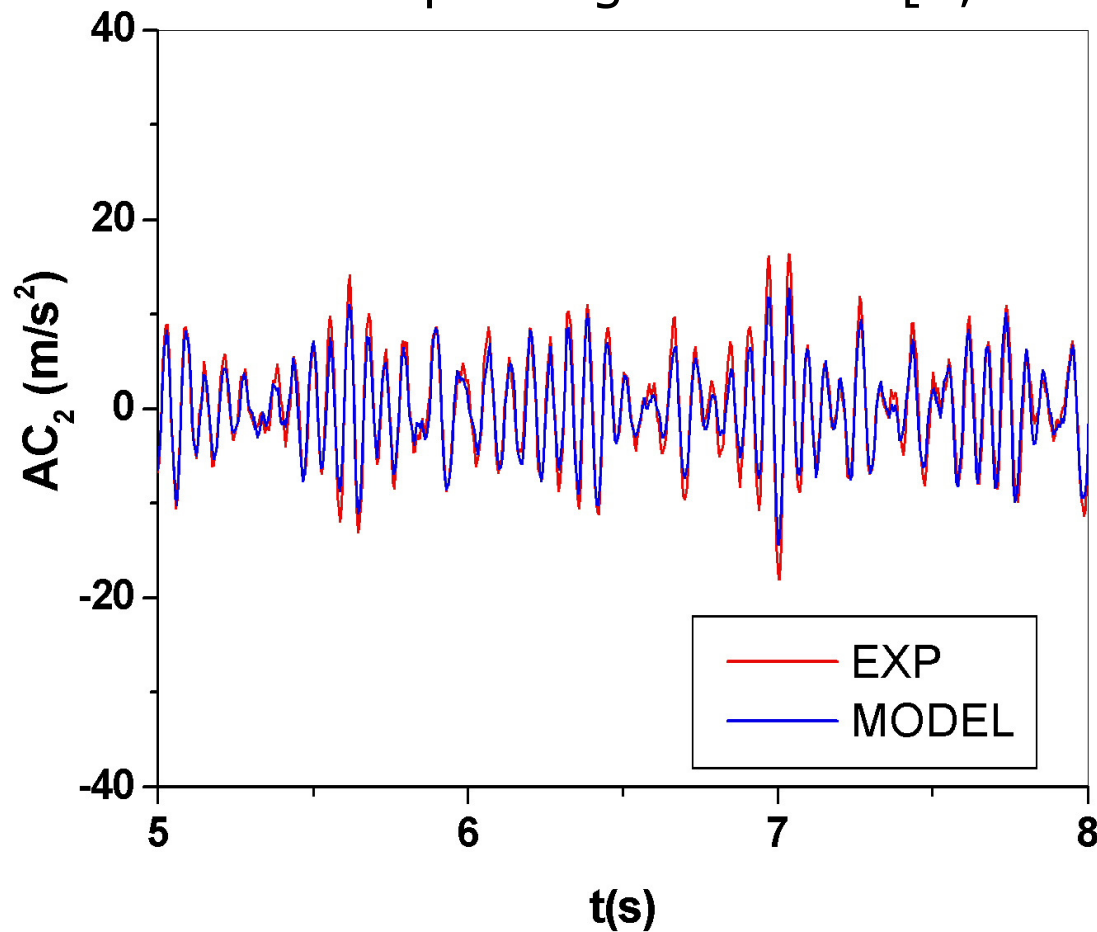
- Validation in time domain with na excitation equal to a white noise encompassing the band [5, 17.5] Hz



# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C2

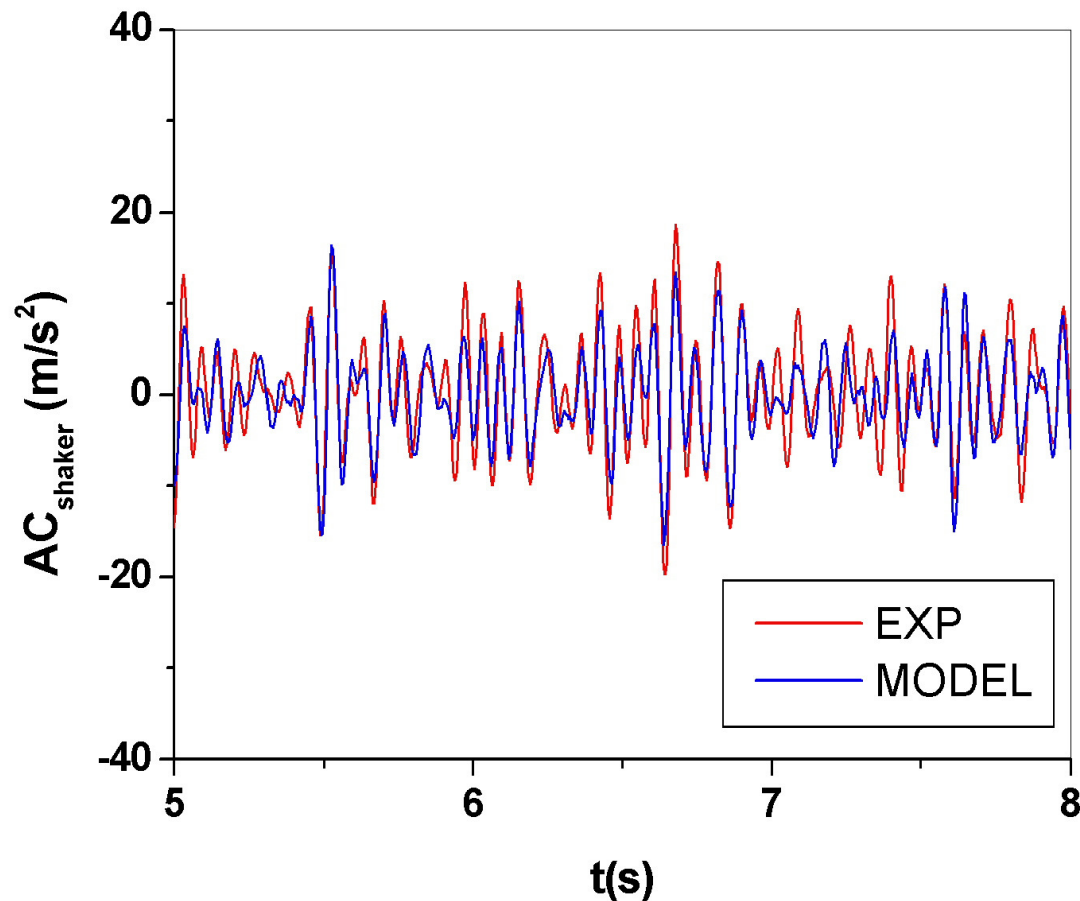
- Validation in time domain with na excitation equal to a white noise encompassing the band [5, 17.5] Hz



# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C2

- Validation in time domain with na excitation equal to a white noise encompassing the band [5, 17.5] Hz





# RESULTS AND CONCLUSIONS



## □ RESULTS: CASE C3

- [17.5, 30] Hz
- Use the results provided by case C1 as *a priori* information.
- Hysteretic damping model. Unknown parameters  $\alpha$  and EI and  $\eta I$ .
- Parameterization:  $EI = p_1 \times 10^3$ ,  $\alpha = p_2$  and  $\eta I = p_3$
- The components of the covariance matrix  $\mathbf{V}$  and vector  $\mathbf{p}_\mu$  associated to EI and  $\alpha$  are obtained from case C1.

# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C3

- What about the components of the covariance matrix  $\mathbf{V}$  and vector  $\mathbf{p}_\mu$  for parameter  $\eta I$  ?
- As we do not have any information concerning  $I$  we simply consider that its mean is a small number and its standard deviation is a large number; therefore we have chosen  $10^{-5}$  and  $10^{10}$ , respectively.

$$\mathbf{p}_\mu = \{0.7419, 0.3495, 10^{-5}\}^T$$

$$\mathbf{V} = \sigma^2 \times \begin{pmatrix} 7.91 \times 10^{-6} & -2.78 \times 10^{-7} & 0 \\ -2.78 \times 10^{-7} & 1.20 \times 10^{-4} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix}$$

# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C3

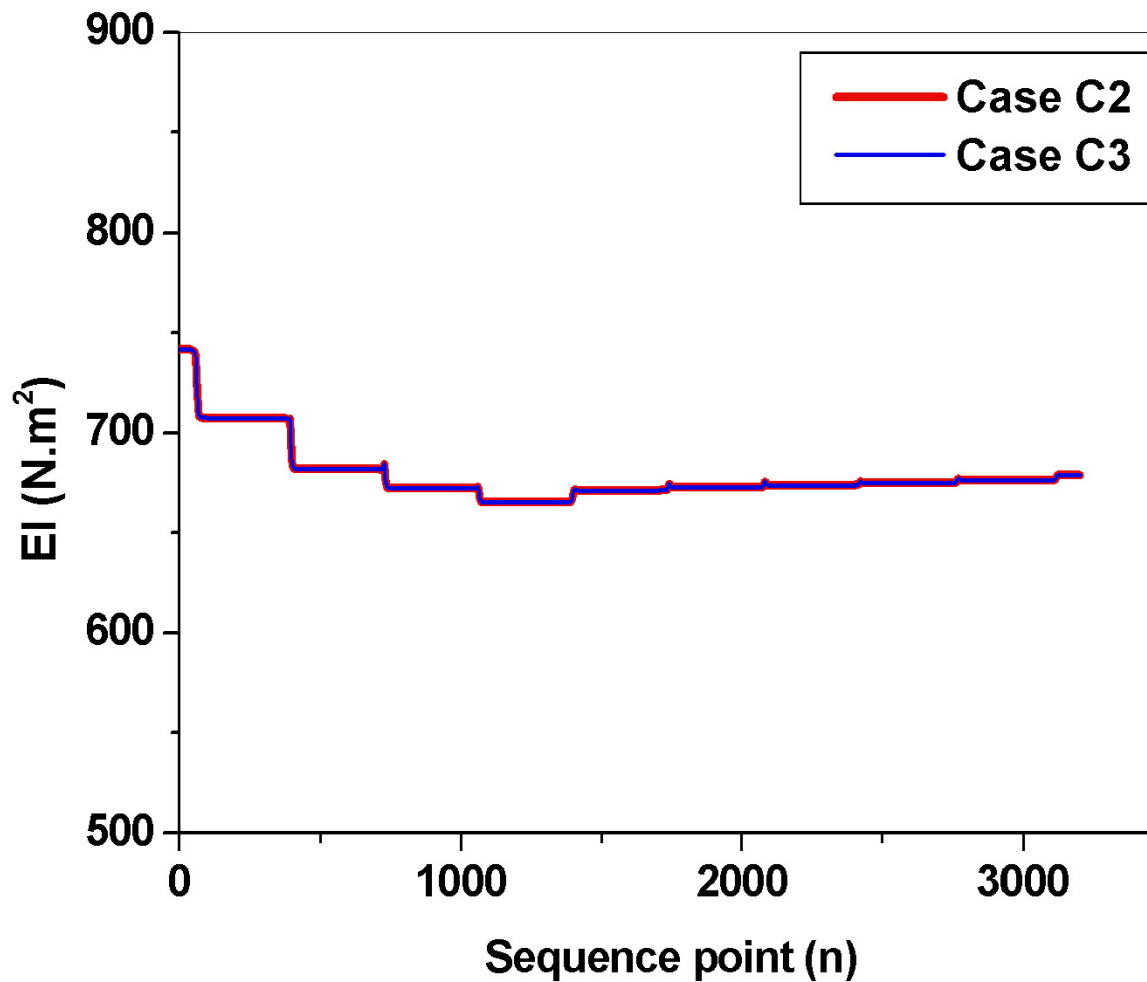
FEM					
$EI$	$\alpha$	$\bar{\eta} I$	$\frac{\sigma_{EI}}{\sigma}$	$\frac{\sigma_{\alpha}}{\sigma}$	$\frac{\sigma_{\eta I}}{\sigma}$
678.9	0.3771	6.0481	0.3504	0.009	0.679

## □ Norms for case C3

Band ( $Hz$ )	$AC1$	$AC2$	$AC3$
[5,17.5]	0.8220	0.8101	0.8339
[17.5,30]	0.4659	0.4407	0.5026

