Sequential Estimation of Bending Stiffness and Damping Parameters of Transmission Line Conductors

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Identificación de Propiedades de Materiales por Métodos Inversos – Mar del Plata
OUTLINE

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  - Motivation
  - Objective
- BRIEF LITERATURE REVIEW
- MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM
  - Direct Problem
  - Inverse Problem
- RESULTS AND CONCLUSIONS
- FINAL REMARKS
- FUTURE WORKS
GENERAL OVERVIEW

- Wind-induced vibrations on transmission line conductors (TLC) caused by vortex-shedding (Rawlins, 1979; Hagedorn, 1982; Meynen et al., 2005)

- Main features: frequency range 5 Hz – 60 Hz and amplitude range $0.01D – 1D$ ($D$ denotes the conductor diameter 15 mm – 30 mm)

- Well-known galloping vibrations of very low frequencies (below 1 Hz), caused by aerodynamic instabilities, are not addressed here

- TLC are composed of wires helically wrapped around a central core
GENERAL OVERVIEW (cont.)

**Aluminum Conductor Steel Reinforced (ACSR Conductors)**
Layers of aluminum wires helically wrapped around a central steel core

photograph and sketch of a typical TLC
GENERAL OVERVIEW (cont.)

- TLC are subjected to very high tensile loads (20 kN – 40 kN) and clamped at the suspension towers

- Frequency spectrum almost continuous; two natural frequencies separated by approximately 0.1 Hz – 0.3 Hz

- Vortex-shedding frequency almost always close to one of the natural frequencies of TLC

- Field and laboratory measurements indicate that TLC have low internal damping, mainly in the frequency range 0 – 30 Hz

- Main damping mechanisms are: (i) interstrand friction among the wires (structural damping); (ii) aerodynamic damping and (iii) material damping
GENERAL OVERVIEW (cont.)

- Wind-induced vibrations on TLC occur for wind speeds in the range 1 m/s to 10 m/s.

- Reynolds number lies in the sub-critical range \((10^3 \text{ to } 10^4)\).

- Vortex-shedding across stationary bluff bodies in this Reynolds range has a well defined frequency, expressed in terms of a nondimensional parameter called Strouhal number \(St\).

- For smooth and circular cylinders \(St = 0.2\).

- For TLC, field measurements indicate that \(0.185 < St < 0.22\) (Kraus and Hagedorn, 1991; Rao, 1995).
GENERAL OVERVIEW (cont.)

- Vortex-shedding across a stationary cylinder is not yet completely understood [Williamson and Govardhan (2004)]
- Concerning wind-induced vibrations on TLC, other complicating factors come into picture
  - (i) the dynamic interaction between wind flow and TLC vibrations;
  - (ii) the turbulent nature of wind flow;
  - (iii) TLC structural vibrations due to lack of information regarding the bending stiffness and damping parameters of TLC

MOTIVATION

- Wind-induced vibrations are a critical problem for safety and reliability of transmission lines
- Bending strains and stresses caused by such vibrations may cause fatigue damages of conductor wires
MOTIVATION (cont.)

- Fatigue damages may lead to complete rupture of the conductor and, consequently, to the interruption on the supply of electric energy

- Therefore, the understanding of wind-induced vibrations on TLC is a relevant issue

- Accurate predictions of such vibrations depend, of course, on the knowledge of stiffness and damping properties of TLC

OBJECTIVE

- Estimate the bending stiffness and damping parameters of a typical TLC based on inverse analysis
ALL IN ALL, WHAT IS THE SYSTEM UNDER ANALYSIS?
WHAT IS THE OBJECTIVE OF THIS PROJECT?

To reduce the vibration levels of TLC.

WHAT ARE THE TARGETS?

Target 1: Determine the bending stiffness and the damping parameters of transmission line cables.

Target 2: Determine a suitable mathematical model for the StockBridge Damper.
WHAT ARE THE TARGETS? (Cont.)

- Target 3: Analyze the fluid Structure Interaction.

