MODELAGEM MULTIESCALA: HOMOGENEIZAÇÃO

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OUTLINE

- Heterogeneous Materials
- Homogenization
- Homogenization Applied to the Heat Conduction Problem (HCP)
- Homogenization Applied to the Elasticity Problem (EP)

HETEROGENEOUS MATERIALS

- Hystory
  - Maxwell, J.C.(1873)
    - Effective conductivity of a dispersion of espheres that is exact for dilute sphere concentrations.
  - Rayleigh, L.(1892)
    - Effective conductivity of regular arrays of espheres (used to this day).
  - Einstein, A.(1906)
    - Effective viscosity of a dilute suspension of espheres.

HETEROGENEOUS MATERIALS

- Definition
  - A heterogeneous material is comprised of domains composed of different materials, such as a composite, or these domains can even be composed of the same material but in different states, such as a polycristal.

HETEROGENEOUS MATERIALS

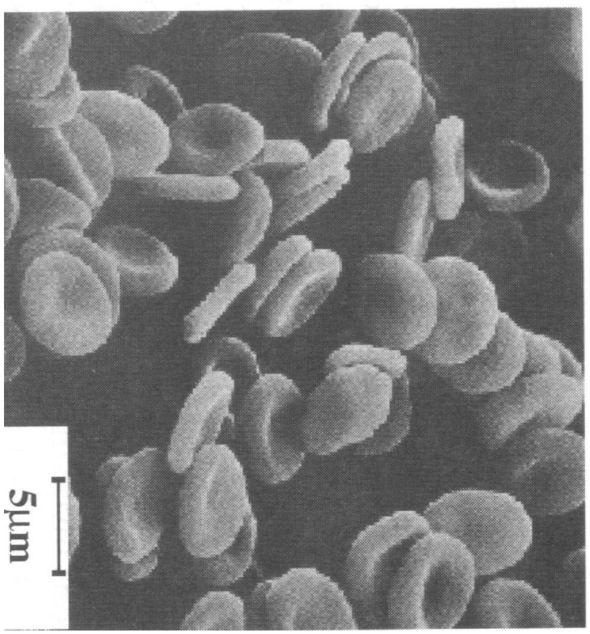
- Examples: synthetic products
  - Gels
  - Foams
  - Concrete
  - Cellular solids
  - Coloids
  - Composites
  - ...

HETEROGENEOUS MATERIALS

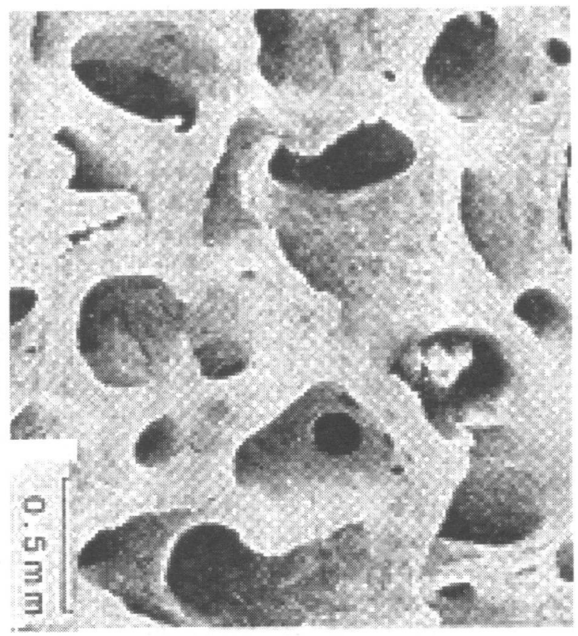
- Examples: in nature
  - Soils
  - Sandstone
  - Wood
  - Bone
  - Lungs
  - Blood
  - Animal and Plant Tissue
  - ...

HETEROGENEOUS MATERIALS

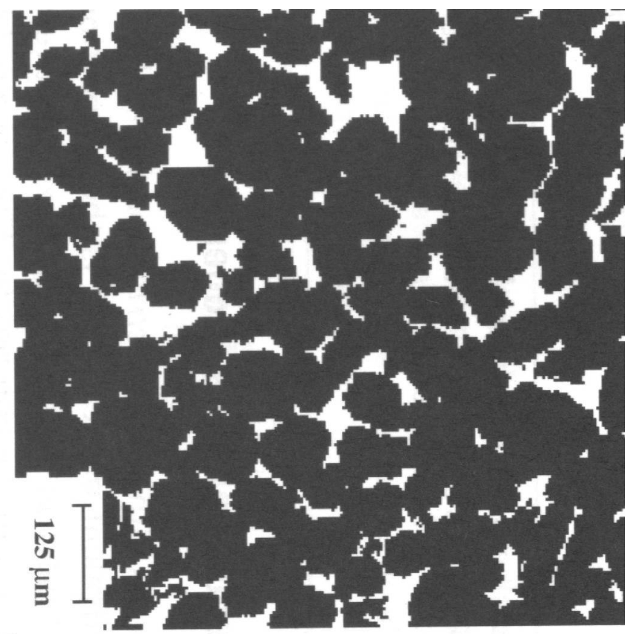
- Examples of natural random heterogeneous materials.(Extracted from S.Torquato, Random Heterogeneous Materials, Springer Verlag, 2001.)



(Red blood cells)



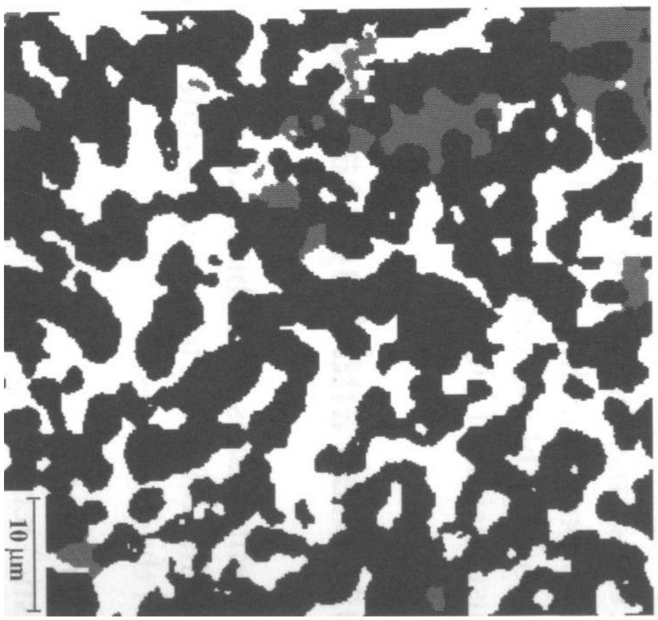
(Celullar Structure of bone)



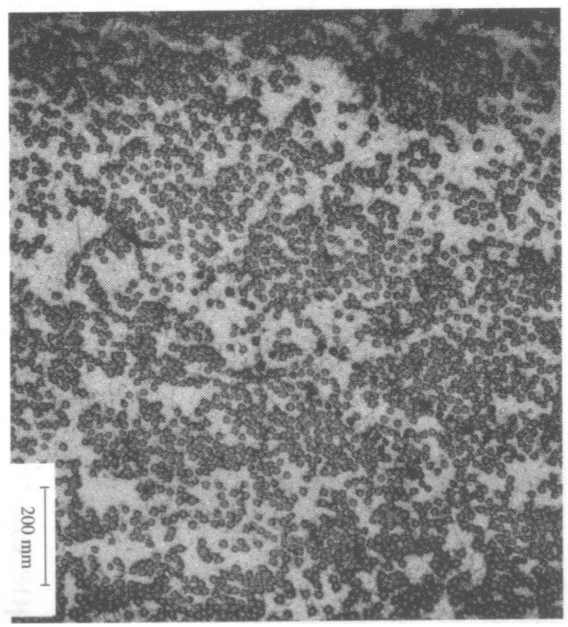
(Fontainebleau sandstone)

HETEROGENEOUS MATERIALS

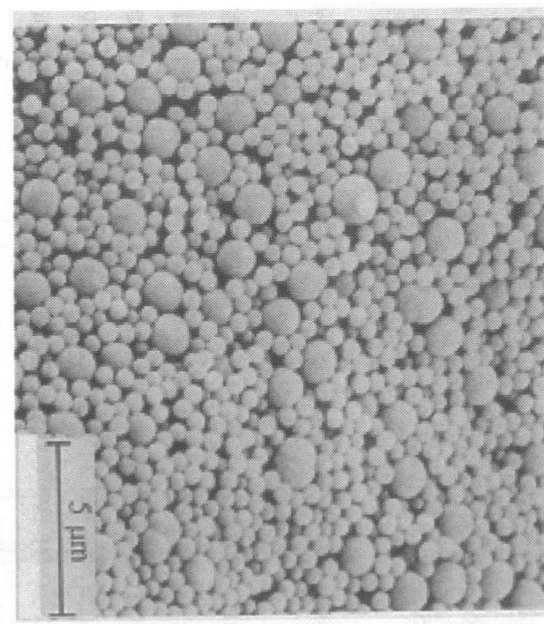
- Examples of synthetic random heterogeous materials. (Extracted from S.Torquato, Random Heterogeneous Materials, Springer Verlag, 2001.)



