HYBRID METHODS IN THERMAL & FLUIDS SCIENCE AND ENGINEERING WITH MULTIPHYSICS

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LABORATÓRIO DE TRANSMISSÃO E TECNOLOGIA DO CALOR

Métodos Inversos Aplicados a Ingeniería

INTEMA
Facultad de Ingeniería-UNMDP
Mar del Plata, Argentina, September 2007

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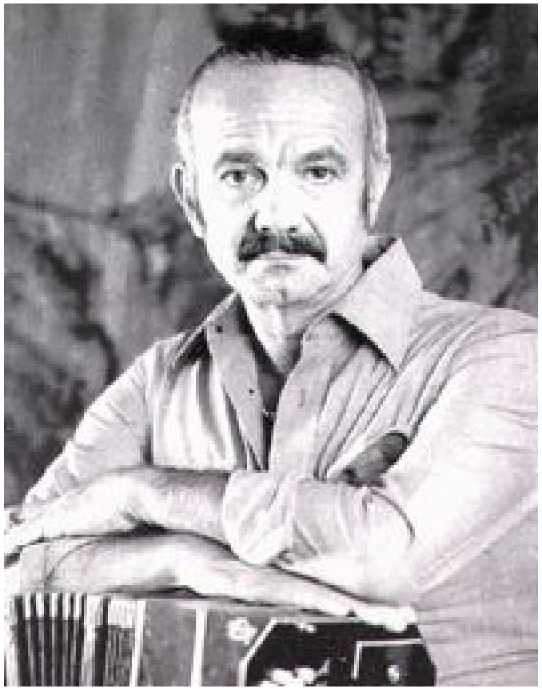
Lecture Contents

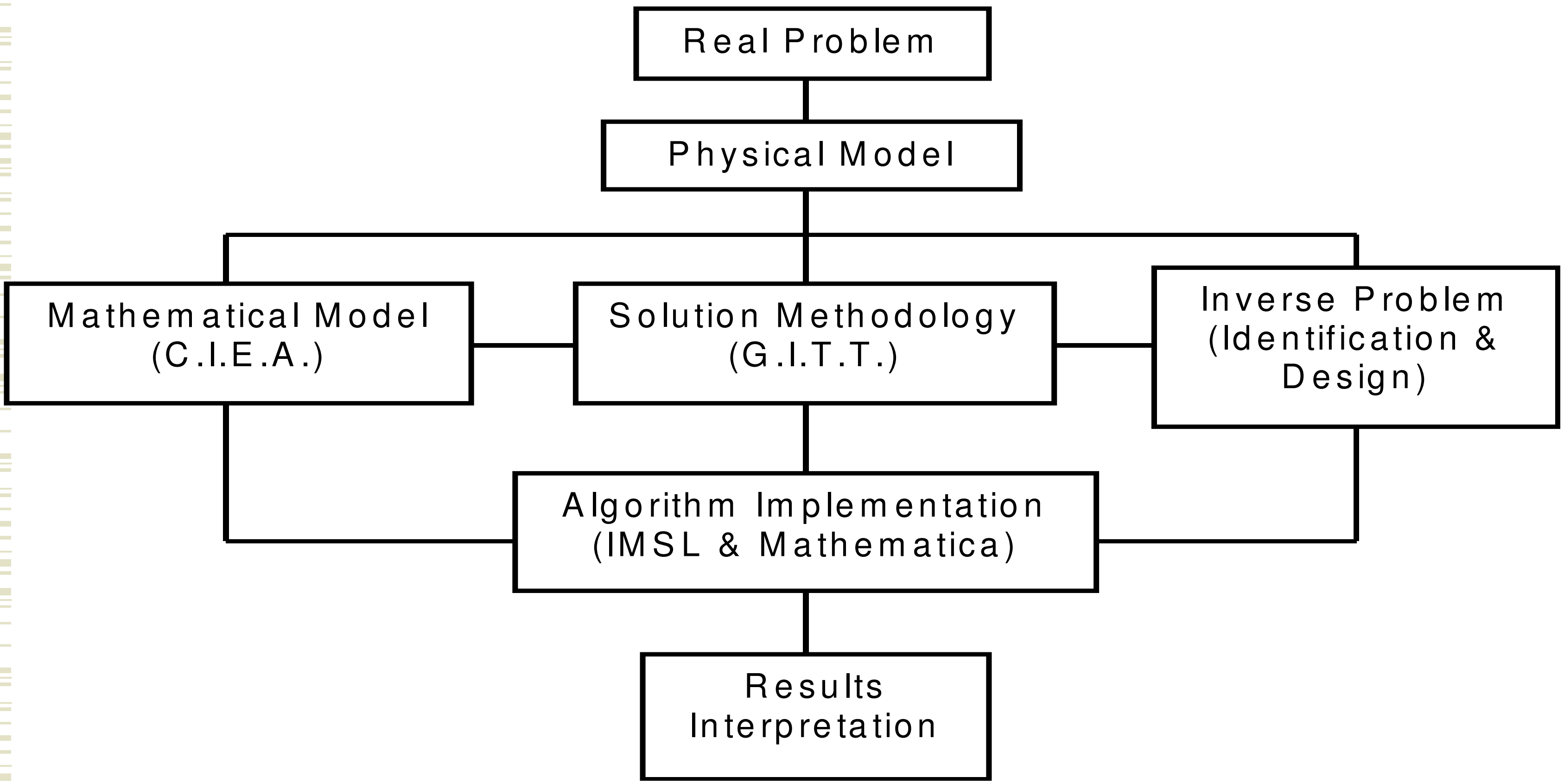
- **Part I**: Overview of Hybrid Methods and Recent Applications