- Target 4: Determine the optimum number of StockBridge Dampers as well as their optimal positions to reduce the amplitude of vibrations of TLC when the system is excited by wind.
WHAT IS THIS PRESENTATION ABOUT?

- Target 1: Estimation of the bending stiffness and damping parameters of transmission line cables

WHAT ARE THE MAIN ISSUES CONCERNING THIS TARGET?

- Which model should we use?
- Which damping model should we use?
- How do we estimate the model parameters?
The majority of theoretical models proposed to predict wind-induced vibrations idealizes TLC structure as a continuous (Claren and Diana, 1969; Dhotarad et al., 1978; Hagedorn et al., 1987; Diana et al., 2000; Vecchiarelli et al., 2000; Barbieri et al., 2004; Meynen et al., 2005).

The simplest models idealize TLC as homogeneous taut strings without bending stiffness; more complex ones idealize TLC as homogeneous elastic beams with structural damping being represented as of hysteretic kind.

Authors rarely report the values adopted for the bending stiffness and damping parameters; there is a current lack in the literature about mechanical properties of typical TLC.

Most of data available refers to the power dissipated by TLC during standard self-damping tests on a laboratory span. Discrepancy among measurements performed by different authors may reach 100%!

Authors rarely compare their theoretical predictions against experimental measurements (Claren and Diana, 1969; Diana et al., 2000; Barbieri et al., 2004).
Few works have attempted to account for the helicoidal structure of TLC and all damping mechanisms, most authors adopts a constant value for the bending stiffness.

Recommendation is to choose such the bending stiffness as a constant value between the minimum and maximum values [CIGRÉ (1989)]
- minimum value (\(EI_{\text{min}}\)) is obtained by considering TLC as a bundle of individual wires free to move relative to each other
- maximum value (\(EI_{\text{max}}\)) is obtained by considering TLC a bundle of individual wires unable to move relative to each other due to contact pressure

Nevertheless, Papailiou (1997) presented a more sophisticated model which accounts for helicoidal geometry of the wires, interlayer friction and slipping during bending (non-linear model)

Such a model leads to a bending stiffness which changes with amplitude and mechanical load applied to TLC

Papailiou (1997) compared his theoretical predictions against experimental measurements performed on laboratory and the agreement was satisfactory
MINIMUM AND MAXIMUM VALUES OF THE BENDING STIFFNESS [CIGRÉ (1989)]

\[ EI_{\text{min}} = \frac{\pi}{64} \left( E_s d_s^4 N_s + E_a d_a^4 N_a \right) \]
\[ EI_{\text{max}} = E_s \sum_{i=1}^{n_s} I_{\text{max},i} + E_a \sum_{j=1}^{n_a} I_{\text{max},j} \]
\[ I_{\text{max},i} = \frac{N_i \pi d_i^2}{8} \left( \frac{d_i^2}{8} + R_i^2 \right) \]

- \( E_s \) Young’s modulus of elasticity for steel
- \( E_a \) Young’s modulus of elasticity for steel
- \( d_s \) Steel wire diameter
- \( d_a \) Aluminum wire diameter
- \( N_s \) Number of steel wires
- \( N_a \) Number of aluminum wires
- \( n_s \) Number of layers of steel wires
- \( n_a \) Number of layers of aluminum wires
- \( N_i \) Number of wires in the \( i^{th} \) layer
- \( R_i \) Radius of \( i^{th} \) layer

ACSR conductor Grosbeak

26 aluminum wires \( d_a = 3.973 \) mm
7 steel wires \( d_s = 3.089 \) mm
\( EI_{\text{min}} = 28.4 \) N.m²
\( EI_{\text{max}} = 1027 \) N.m²

What value of \( EI \) should we use?
HYPOTHESIS

- TLC modelled as an Euler-Bernoulli beam with constant bending stiffness and subjected to a constant axial load
- Small displacements
- Aerodynamic damping of viscous type
- Material damping modelled from constitutive equation

Sketch of a differential element of the TLC
MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

- EQUILIBRIUM EQUATIONS

\[ H(x,t) + \frac{\partial}{\partial x} \left( T \frac{\partial w}{\partial x} \right) - \frac{\partial^2 M}{\partial x^2} - \alpha \frac{\partial w}{\partial t} = \mu \frac{\partial^2 w}{\partial t^2} \]

\[ M(x,t) = \int_{A} -y \sigma(x,t) \, dA \]

- CONSTITUTIVE EQUATION?