Aluminum (white) and another ceramic phase (gray)



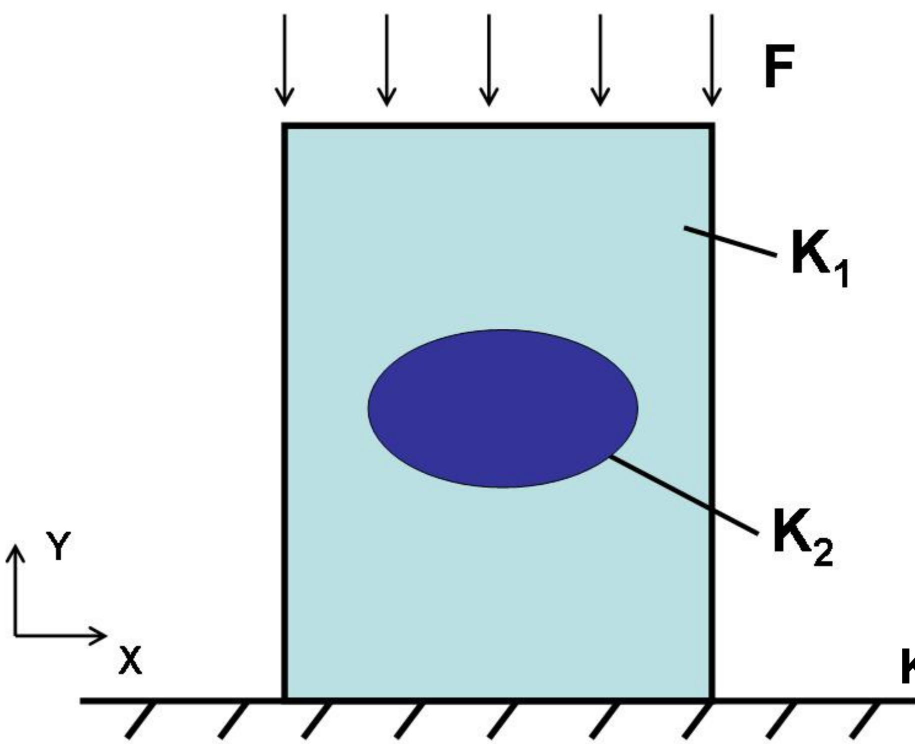
Fiber-reinforce cermet



Colloidal system of hard Spheres.

HETEROGENEOUS MATERIALS

- **P1**: Given $\mathbf{K}_1$, $\mathbf{K}_2$ and **F** can we determine the displacement field $\mathbf{u}(\mathbf{x})$ of this body ?



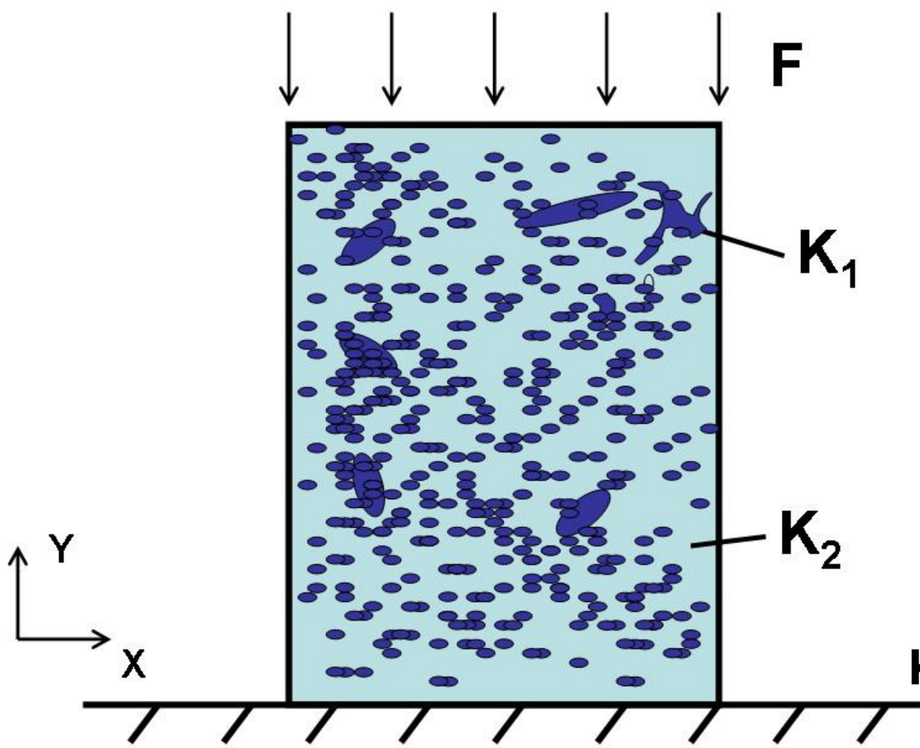
$\mathbf{K}_1$ and $\mathbf{K}_2$: elastic properties

HETEROGENEOUS MATERIALS

- **P1**: How can we solve problem P1 ?
  - Analytical solution (Depending on the shape of the inclusion)
  - Finite Element Method
  - ...?

HETEROGENEOUS MATERIALS

- **P2**: Given $\mathbf{K}_1$, $\mathbf{K}_2$ and **F** can we determine the displacement field $\mathbf{u}(\mathbf{x})$ of this body ?



$\mathbf{K}_1$ and $\mathbf{K}_2$: elastic properties

HETEROGENEOUS MATERIALS

- **P2**: How can we solve problem P2 ?
  - ~~Analytical Solution~~
  - ~~Finite Element Method~~
  - Black Box Approach : $\mathbf{u}(\mathbf{x_m}) = g(\mathbf{F},\alpha_1,\ldots,\alpha_N)$
  - Phenomenological Approach:

  $\mathbf{u}(\mathbf{x}) = g(\mathbf{F},p_1,\ldots,p_N)$
  - Homogenization

  $\mathbf{u}(\mathbf{x}) = g(\mathbf{F},\mathbf{K}_1,\ldots,\mathbf{K}_N,?)$

HETEROGENEOUS MATERIALS

- **P2**: Phenomenological Approach

  - Hypothesis:

  There is an equivalent macroscopic description.

  - Parameters:

  Through **experiments** one can obtain the equivalent physical parameters $p_1, \ldots, p_N$.

HETEROGENEOUS MATERIALS

- **P2**: Homogenization

  - Hypothesis:

  There is an equivalent macroscopic description.

  - Governing Equations:

  Build the macroscopic governing equations starting at the heterogeneity scale (microscopic).

HOMOGENIZATION

- DEFINITION
  - The homogenization theory is aimed at determining the macroscopic governing partial differential equations of a heterogeneous medium when the characteristic length of the heterogeneities is much times lower than the characteristic length of the medium.

HOMOGENIZATION

- METHODS (Bensoussan et. al)

  - Based on the construction of asymptotic expansions using multiple scales.
  - Based on energy estimates.
  - Based on probabilistic arguments and works whenever the problem admits probabilistic formulation or has a probabilistic origin.
  - Based on the spectral decomposition of operators with periodic coefficients, the so-called expansion Bloch waves (problems involving high frequency wave propagation in rapidly varying periodic media) .