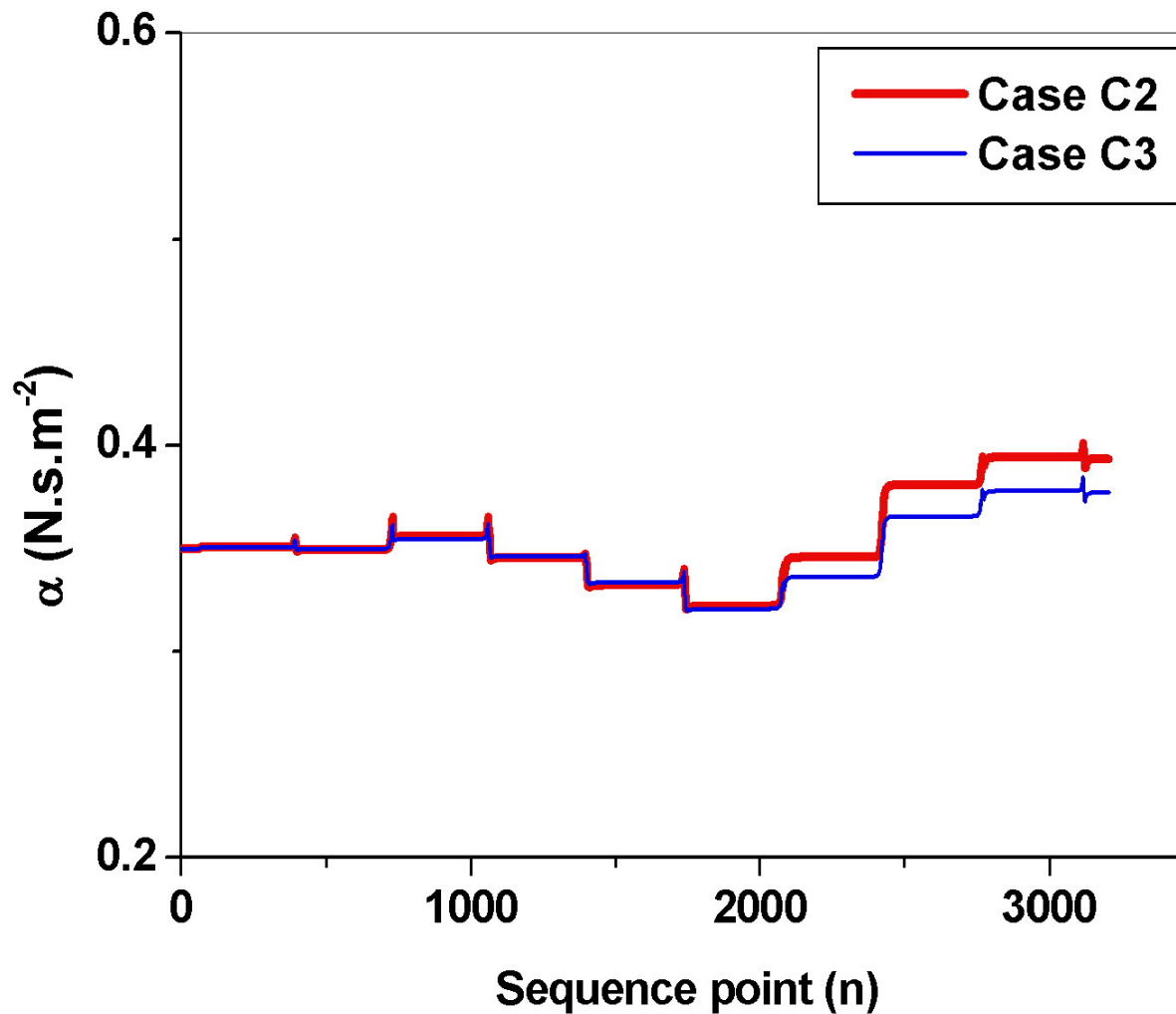
# RESULTS AND CONCLUSIONS

- Sequential evolution of the estimated parameters for cases C2 e C3



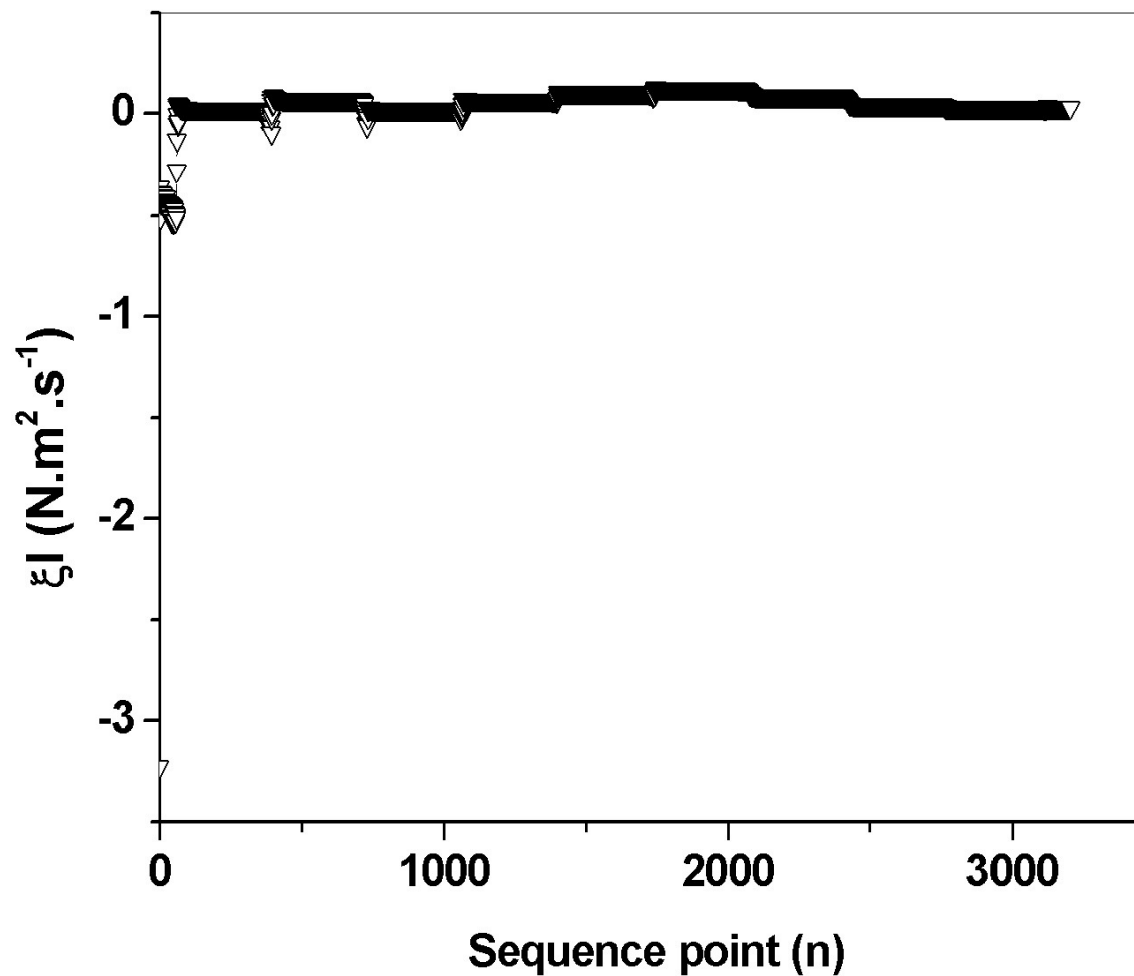
# RESULTS AND CONCLUSIONS

- Sequential evolution of the estimated parameters for cases C2 e C3



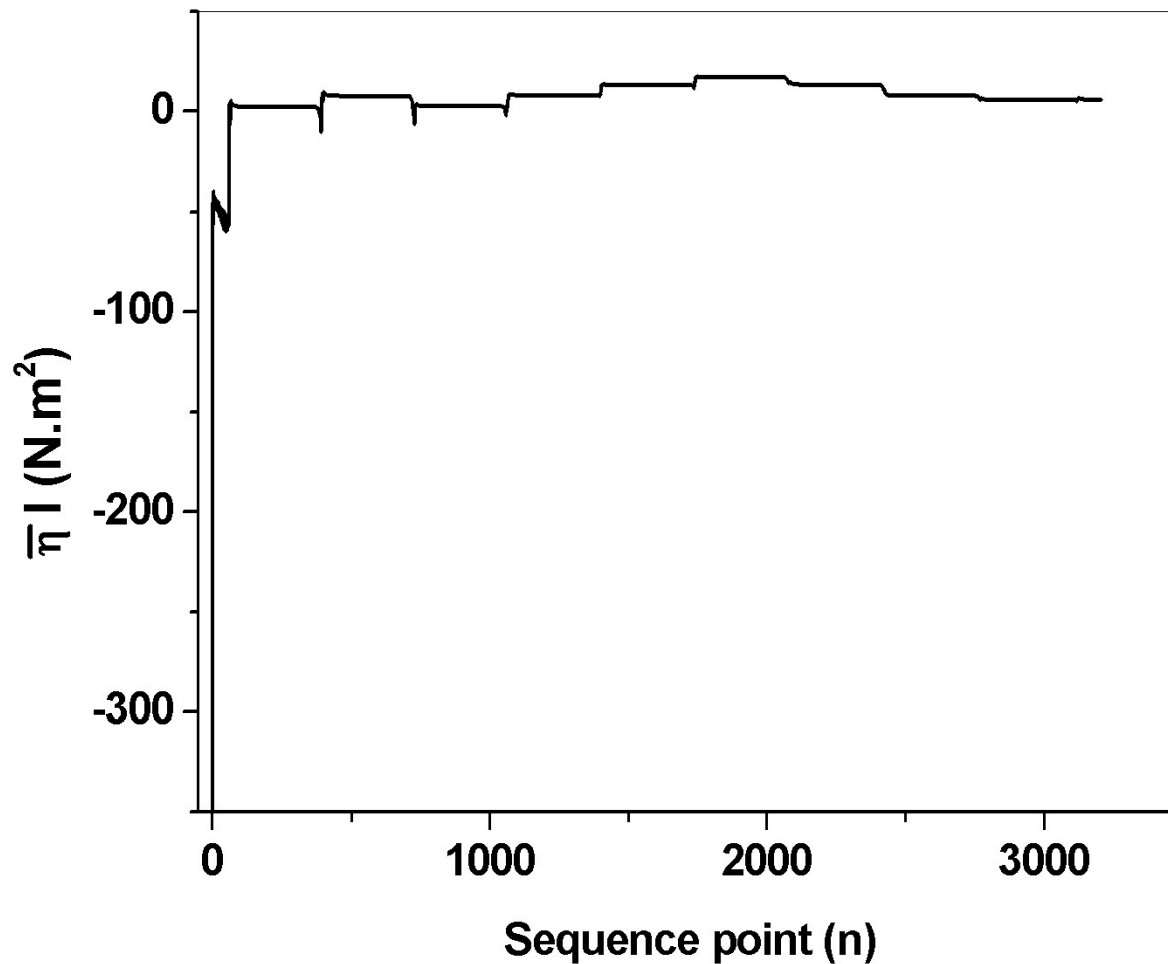
# RESULTS AND CONCLUSIONS

- Sequential evolution of the estimated parameters for cases C2 e C3



# RESULTS AND CONCLUSIONS

- Sequential evolution of the estimated parameters for cases C2 e C3



# RESULTS AND CONCLUSIONS

## □ RESULTS: CASE C4

- Exactly equal to case C1 e C2 in sequence but with axial tension equal to  $T = 27468.0 \text{ N}$
- $EI = 615.8 \text{ N.m}^2$
- $\alpha = 0.3674 \text{ N.s.m}^{-2}$
- $\xi I = 2.8186 \times 10^{-2} \text{ N.m}^2.\text{s}^{-1}$