  - Linear or nonlinear?

  - Candidate models?
CONSTITUTIVE EQUATION – LINEAR?

STICK-SLIP BETWEEN THE STRANDS (PAPAILIOU, 1997)

FOR TLC THE AXIAL TENSION IS EXTREMELY HIGH

EXPERIMENTS AIMED AT ASSESSING WHETHER TLC POSSES LINEAR BEHAVIOUR WERE PERFORMED.

BASED ON EXPERIMENTAL DATA WE DECIDED TO ADOPT A LINEAR MODEL FOR THE TLC USED (HORIZONTAL, HIGH AXIAL TENSION AND LOW SAG)
CONSTITUTIVE EQUATION

- Generalized damping model (time domain)
  \[ \sigma(x,t) = E \varepsilon(x,t) + \int_0^t K(x,\tau-t) \frac{\partial \varepsilon}{\partial \tau} \, d\tau \]

- Kelvin-Voigt model (time domain)
  \[ \sigma = E \varepsilon + \xi \frac{\partial \varepsilon}{\partial t} \]

- Hysteretic damping model (frequency domain)
  \[ \hat{\sigma}(x, j\omega) = (EI + j \eta l) \hat{\varepsilon}(x, j\omega) \]
KELVIN-VOIGT MODEL

- Constitutive equation defined in time domain what enables one to obtain the governing partial differential equations also in time domain.

Governing equation

\[ EI \frac{\partial^4 w}{\partial x^4} - T \frac{\partial^2 w}{\partial x^2} + \xi I \frac{\partial}{\partial t} \left( \frac{\partial^4 w}{\partial x^4} \right) + \alpha \frac{\partial w}{\partial t} + \mu \frac{\partial^2 w}{\partial t^2} = H(x,t) \]

- Boundary conditions
HYSTERETIC DAMPING MODEL

- Constitutive equation defined in frequency domain. Corresponds to the dissipation model mostly adopted for the analysis of TLC.

- Characterized by a constant loss factor.

- Non-causal. Therefore, we cannot obtain the governing equations in time domain.
DIRECT PROBLEM

- Comprises the solution of the above equations subjected to appropriate boundary and initial conditions, assuming that parameters $EI$, $\xi$, $\eta$, and $a$, and the excitation $H(x,t)$ are known.

- Two techniques were used to solve the direct problem: (i) the finite-element method FEM (Hughes, 2000; Reddy, 1993) and (ii) the generalized integral transform GITT (Cotta, 1993; Özişik, 1993)
MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

- DIRECT PROBLEM BY FEM: KELVIN-VOIGT DAMPING MODEL

  - Weak Form

  $$\int_{x_e}^{x_{e+1}} \left( -T \frac{\partial w}{\partial x} \frac{\partial u}{\partial x} - EI \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 u}{\partial x^2} - \xi I \frac{\partial^3 w}{\partial x^2 \partial t} \frac{\partial^2 u}{\partial x^2} - \alpha \frac{\partial w}{\partial t} u - \mu \frac{\partial^2 w}{\partial t^2} u + Hu \right) dx = 0$$

  - Galerkin Discretization

  $$w(x, t) = N(x)\hat{w}_h(t)$$

  - System of Ordinary Differential Equations

  $$M^e\ddot{\hat{w}}_h + D^e\dot{\hat{w}}_h + K^e\hat{w}_h = f^e$$
MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