HOMOGENIZATION

- MACROSCOPIC DESCRIPTION ?
  - What are we really looking for when we say that we are searching the macroscopic governing equations for the medium ?
  - R.: Ariault states that we are looking for an equivalent boundary value problem, i.e., we are looking for the relations between the *macroscopic variables* of the problem and its associated *effective properties.*

HOMOGENIZATION

- WHAT DO WE MEAN BY MACRO AND MICRO ?

    - The physical phenomena under analysis occur on the *microscopic* length scales that span from tens of nanometers in the case of gels to meters in the case of geological media. (Extracted from S.Torquato, Random Heterogeneous Materials, Springer Verlag, 2001.)

HOMOGENIZATION

- EFFECTIVE PROPERTIES ?

  - What do we mean by effective properties ?

  - R.: The effective properties of heterogeneous materials are determined by suitable averages of local fields derived from appropriate governing continuum equations of the problem under analysis.

HOMOGENIZATION

- EFFECTIVE PROPERTIES (Key-Questions)

  - Given a medium composed of phases such that each phase is characterized by its properties denoted by $\mathbf{K}_1$, ..., $\mathbf{K}_N$ and its volumetric fractions denoted by , ϕ_1, ..., ϕ_N, respectively, how are its effective properties mathematically defined ?
  - Do we have to take the microstructure into account ?

HOMOGENIZATION

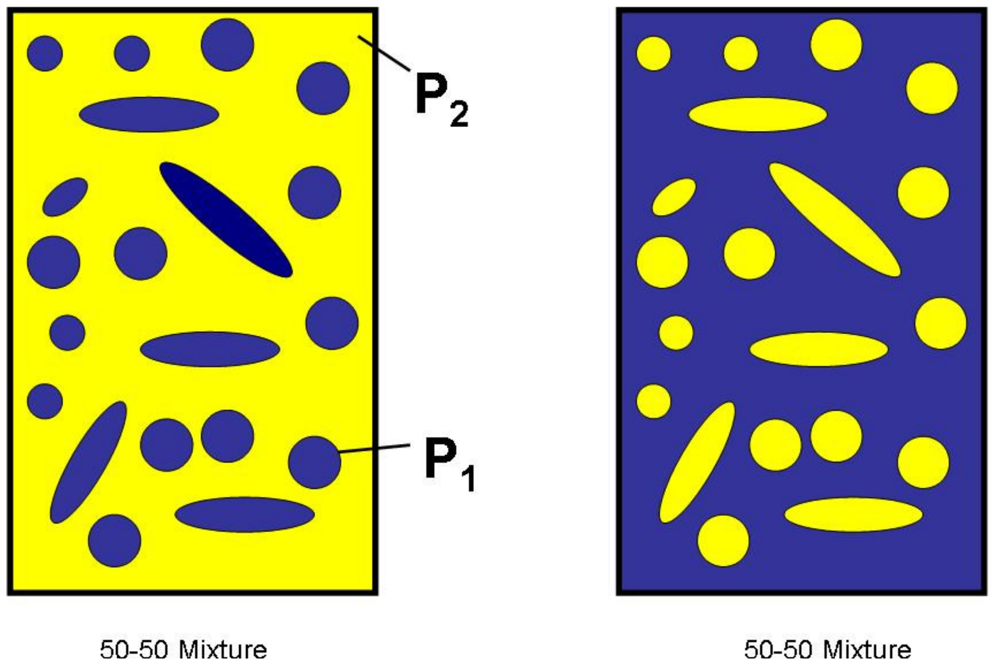
- MICROSTRUCTURE AND EFFECTIVE PROPERTIES

  - In order to evaluate if the analysis of the microstructure is important for the effective calculation of the effective properties lets consider a simple example.

  - The example consists of a heterogeneous medium composed of two phases with very diffrent elastic properties but with ϕ_1=ϕ_2= 0.5.

HOMOGENIZATION

- EXAMPLE



- Lets consider the blue phase highly stiff compared to the yellow one.

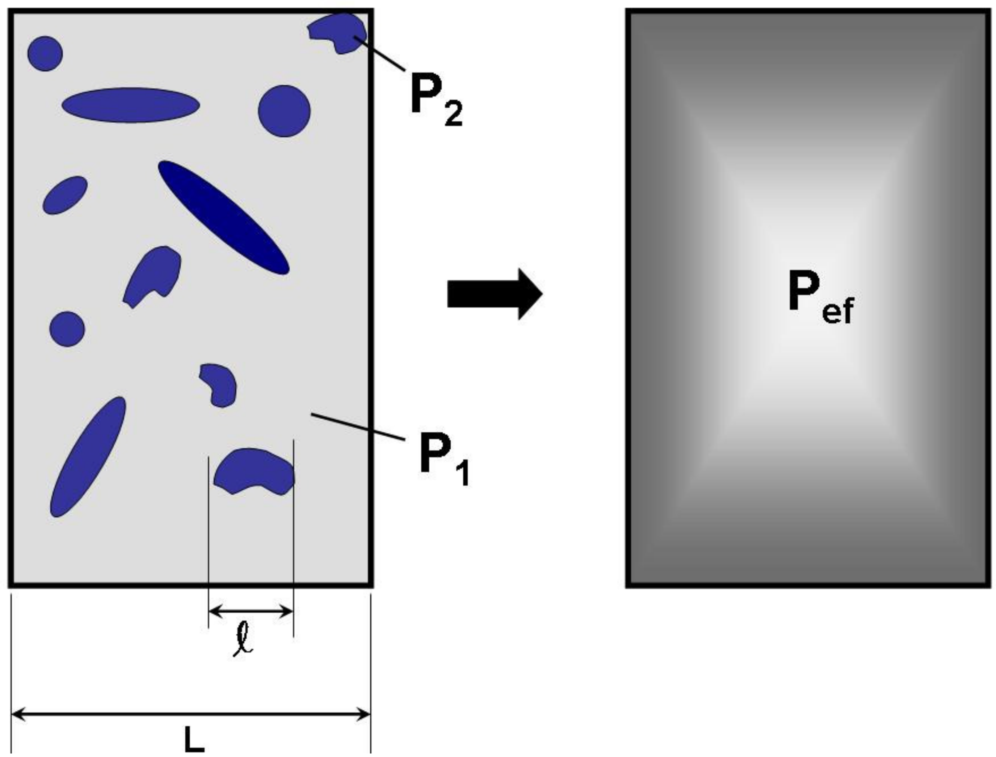
HOMOGENIZATION

- QUESTION
  - Which one possesses the higher effective stiffness ?
  - R.: The right one.
  - Even though $\phi_1 = \phi_2$ for both composites, their effective properties will be extremely different, therefore

$$\mathbf{K}_e = f(\mathbf{K}_1,\ldots,\mathbf{K}_N,\phi_1,\ldots,\phi_N,\Omega_Y)$$

HOMOGENIZATION

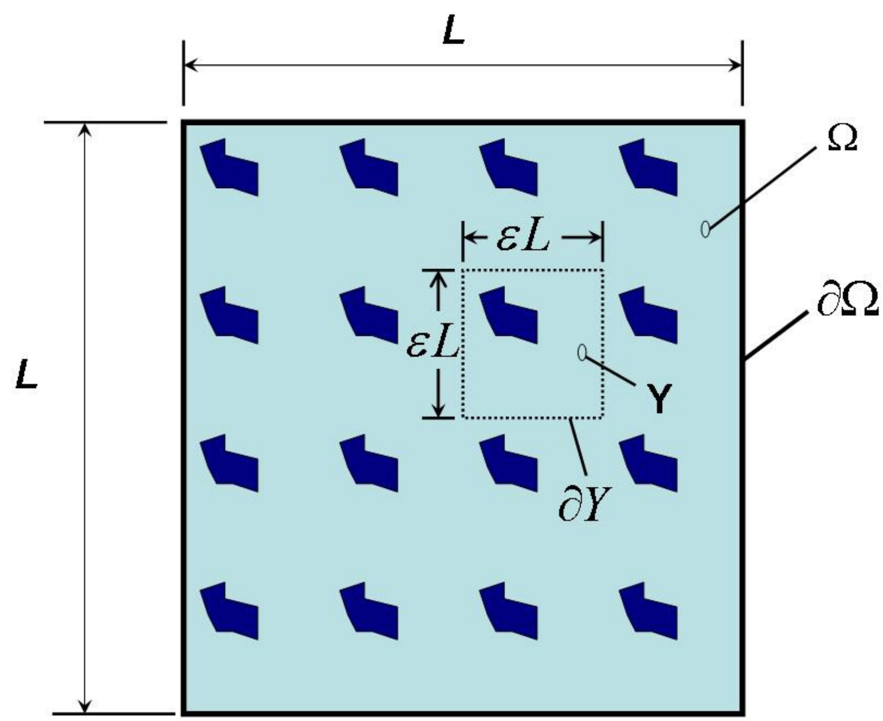
- TARGET ?