- **Part II**: Tutorial on the Generalized Integral Transform Technique (GITT)



Part I - Hybrid Approach

THE SIMULATION PROCESS



Motivation

- Develop **improved lumped-differential** formulations in heat and fluid flow.
- Advance a **hybrid numerical-analytical** solution methodology for PDE's.
- Exploit new concepts on algorithm implementation, based on mixed **symbolic-numerical computation**.
- Construct new algorithms for **inverse problem** analysis based on such hybrid paths.

Hybrid Tools

- The **C**oupled **I**ntegral **E**quations **A**pproach (Improved Formulations) - **CIEA**
- The **G**eneralized **I**ntegral **T**ransform **T**echnique (Hybrid Methods) - **GITT**
- The *Mathematica* System (Mixed Computations)
- Inverse Problems (Identification & Design)

The Generalized Integral Transform Technique - GITT

- Choose the associated **eigenvalue problem**.
- Develop the **integral transform pair**.
- **Integral transform** the original PDE.
- Numerically (or analytically) **solve** the resulting coupled ODE system for **the transformed potentials**.
- Recall the analytical **inversion formula** to reconstruct the hybrid solution of the desired potential.

Classes of Problems
(Linear and Nonlinear)

- Diffusion
- Convection-Diffusion
- Eigenvalue Problems
- Boundary Layer Equations
- Navier-Stokes Equations

Advantages - GITT

- Time-consuming numerical task is always in one single independent variable (**ODEs**).
- Reasonably **simple** computational **implementation** (subroutines libraries).
- Handles **irregular domains** directly.
- Automatic **global error control**.
- Mild increase in computational cost for increasing number of space variables.

Total Transformation

PARABOLIC & PARABOLIC-HYPERBOLIC:

1 D- 3 D PDE $\xrightarrow{\int}$ **System of ODE's (IVP)**

DIVPAG/IMSL, NDSolve

ELLIPTIC:

2 D- 3 D PDE $\xrightarrow{\int}$ **System of ODE's (BVP)**

DBVPFD/IMSL

Partial Transformation

PARABOLIC & PARABOLIC-HYPERBOLIC:

2 D- 3 D PDE $\xrightarrow{\int}$ **1D – System of PDE's (DMOLCH/IMSL, NDSolve)**

ELLIPTIC:

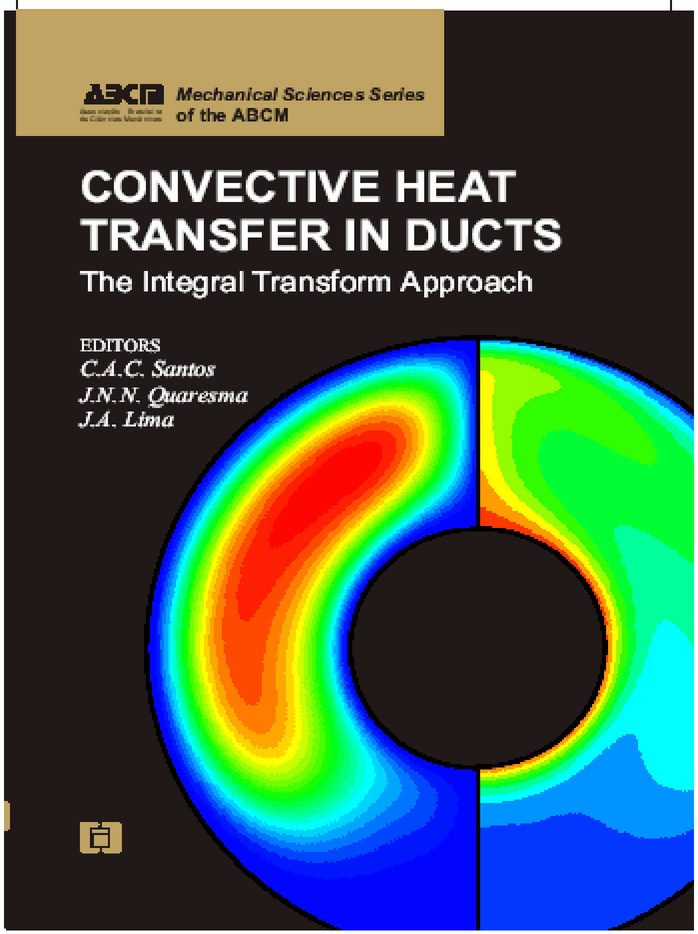
3 D PDE $\xrightarrow{\int}$ **2D – System of PDE's**