DIRECT PROBLEM BY FEM: KELVIN-VOIGT DAMPING MODEL

- Mass, Stiffness and Damping Matrices

\[
M^e = \int_{x_e}^{x_{e+1}} \mu N^T N \, dx
\]

\[
K^e = \int_{x_e}^{x_{e+1}} \left( EI \frac{\partial^2 N^T}{\partial x^2} \frac{\partial^2 N}{\partial x^2} + T \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial x} \right) \, dx
\]

\[
D^e = \frac{\alpha}{\mu} M^e + \int_{x_e}^{x_{e+1}} \xi I \frac{\partial^2 N^T}{\partial x^2} \frac{\partial^2 N}{\partial x^2} \, dx = \frac{\alpha}{\mu} M^e + \frac{\xi}{E} K^e - \frac{\xi T}{E} \int_{x_e}^{x_{e+1}} \frac{\partial N^T}{\partial x} \frac{\partial N}{\partial x} \, dx
\]
DIRECT PROBLEM BY FEM: HYSTERETIC DAMPING MODEL

The relationship between the excitation and the response of the system can be written only in the frequency domain

\[
\left[ (j\omega)^2 M + (j\omega)D_\alpha + \left\{ K + jD_\eta \right\} \right] \mathbf{w}(j\omega) = \mathbf{F}(j\omega)
\]

Elemental matrices

\[
D^e_\alpha = \frac{\alpha}{\mu} M^e
\]

\[
D^e_\eta = \int_{x_e}^{x_{e+1}} \eta I \left( \frac{\partial^2 N^T}{\partial x^2} \right) \left( \frac{\partial^2 N}{\partial x^2} \right) dx
\]
MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

- DIRECT PROBLEM: HYBRID SOLUTION BY GITT – KELVIN-VOIGT
  - Auxiliary Eigenvalue Problem
    + homogeneous B.C.’s

  - Inverse-Transform Pair

  \[
  \bar{w}_m(t) = \int_0^L \Psi_m(\lambda_m, x) w(x, t) \, dx \quad \text{(transform)}
  \]

  \[
  w(x, t) = \sum_{m=1}^{\infty} \frac{\Psi_m(\lambda_m, x)}{N(\lambda_m)} \bar{w}_m(t) \quad \text{(inverse)}
  \]

  - Transformation of Original Problem - System of Coupled Ordinary Differential Equations

  \[
  \xi_m = \frac{\alpha}{2\mu \lambda_m}, \quad \xi_m^* = \xi_m + \frac{1}{2} \frac{\xi I}{EI} \lambda_m
  \]

  \[
  \frac{d^2 \bar{w}_m}{dt^2} + 2 \xi_m \lambda_m \frac{d\bar{w}_m}{dt} + \xi I \frac{T}{EI} \sum_{n=1}^{\infty} \Lambda_{mn} \frac{d\bar{w}_n}{dt} + \lambda_m^2 \bar{w}_m = \frac{H_0}{\mu} \Psi_m(\lambda_m, x_s) e^{i\Omega t}
  \]

  \[
  \Lambda_{mn} = \frac{1}{N(\lambda_n)} \int_0^L \Psi_n(\lambda_n, x) \frac{d^2 \Psi_m}{dx^2} \, dx = -\frac{1}{N(\lambda_n)} \int_0^L \frac{d\Psi_n}{dx} \frac{d\Psi_m}{dx} \, dx.
  \]
MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

INVERSE PROBLEM

- The unknown parameters are the bending stiffness $EI$, the viscous damping coefficient $\alpha$, and the constitutive damping parameters $\eta I$ and $\xi I$.

- Additional information used to estimate these two parameters are the complex frequency response functions ‘measured’ at prescribed locations $x_p$, $p = 1, 2, \ldots, N_s$, and circular frequencies $W_q$, $q = 1, 2, \ldots, N_f$ ($N_s$ and $N_f$ denote, respectively, the number of sensors and frequency data).
INVERSE PROBLEM

- Solution of inverse problem comprises the minimization of a suitable error function \( S(p) \), viz.

\[
S(p) = f(p, H^{Exp}, H^{Est}(p))
\]

- What is a suitable choice for \( S(p) \) ?