- HOW ?

HOMOGENIZATION-HCP

- Heat Conduction Problem: Periodic Structure



Ω: Macroscopic Domain

Y:Microscopic Domain

ε: Scale parameter

L:Characteristic lenght of the macroscopic scale

HOMOGENIZATION-HCP

- Governing Equations

$$\frac{\partial}{\partial x_i}\left[-K_{ij}^{\varepsilon}(\mathbf{x})\frac{\partial T^{\varepsilon}(\mathbf{x})}{\partial x_j}\right]=g(\mathbf{x}),\ \forall\ \mathbf{x}\in\Omega$$

- Boundary Conditions

$$T^{\varepsilon}=C_1,\quad \forall\,\mathbf{x}\in\Gamma_1$$

$$-K_{ij}^{\varepsilon}\frac{\partial T^{\varepsilon}}{\partial x_j}n_i=C_2,\quad \forall\,\mathbf{x}\in\Gamma_2$$

$$\Gamma_1\bigcup\Gamma_2=\partial\Omega$$

K$^{\varepsilon}$: Conductivity tensor

T$^{\varepsilon}$:Temperature field

HOMOGENIZATION-HCP

- Continuity Conditions (Perfect contact)

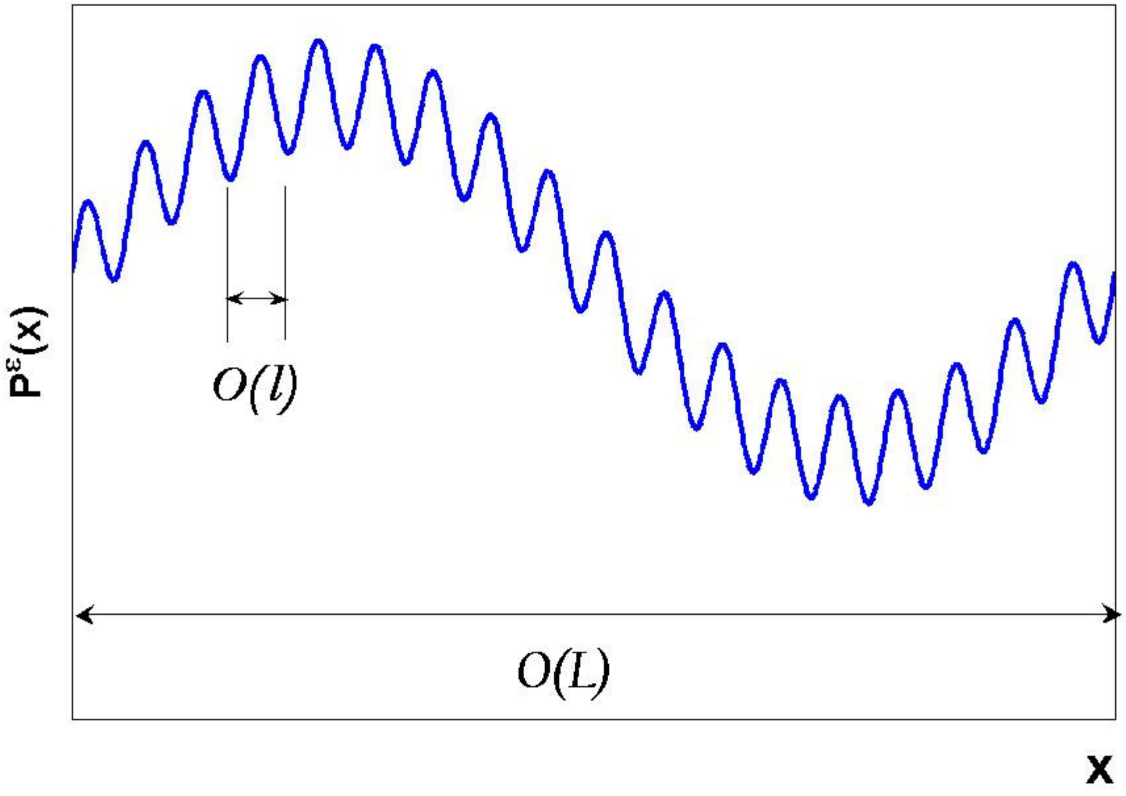
$$T^{+}(\mathbf{x}) = T^{-}(\mathbf{x}), \forall\, \mathbf{x} \in \partial Y$$

$$\frac{\partial}{\partial x_i}\left[-K_{ij}^{\varepsilon}(\mathbf{x})\frac{\partial T^{\varepsilon}(\mathbf{x})}{\partial x_j}\right]^{+} = \frac{\partial}{\partial x_i}\left[-K_{ij}^{\varepsilon}(\mathbf{x})\frac{\partial T^{\varepsilon}(\mathbf{x})}{\partial x_j}\right]^{-}, \forall, \mathbf{x} \in \partial Y$$

- Hypothesis : Separation of Scales is coherent.

HOMOGENIZATION-HCP

- SEPARATION OF SCALES ?
  - Lets consider the property $P^{\varepsilon}(\mathbf{x})$ described below:



Pe(x) *suggests* a possible use of two spatial scales !

HOMOGENIZATION-HCP

- HOW CAN WE MATHEMACALLY PERFORM THE SEPARATION OF SCALES ?

  - Concerning the separation of scales, Joseph B. Keller in [6] states that in order to allow a variable to vary in a macroscale (O(L)) and also in a microscale (O(ℓ)) we can represent it as a function of the macro/slow variable **x** and of the micro/rapid variable **y**. HOW ?

$$\mathbf{y} = \varepsilon^{-1}\mathbf{x} \qquad \varepsilon = \frac{l}{L} \quad \text{(Scale parameter)}$$

HOMOGENIZATION-HCP

- WHY **y** IS CALLED FAST VARIABLE ?
  - Lets consider a function g

$$g^{\varepsilon}(\mathbf{x}) = g(\mathbf{y}) = g(\mathbf{x}/\varepsilon)$$

$$\frac{\partial g}{\partial x_j} = \varepsilon^{-1}\frac{\partial g}{\partial y_j}(\mathbf{y})$$

HOMOGENIZATION-HCP

- AFTER ADOPTING THE HYPOTHESIS OF TWO SEPARATE SCALES HOW CAN WE TAKE THIS INTO ACCOUNT TO FIND THE SOLUTION $T^{\varepsilon}(\mathbf{x})$?

  - The presence of two disparate scales motivates the method of asymptotic expansions using multiple scales.

  - How is the general dependence of $T^{\varepsilon}(\mathbf{x})$ with these scales ?

HOMOGENIZATION-HCP

- WHAT IS THE GENERAL FORM OF $T^{\varepsilon}(\mathbf{x})$?

$$T^{\varepsilon}(\mathbf{x}) = v(\mathbf{x}, \mathbf{y}, \varepsilon)$$

- J.B. Keller also mentions that the fact that the fast variable was considered explicitly in v is that he believes the dependence of $T^{\varepsilon}(\mathbf{x})$ in ε is due to **y** and moreover, he also believes that v depends regularly on ε, viz.

$$T^{\varepsilon}(\mathbf{x}) = v(\mathbf{x}, \mathbf{y}, \varepsilon) = T^{(0)}(\mathbf{x}, \mathbf{y}) + \varepsilon T^{(1)}(\mathbf{x}, \mathbf{y}) + \varepsilon^{2} T^{(2)}(\mathbf{x}, \mathbf{y}) + \ldots$$

HOMOGENIZATION-HCP

- ASPECTS OF THE PROPOSED ASYMPTOTIC FORM OF $T^{\varepsilon}(\mathbf{x})$?