# FINAL REMARKS



- ❑ TLC's were modelled as homogeneous beams with viscous and structural damping
  
- ❑ Based on experimental data it was chosen a linear model to represent the system.
  
- ❑ Two classical damping models have been used
  
- ❑ Their bending stiffness and damping parameters were estimated based on inverse analysis
  
- ❑ Direct problem associated to estimation process was solved by two approaches: FEM and GITT
  
- ❑ Inverse problem was solved through Levenberg-Marquardt iterative procedure and the sequential estimation technique.

# FINAL REMARKS (cont.)



## □ MAIN CONCLUSIONS

- Objective function much more sensitive to  $\alpha$  than to  $EI$  in the frequency range (0,20 Hz)
- Estimates for  $\alpha$  were in good agreement with its true value
- Estimated parameters practically unaffected by noise level
- Uncertainty in span length affects much more  $EI$
- Uncertainty in mechanical load largely affects both parameters

## □ MAIN CONTRIBUTIONS

- The estimation of bending stiffness and damping parameter of TLC
- Use of GITT approach to solve direct problem associated to estimation process
- Numerical analysis of the effects of model uncertainties on estimated parameters what, to authors belief, are not considered previously in the literature for this specific problem

# FUTURE WORKS



- Estimate these parameters for different span lengths and different axial tractions
- Investigate a suitable mechanical model for the StockBridge damper
- Analyze the coupled system TLC and damper based on the estimated models and evaluate it based on experimental data.