- What are the characteristics of our experimental data and what is our interest?
INVERSE PROBLEM - FIRST STAGE

- Use the experimental data concerning the lowest frequency bands

- Estimate EI and one of the damping parameters by the classical Levenberg-Marquardt iterative procedure [Beck and Arnold, 1977; Özişik and Orlande (2000)].
INVERSE PROBLEM - FIRST STAGE

- Error function

\[
S(p) = \left[ H^{exp}(p) - H^{est}(p) \right]^T \left[ H^{exp}(p) - H^{est}(p) \right]
\]

- Iterative procedure

\[
\Delta p^k = J^k [J^k]^T + \lambda^k \Lambda^k \right]^{-1} J^k [H^{exp} - H^{est}(p^k)]
\]

\[
\Delta p^k = p^{k+1} - p^k
\]
INVERSE PROBLEM - SECOND STAGE

Use the experimental data containing information of the higher frequency bands

Use the estimated parameters obtained in the first stage as *a priori* information for the estimation of the unknown parameters.

Error function

\[
S_{MAP}(p) = \left( H^{exp} - H^{est}(p) \right)^T W \left( H^{exp} - H^{est}(p) \right) + \left( p_{\mu} - p \right)^T V^{-1} \left( p_{\mu} - p \right)
\]
INVERSE PROBLEM - SECOND STAGE

- Error function

\[ S_{MAP}(p) = \left[ H^{exp} - H^{est}(p) \right]^T W \left[ H^{exp} - H^{est}(p) \right] \\
+ [p_\mu - p]^T V^{-1} [p_\mu - p] \]

- Iterative procedure

\[ \Delta p^k = \left( J^k W J^k + V^{-1} \right)^{-1} \left\{ J^k W [H^{exp}(p) - H^{est}(p^k)] + V^{-1} (p_\mu - p^k) \right\} \]
In the present work the iterative procedure for the second stage was implemented in a convenient form for computational purposes, which avoids matrix inversions. For this we have employed the sequential estimation technique [Beck and Arnold, 1977, Orlande, 2002, Beck, 2003].
Step 1: Initialize the iterative procedure by setting the iteration index $k$ to 0 and $p^{(0)} = p_\mu$.

Compute the estimate for the vector of unknown parameters sequentially, by using

$$A = V_n J_{n+1}^T$$
$$\Delta = J_{n+1} A + W_{n+1}^{-1}$$
$$\Gamma = \Delta^{-1} A$$
$$E_{n+1} = H_{n+1}^{\text{exp}} - H_{n+1}^{\text{est}}(p^k)$$
$$p^{k+1}_{n+1} = p^{k+1}_n + \Gamma [E_{n+1} - J_{n+1}(p^{k+1}_n - p_k)]$$
$$V_{n+1} = V_n - \Gamma J_{n+1} V_n$$

$$V_0 = V, \ p_0 = \mu, \ W_{n+1} = \sigma_{n+1}^{-2}$$

$$J_n = \left. \frac{\partial H_n^{\text{est}}}{\partial p} \right|_{p=p^k}$$
EXPERIMENTAL SET-UP (Sketch)
RESULTS AND CONCLUSIONS

EXPERIMENTAL SET-UP – CEPEL’s LABORATORY SPAN
RESULTS AND CONCLUSIONS

EXPERIMENTAL SET-UP – SHAKER
RESULTS AND CONCLUSIONS

- EXPERIMENTAL SET-UP – LOAD CELL
RESULTS AND CONCLUSIONS

- EXPERIMENTAL SET-UP – ACCELEROMETER AND LOAD CELL
EXPERIMENTAL SET-UP

- Grosbeak ACSR conductor
- \( \mu = 1.30271 \, \text{kg/m} \)
- \( T = 21778.2 \, \text{N} \) and \( T = 27468.0 \, \text{N} \)
- \( L = 51.905 \, \text{m} \)
- \( X_1 = 1.39 \, \text{m} \)
- \( X_2 = 0.70 \, \text{m} \)
- \( X_3 = 1.61 \, \text{m} \)
### EXPERIMENTAL SET-UP: FRFs