  - Now the unknown fields become $T^{(0)}(\mathbf{x},\mathbf{y})$, $T^{(1)}(\mathbf{x},\mathbf{y})$, $T^{(2)}(\mathbf{x},\mathbf{y})$, ...

  - It is shown in Bensoussan *et al* [2] that $T^{\varepsilon}(\mathbf{x})$ converges weakly to $T^{(0)}(\mathbf{x},\mathbf{y})$ as ε tends to zero

HOMOGENIZATION-HCP

- HOW DO WE PERFORM THE SPATIAL DERIVATIVES ?

  - As we have adopted the hypothesis of separation of scales we must first treat **x** and **y** as independent variables and subsequently replace **y** by $\varepsilon^{-1}\mathbf{x}$.

  - Example ?

HOMOGENIZATION-HCP

- EXAMPLE OF SPATIAL DERIVATIVE WITH A FUNCTION WHICH DEPENDS ON TWO SCALES
  - If $f^{\varepsilon}(\mathbf{x}) = f(\mathbf{x},\mathbf{y})$, then

$$\frac{\partial f^{\varepsilon}}{\partial x_i} = \frac{\partial f}{\partial x_i} + \frac{\partial f}{\partial y_i}\frac{\partial y_i}{\partial x_i} = \frac{\partial f}{\partial x_i} + \frac{1}{\varepsilon}\frac{\partial f}{\partial y_i}$$

$$\nabla_x f^{\varepsilon} = \nabla_x f + \frac{1}{\varepsilon}\nabla_y f$$

HOMOGENIZATION-HCP

- GOVERNING EQUATIONS OF THE HCP

$$\frac{\partial}{\partial x_i}\left[-K_{ij}^{\varepsilon}(\mathbf{x})\frac{\partial T^{\varepsilon}(\mathbf{x})}{\partial x_j}\right]=g(\mathbf{x}), \qquad \forall\, \mathbf{x}\in\Omega$$

$$\left[\frac{\partial}{\partial x_i}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_i}\right]\left[-K_{ij}^{\varepsilon}\left(\frac{\partial}{\partial x_i}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_i}\right)\left(T^{(0)}+\varepsilon T^{(1)}+\varepsilon^2 T^{(2)}+\ldots\right)\right]=g$$

HOMOGENIZATION-HCP

- REARRANGING THE GOVERNING EQUATIONS IN POWERS OF ε:

$$O(\varepsilon) + \varepsilon^0 \left[\frac{\partial}{\partial x_i} \left\{ -K_{ij}^{\varepsilon} \left(\frac{\partial T^{(0)}}{\partial x_j} + \frac{\partial T^{(1)}}{\partial y_j} \right) \right\} + \frac{\partial}{\partial y_i} \left\{ -K_{ij}^{\varepsilon} \left(\frac{\partial T^{(1)}}{\partial x_j} + \frac{\partial T^{(2)}}{\partial y_j} \right) \right\} \right] +$$

$$+\varepsilon^{-1} \left[\frac{\partial}{\partial x_i} \left\{ -K_{ij}^{\varepsilon} \left(\frac{\partial T^{(0)}}{\partial y_j} \right) \right\} + \frac{\partial}{\partial y_i} \left\{ -K_{ij}^{\varepsilon} \left(\frac{\partial T^{(0)}}{\partial x_j} + \frac{\partial T^{(1)}}{\partial y_j} \right) \right\} \right] +$$

$$+\varepsilon^{-2} \left[\frac{\partial}{\partial y_i} \left\{ -K_{ij}^{\varepsilon} \left(\frac{\partial T^{(0)}}{\partial y_j} \right) \right\} \right] = g$$

HOMOGENIZATION-HCP

- DIFFERENTIAL OPERATORS (K^{ε} periodic)

$$A_0 = -\frac{\partial}{\partial y_i}\left[K_{ij}^{\varepsilon}(\mathbf{y})\frac{\partial}{\partial y_j}\right]$$

$$A_1 = -\frac{\partial}{\partial x_i}\left[K_{ij}^{\varepsilon}(\mathbf{y})\frac{\partial}{\partial y_j}\right]-\frac{\partial}{\partial y_i}\left[K_{ij}^{\varepsilon}(\mathbf{y})\frac{\partial}{\partial x_j}\right]$$

$$A_2 = -\frac{\partial}{\partial x_i}\left[K_{ij}^{\varepsilon}(\mathbf{y})\frac{\partial}{\partial x_j}\right]$$

$$\left.\begin{array}{l} A_0\, T^{(0)} = 0 \\ A_0\, T^{(1)} + A_1\, T^{(0)} = 0 \\ A_0\, T^{(2)} + A_1\, T^{(1)} + A_2\, T^{(0)} = g \end{array}\right.$$

HOMOGENIZATION-HCP

- HOW DO WE DEFFINE A SPATIAL AVERAGE OF A Y-PERIODIC FUNCTION ?

  - The spatial average of a Y-periodic function F(**y**) over a unit cell denoted by <F(**y**)> is defined as follows

$$\left\langle \mathbf{F}(\mathbf{y}) \right\rangle = \frac{1}{Y} \int_{|Y|} \mathbf{F}(\mathbf{y})\, d\mathbf{y}$$

  - where |Y| is a d-dimensional volume (measure) of Y . The quantity may represent a tensor of any order.

HOMOGENIZATION-HCP

- THEOREM 1

  - Let F(**y**) be a Y-periodic function that is square integrable. For the boundary value problem

$$A_0\,\Phi(\mathbf{y}) = \mathbf{F}(\mathbf{y}), \qquad \forall\,\mathbf{y} \in Y$$

where Φ(**y**) is Y-periodic the following hold:

(i) There exists a solution if and only if

(ii) If a solution exists, it is unique up to an additive constant.

HOMOGENIZATION-HCP

- HOW DO WE SOLVE $A_0T^{(0)}=0$?

  - The first step is simply to note that equation this equation fulfills the conditions specified in Theorem 1 and hence there exists a solution $T^{(0)}$.

  - The second step is to note that the operator A_0 involves only derivatives with respect to **y** what means that in this equation the variable **x** works simply as a parameter.

HOMOGENIZATION-HCP

- SOLUTION ?

  - Among a myriad of possible solutions are the constant ones, i.e., those solutions which depend only on **x**. J.B. Keller states that A_0 is such that these constants are the only solutions which are defined in the entire **Y**-pace and are *bounded*, therefore

$$T^{(0)} = T^{(0)}(\mathbf{x})$$

HOMOGENIZATION-HCP

- DOMAIN OF INTEGRATION ?
  - As we are considering spatial periodic properties we perform integrations over the domain of a periodic cell.
  - *Note*: Another interesting comment related to the periodicity hypothesis is that we may consider only solutions that satisfy the same periodicity !

HOMOGENIZATION-HCP

- HOW DO WE SOLVE $A_0 T^{(1)} + A_1 T^{(0)} = 0$?
  - First question : Does $T^{(1)}$ exist ?

$$A_0 T^{(1)} = -A_1 T^{(0)} = \frac{\partial K_{ij}(\mathbf{y})}{\partial y_i} \frac{\partial T^{(0)}(\mathbf{x})}{\partial x_j}$$

  - As A_0 involves only derivatives with respect to **y**, in right side of the equation above **x** acts as a parameter.

HOMOGENIZATION-HCP

- ADJOINT PROBLEM

  - Lets consider the following adjoint problem defined in the cell domain

$$A_0 \chi_j(\mathbf{y}) = \frac{\partial K_{ij}}{\partial y_i}, \qquad \forall \, \mathbf{y} \in Y$$

  - We can prove that this auxiliary problem has solution if it fulfills the requirements states in Theorem 1.

HOMOGENIZATION-HCP

- ADJOINT PROBLEM
  - From the divergence theorem one gets

$$\int_Y \frac{\partial K_{ij}}{\partial y_i} d\mathbf{y} = \int_{\partial Y} K_{ij}\, n_j dS$$

  - Due to the Y-periodicity of K_{ij} the right side of this equation is zero. Therefore,

$$\frac{\partial K_{ij}}{\partial y_i} \;\; is \;\; Y - periodic$$

  - And $\chi_j(y)$ exists.