<table>
<thead>
<tr>
<th>Band (Hz)</th>
<th>( n_d ) (Number of averages)</th>
<th>Accelerometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5, 17.5]</td>
<td>30</td>
<td>AC1, AC2 and AC3</td>
</tr>
<tr>
<td>[17.5, 30]</td>
<td>30</td>
<td>AC1, AC2 and AC3</td>
</tr>
</tbody>
</table>

- All the FRFs are measured with 801 equally spaced frequency points.
RESULTS AND CONCLUSIONS

EXPERIMENTAL SET-UP: FRFs
RESULTS AND CONCLUSIONS

RESULTS: CASE C1

- $T = 21778.2$ N

- $[5, 17.5]$ Hz

- Kelvin-Voigt model

- Unknown parameters $\alpha$ and $EI$ and for case C1 it is considered that $\xi I = 10^{-4}$ N.m$^2$s$^{-1}$.

- Initial Guess for $EI$ and $\alpha$?
RESULTS: CASE C1

We do not have any information about a suitable intial guess for $\alpha$, therefore we have considered $\alpha^{(0)} = 10^{-2}\text{N.s.m}^{-2}$.

Concerning the parameter $EI$, the reports by CIGRE provide the maximum and minimum values: $\text{EI}_{\text{min}} = 28 \text{ N.m}^2$ and $\text{EI}_{\text{max}} = 1027 \text{ N.m}^2$.

Parameterization $\text{EI}=p_1\times10^3$ and $\alpha = p_2$. 
RESULTS AND CONCLUSIONS

RESULTS: CASE C1

Estimated bending stiffness and aerodynamic damping coefficient (Case C1).

<table>
<thead>
<tr>
<th>GITT</th>
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<th></th>
<th>FEM</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$EI^{(0)}$</td>
<td>$EI$</td>
<td>$\alpha$</td>
<td>$\frac{\sigma_{EI}}{\sigma}$</td>
<td>$\frac{\sigma_{\alpha}}{\sigma}$</td>
<td>$EI^{(0)}$</td>
<td>$EI$</td>
<td>$\alpha$</td>
<td>$\frac{\sigma_{EI}}{\sigma}$</td>
</tr>
<tr>
<td>28</td>
<td>748.9</td>
<td>0.355</td>
<td>2.8</td>
<td>0.012</td>
<td>28</td>
<td>742.2</td>
<td>0.3494</td>
<td>2.8</td>
</tr>
<tr>
<td>527.5</td>
<td>748.9</td>
<td>0.355</td>
<td>2.8</td>
<td>0.012</td>
<td>527.5</td>
<td>741.9</td>
<td>0.3495</td>
<td>2.8</td>
</tr>
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<td>1027</td>
<td>748.9</td>
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<td>1027</td>
<td>741.9</td>
<td>0.3495</td>
<td>2.8</td>
</tr>
</tbody>
</table>

DEFINITION: Normalized norm

$$|q_{\text{est}}^N| = \frac{(q^{\text{exp}} - q^{\text{est}})H(q^{\text{exp}} - q^{\text{est}})}{(q^{\text{exp}})^Hq^{\text{exp}}}$$
RESULTS AND CONCLUSIONS

- RESULTS: CASE C1

- Definition of a normalized norm of the FRFs

\[ |q^{est}|_N = \frac{(q^{exp} - q^{est})^H(q^{exp} - q^{est})}{(q^{exp})^Hq^{exp}} \]

- Norms for case C1

<table>
<thead>
<tr>
<th>Band (Hz)</th>
<th>AC1</th>
<th>AC2</th>
<th>AC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5,17.5]</td>
<td>0.6761</td>
<td>0.6665</td>
<td>0.6894</td>
</tr>
</tbody>
</table>
RESULTS AND CONCLUSIONS

- RESULTS: CASE C2

- [17.5, 30] Hz

- Use the results provided by case C1 as *a priori* information.