HOMOGENIZATION-HCP

- SOLUTION OF $T^{(1)}(\mathbf{x},\mathbf{y})$

$$T^{(1)}(\mathbf{x},\mathbf{y}) = \chi_j(\mathbf{y})\frac{\partial T^{(0)}}{\partial x_j} + u(\mathbf{x})$$

HOMOGENIZATION-HCP

- HOW DO WE SOLVE $A_0T^{(2)}+A_1T^{(1)}+A_2T^{(0)} = g$?

  - Which conditions should be fulfilled to assure that $T^{(2)}$ exists ?

$$A_0T^{(2)} = g - A_1T^{(1)} - A_2T^{(0)}$$

  - What do we need to to apply Theorem 1 ?

$$\left\langle g - A_1T^{(1)} - A_2T^{(0)} \right\rangle = 0$$

HOMOGENIZATION-HCP

- Y-PERIODICITY CONDITIONS

$$\frac{1}{|Y|}\int_Y \left[g + \frac{\partial}{\partial y_i}\left(K_{ij}\chi_q\right)\frac{\partial^2 T^{(0)}}{\partial x_j \partial x_q} + \frac{\partial K_{ij}}{\partial y_i}\frac{\partial u}{\partial x_j}\right] d\mathbf{y} +$$

$$\frac{1}{|Y|}\int_Y K_{ij}\left(\frac{\partial \chi_q}{\partial y_j}\frac{\partial^2 T^{(0)}}{\partial x_i \partial x_q} + \frac{\partial^2 T^{(0)}}{\partial x_i \partial x_j}\right) d\mathbf{y} = 0$$

Simplifications

HOMOGENIZATION-HCP

- SIMPLIFICATIONS

$$\frac{1}{|Y|}\int_Y \frac{\partial K_{ij}(\mathbf{y})}{\partial y_i}\frac{\partial u(\mathbf{x})}{\partial x_j}d\mathbf{y} = \frac{1}{|Y|}\frac{\partial u(\mathbf{x})}{\partial x_j}\int_Y \frac{\partial K_{ij}(\mathbf{y})}{\partial y_i}d\mathbf{y} = 0$$

$$\frac{1}{|Y|}\int_Y \frac{\partial}{\partial y_i}(K_{ij}\chi_q)\frac{\partial^2 T^{(0)}}{\partial x_j \partial x_q}d\mathbf{y} = \frac{1}{|Y|}\frac{\partial^2 T^{(0)}(\mathbf{x})}{\partial x_j \partial x_q}\int_Y \frac{\partial}{\partial y_i}(K_{ij}(\mathbf{y})\chi_q(\mathbf{y}))d\mathbf{y} = 0$$

HOMOGENIZATION-HCP

- RESULTING EQUATIONS

CONSTANT

$$\frac{1}{|Y|}\int_Y g\,d\mathbf{y} + \left[\frac{1}{|Y|}\int_Y K_{ij}(\mathbf{y}) + K_{ir}(\mathbf{y})\frac{\partial \chi_j(\mathbf{y})}{\partial y_r}\,d\mathbf{y}\right]\frac{\partial^2 T^{(0)}(\mathbf{x})}{\partial x_i \partial x_j} = 0$$

FUNCTION WHICH DEPENDS ONLY ON THE MACRO SCALE

HOMOGENIZATION-HCP

- STANDARD FASHION

$$-K_{ij}^{eq}\frac{\partial^2 T^{(0)}}{\partial x_i \partial x_j}(\mathbf{x}) = \langle g \rangle$$

$$K_{ij}^{eq} = \langle K_{ij} \rangle + \left\langle K_{ir}\frac{\partial \chi_j}{\partial y_r} \right\rangle$$

HOMOGENIZATION-EP

- Elasticity Problem: Periodic Structure

  - Governing Equation

$$-\frac{d}{dx}\left[E^{\varepsilon}.\frac{du^{\varepsilon}}{dx}\right]=f(x), \qquad x\in \left]0,1\right[$$

  - Boundary Conditions

$$u^{\varepsilon}(0)=u^{\varepsilon}(1)=0$$

HOMOGENIZATION-EP

- Periodic Structure

$$E^{\varepsilon}(x) = E^{\varepsilon}(x+l)$$

  - Hypothesis: It is possible to adopt a separation of scales

$$u^{\varepsilon}(\mathbf{x}) = u^{(0)}(\mathbf{x},\mathbf{y}) + \varepsilon \; u^{(1)}(\mathbf{x},\mathbf{y}) + \varepsilon^{2} \; u^{(2)}(\mathbf{x},\mathbf{y}) + ...$$

HOMOGENIZATION-EP

- Homogenization

$$-E_{eq}\frac{d^2u^{(0)}}{dx^2}=\langle f\rangle$$

$$\frac{1}{E_{eq}}=\frac{1}{l}\int_0^l\frac{1}{E(Z)}dz$$

$$\langle f\rangle=\frac{1}{l}\int_0^l f\,dy$$

HOMOGENIZATION-EP

- This one-dimensional problem has an analytic solution

$$c_0 = \frac{1}{\int_0^1 E(z/\varepsilon)dz} \int_0^1 \left[\frac{1}{E(z/\varepsilon)} \int_0^z f(t)dt \right] dz$$

$$u^\varepsilon(x) = -\int_0^x \left[\frac{1}{E(z/\varepsilon)} \int_0^z f(t)dt + c_0 \right] dz$$