- Unknown parameters $\alpha$ and $EI$ and $\xi_I$.

- Parameterization: $EI = p_1 \times 10^3$, $\alpha = p_2$ and $\xi_I = p_3 \times 10^{-2}$

- The components of the covariance matrix $\mathbf{V}$ and vector $\mathbf{p}_\mu$ associated to $EI$ and $\alpha$ are obtained from case C1.
RESULTS AND CONCLUSIONS

RESULTS: CASE C2

What about the components of the covariance matrix \( V \) and vector \( p_\mu \) for parameter \( \xi I \)?

As we do not have any information concerning \( I \) we simply consider that its mean is a small number and its standard deviation is a large number; therefore we have chosen \( 10^{-5} \) and \( 10^{10} \), respectively.

\[
p_\mu = \{0.7419, 0.3495, 10^{-5}\}^T
\]

\[
V = \sigma^2 \times \begin{pmatrix}
7.91 \times 10^{-6} & -2.78 \times 10^{-7} & 0 \\
-2.78 \times 10^{-7} & 1.20 \times 10^{-4} & 0 \\
0 & 0 & 10^{10}
\end{pmatrix}
\]
## RESULTS AND CONCLUSIONS

### RESULTS: CASE C2

<table>
<thead>
<tr>
<th></th>
<th>FEM</th>
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</thead>
<tbody>
<tr>
<td>$EI$</td>
<td>$\alpha$</td>
<td>$\xi I$</td>
<td>$\frac{\sigma_{EI}}{\sigma}$</td>
<td>$\frac{\sigma_{\alpha}}{\sigma}$</td>
</tr>
<tr>
<td>678.9</td>
<td>0.393</td>
<td>0.029</td>
<td>0.347</td>
<td>0.008</td>
</tr>
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</table>

<table>
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<tr>
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<td>$\xi I$</td>
<td>$\frac{\sigma_{EI}}{\sigma}$</td>
<td>$\frac{\sigma_{\alpha}}{\sigma}$</td>
</tr>
<tr>
<td>682.4</td>
<td>0.393</td>
<td>0.026</td>
<td>0.336</td>
<td>0.013</td>
</tr>
</tbody>
</table>

### Norms for case C2

<table>
<thead>
<tr>
<th>Band (Hz)</th>
<th>$AC1$</th>
<th>$AC2$</th>
<th>$AC3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5,17.5]</td>
<td>0.8201</td>
<td>0.8084</td>
<td>0.8320</td>
</tr>
<tr>
<td>[17.5,30]</td>
<td>0.4663</td>
<td>0.4409</td>
<td>0.5032</td>
</tr>
</tbody>
</table>
RESULTS AND CONCLUSIONS

- RESULTS: CASE C2
  - Validation in time domain with na excitation equal to a white noise encompassing the band \([5, 17.5]\) Hz

![Graph showing AC_1 (m/s^2) vs. time (t(s)) for EXP and MODEL.]
RESULTS AND CONCLUSIONS

RESULTS: CASE C2

- Validation in time domain with no excitation equal to a white noise encompassing the band [5, 17.5] Hz.
RESULTS AND CONCLUSIONS

RESULTS: CASE C2

- Validation in time domain with a white noise encompassing the band [5, 17.5] Hz
RESULTS AND CONCLUSIONS

- RESULTS: CASE C3

- [17.5, 30] Hz

- Use the results provided by case C1 as *a priori* information.

- Hysteretic damping model. Unknown parameters $\alpha$ and EI and $nI$.