HOMOGENIZATION-EP

- Parameters

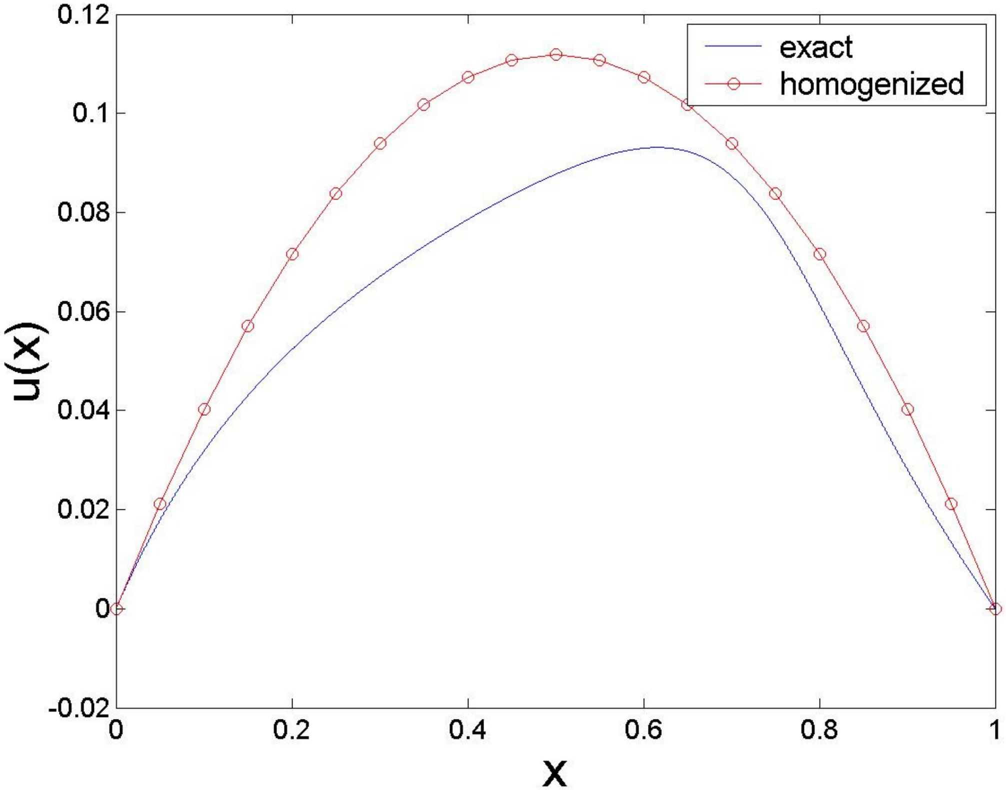
$$f(x) = 1$$

$$E(z) = \frac{1}{2}(\beta - \alpha)(1 + \sin(2\pi z)) + \alpha$$

$$\alpha = \frac{1}{2} \qquad \beta = \frac{5}{2}$$

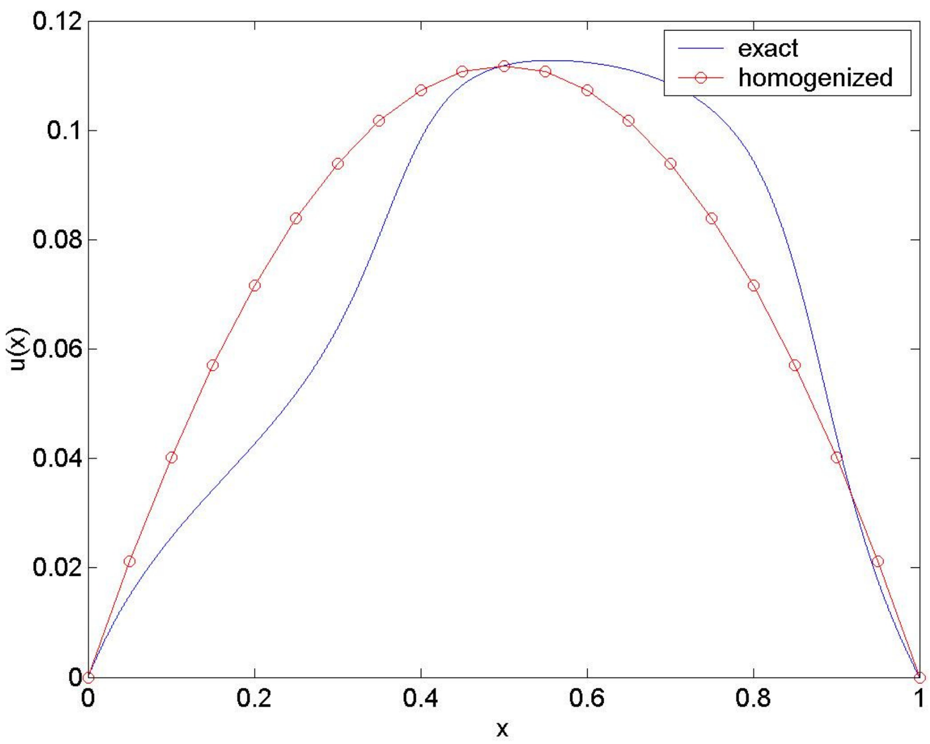
HOMOGENIZATION-EP

- Solution ε =1



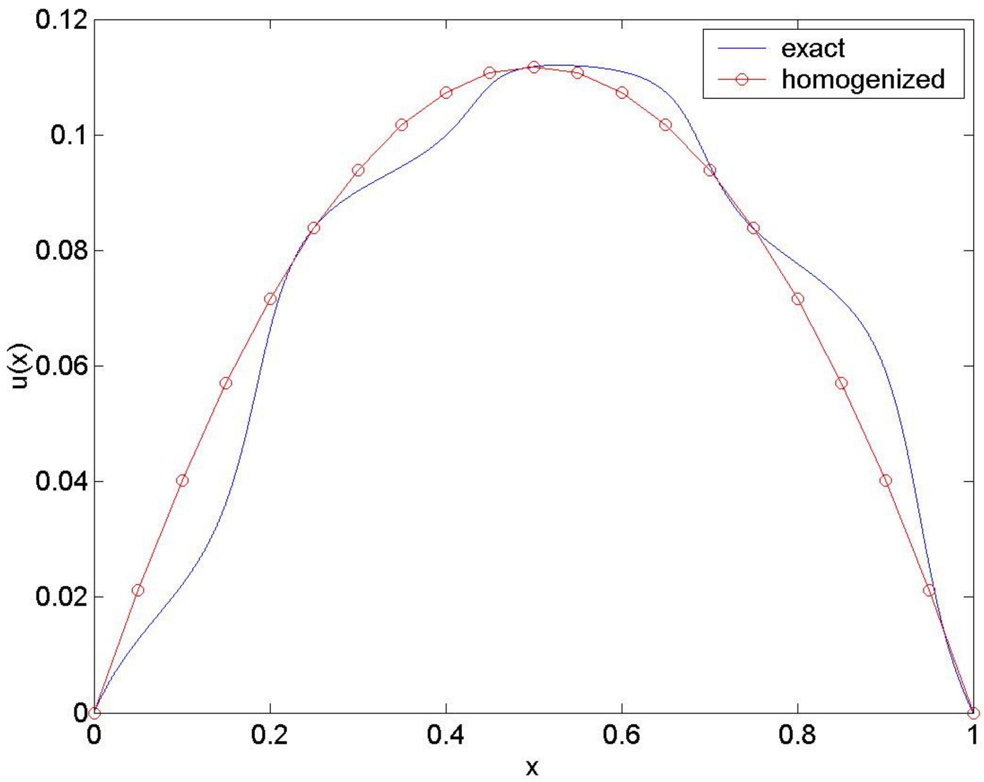
HOMOGENIZATION-EP

- Solution $\varepsilon = 1/2$



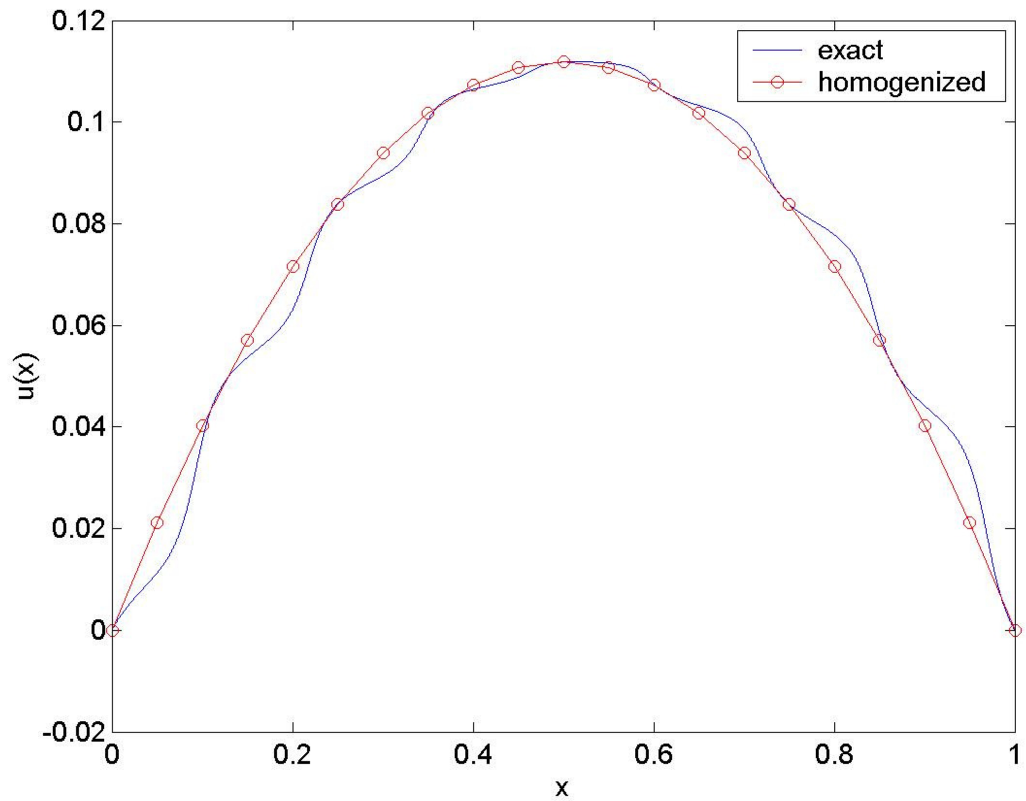
HOMOGENIZATION-EP

- Solution ε =1/4