- Parameterization: $EI = p_1 \times 10^3$, $\alpha = p_2$ and $\eta I = p_3$

- The components of the covariance matrix $V$ and vector $p_\mu$ associated to EI and $\alpha$ are obtained from case C1.
RESULTS AND CONCLUSIONS

RESULTS: CASE C3

What about the components of the covariance matrix $\mathbf{V}$ and vector $\mathbf{p}_\mu$ for parameter $\eta I$?

As we do not have any information concerning $I$ we simply consider that its mean is a small number and its standard deviation is a large number; therefore we have chosen $10^{-5}$ and $10^{10}$, respectively.

$$\mathbf{p}_\mu = \{0.7419, 0.3495, 10^{-5}\}^T$$

$$\mathbf{V} = \sigma^2 \times \begin{pmatrix} 7.91 \times 10^{-6} & -2.78 \times 10^{-7} & 0 \\ -2.78 \times 10^{-7} & 1.20 \times 10^{-4} & 0 \\ 0 & 0 & 10^{10} \end{pmatrix}$$
RESULTS AND CONCLUSIONS

- RESULTS: CASE C3

<table>
<thead>
<tr>
<th>EI</th>
<th>$\alpha$</th>
<th>$\eta I$</th>
<th>$\sigma_{EI}/\sigma$</th>
<th>$\sigma_{\alpha}/\sigma$</th>
<th>$\sigma_{\eta I}/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>678.9</td>
<td>0.3771</td>
<td>6.0481</td>
<td>0.3504</td>
<td>0.009</td>
<td>0.679</td>
</tr>
</tbody>
</table>

- Norms for case C3

<table>
<thead>
<tr>
<th>Band (Hz)</th>
<th>AC1</th>
<th>AC2</th>
<th>AC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5,17.5]</td>
<td>0.8220</td>
<td>0.8101</td>
<td>0.8339</td>
</tr>
<tr>
<td>[17.5,30]</td>
<td>0.4659</td>
<td>0.4407</td>
<td>0.5026</td>
</tr>
</tbody>
</table>
Sequential evolution of the estimated parameters for cases C2 e C3
Sequential evolution of the estimated parameters for cases C2 and C3
Sequential evolution of the estimated parameters for cases C2 and C3.
Sequential evolution of the estimated parameters for cases C2 e C3
RESULTS AND CONCLUSIONS

- RESULTS: CASE C4

- Exactly equal to case C1 e C2 in sequence but with axial tension equal to $T = 27468.0 \text{ N}$

- $EI = 615.8 \text{ N.m}^2$

- $\alpha = 0.3674 \text{ N.s.m}^{-2}$

- $\xi I = 2.8186 \times 10^{-2} \text{ N.m}^2\text{s}^{-1}$
FINAL REMARKS

- TLC’s were modelled as homogeneous beams with viscous and structural damping

- Based on experimental data it was chosen a linear model to represent the system.

- Two classical damping models have been used

- Their bending stiffness and damping parameters were estimated based on inverse analysis

- Direct problem associated to estimation process was solved by two approaches: FEM and GITT

- Inverse problem was solved through Levenberg-Marquardt iterative procedure and the sequential estimation technique.
FINAL REMARKS (cont.)

- MAIN CONCLUSIONS
  - Objective function much more sensitive to $\alpha$ than to $EI$ in the frequency range (0,20 Hz)
  - Estimates for $\alpha$ were in good agreement with its true value
  - Estimated parameters practically unaffected by noise level
  - Uncertainty in span length affects much more $EI$
  - Uncertainty in mechanical load largely affects both parameters

- MAIN CONTRIBUTIONS
  - The estimation of bending stiffness and damping parameter of TLC
  - Use of GITT approach to solve direct problem associated to estimation process
  - Numerical analysis of the effects of model uncertainties on estimated parameters what, to authors belief, are not considered previously in the literature for this specific problem
FUTURE WORKS

- Estimate these parameters for different span lengths and different axial tractions.

- Investigate a suitable mechanical model for the StockBridge damper.

- Analyze the coupled system TLC and damper based on the estimated models and evaluate it based on experimental data.