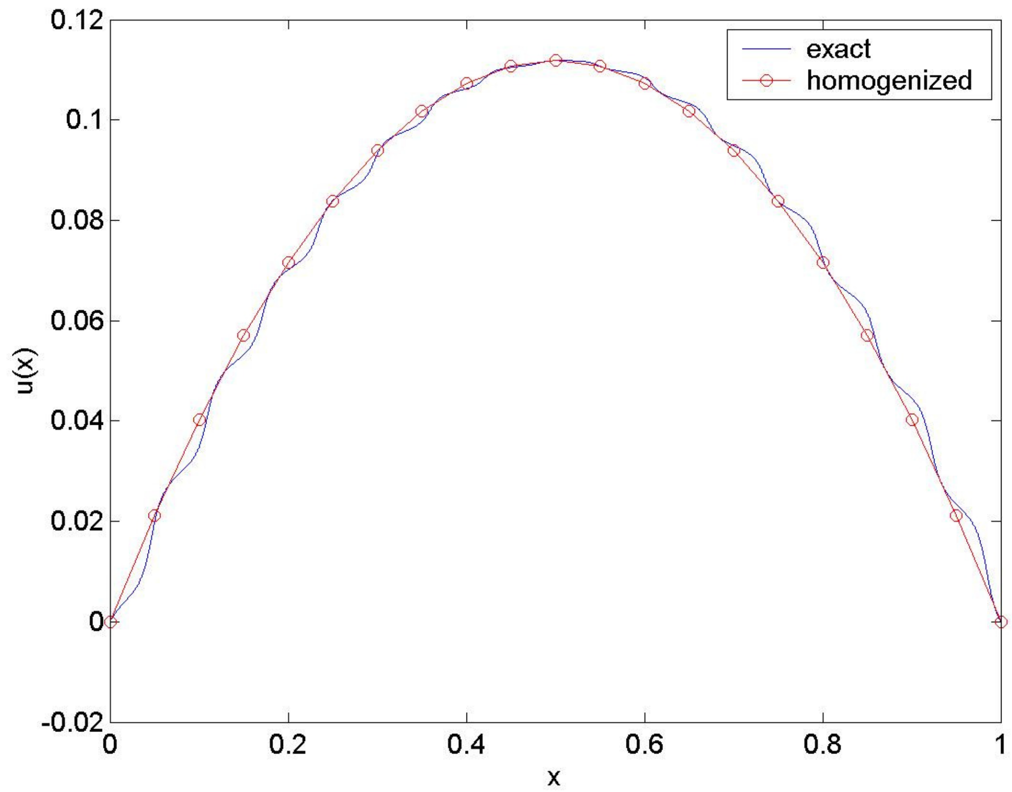
HOMOGENIZATION-EP

- Solution $\varepsilon = 1/8$



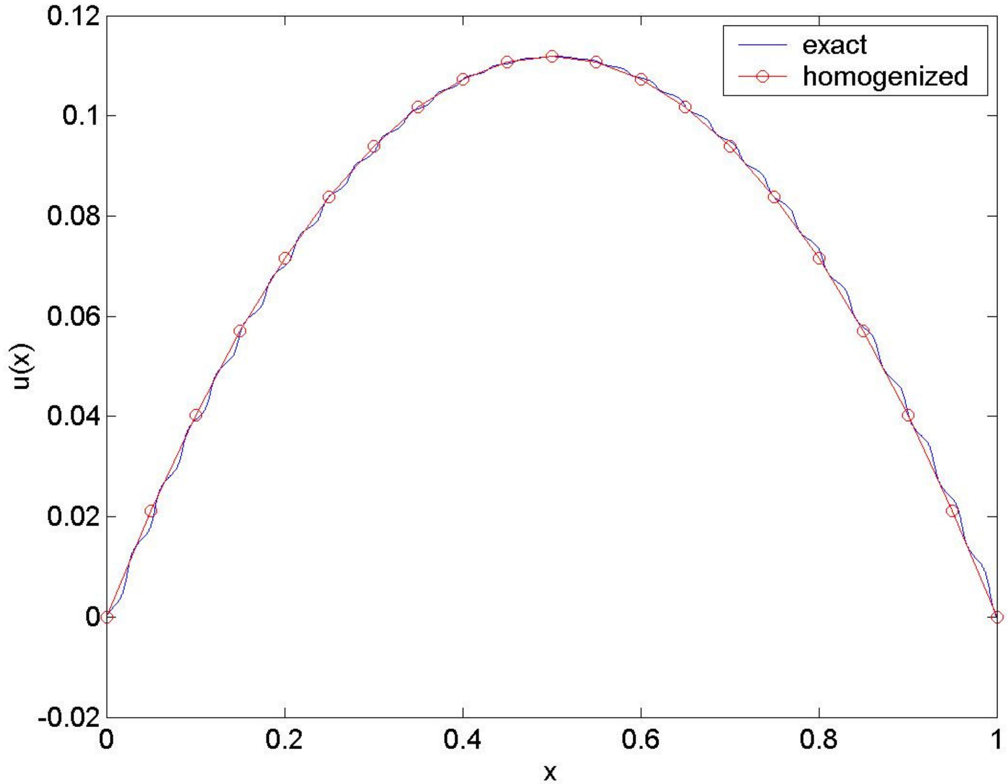
HOMOGENIZATION-EP

- Solution $\varepsilon = 1/16$



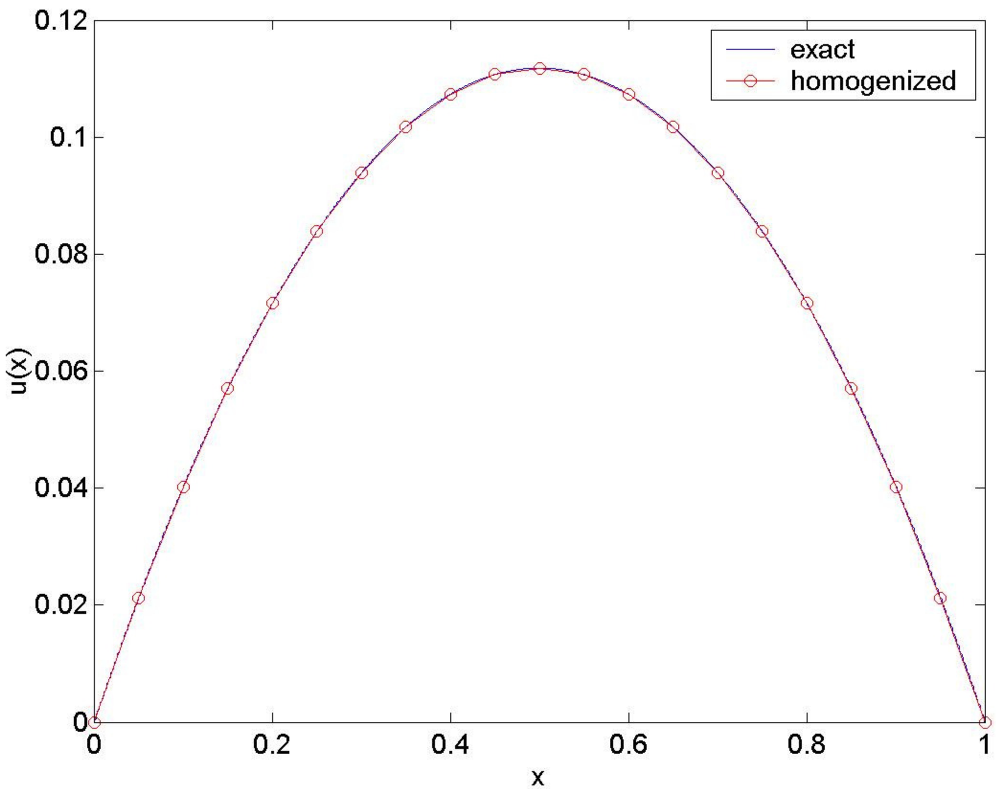
HOMOGENIZATION-EP

- Solution ε =1/32



HOMOGENIZATION-EP

- Solution ε =1/256



MODELAGEM MULTIESCALA: HOMOGENEIZAÇÃO

- Heterogeneous Materials
- Separation of Scales
- Homogenized Solution
- Examples

MODELAGEM MULTIESCALA: HOMOGENEIZAÇÃO

¡ Muchas Gracias !