Convergence of Integral Transforms
Acceleration Techniques

- **Filtering**
  - Single
  - Multiple
  - Local-Instantaneous
- **Integral Balance**
  - A posteriori
  - A priori
  - Combined with Filtering
- **Series Transformation**
  - Aitkens/Lanczos
  - Shanks
  - Nonlinear Transf.

Benchmarks

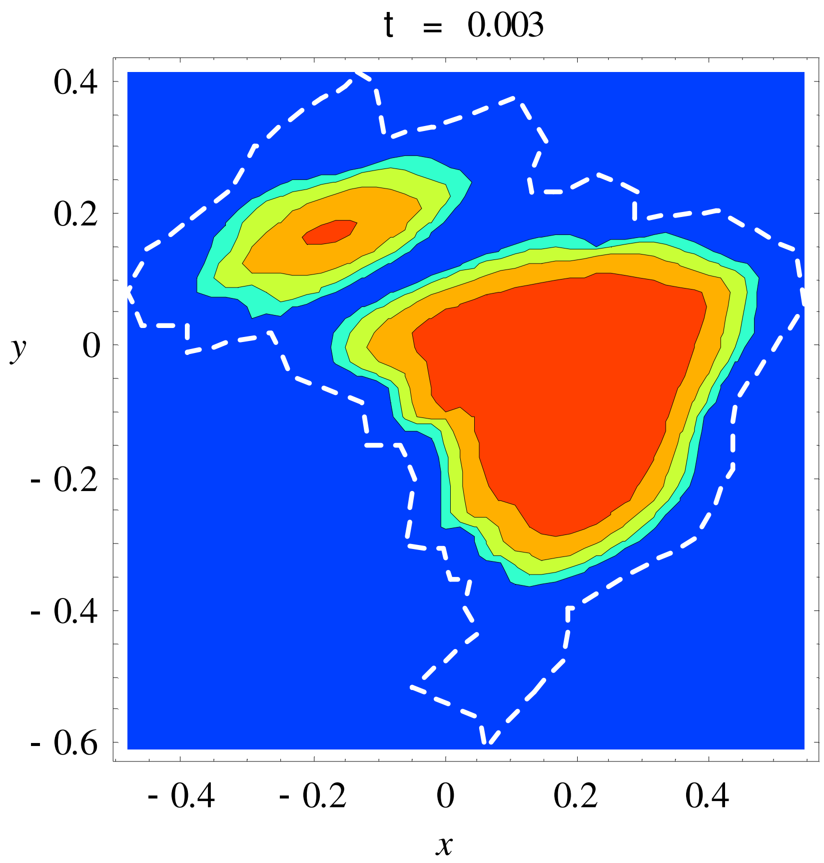
Cotta, R.M., and M.D. Mikhailov, "Hybrid Methods and Symbolic Computations", in: **Handbook of Numerical Heat Transfer**, 2nd edition, Chapter 16, Eds. W.J. Minkowycz, E.M. Sparrow, and J.Y. Murthy, John Wiley, New York, pp.493-522, 2006.



Application in Strategic Areas

- **Space**: Thermomechanical design of recoverable orbital platforms
- **Nuclear**: Optimization of ultracentrifuges for uranimum enrichment & Analysis of nuclear fuel with high burnup
- **Environmental:** Dispersion of waste from electricity generation within the paths soil-water-atmosphere & Environmental Impact Assessment from Mining and Milling Industries.
- **Natural Gas**: Rapid refueling of vehicular natural gas tanks & Adsorption storage in virtual gasodutes.
- **Petroleum**: Simulation of tracers injection in petroleum reservoirs & Analysis of Pipe-in-pipe designs for ultra-deep petroleum exploration.
- **Nanotechnology**: Fabrication, charactherization and convection behavior of nanofluids.

UNIT Project



I. Compilation and organization of available codes and developments

II. UNIT Code (Unified Integral Transforms) design and construction

III. Hybrid solutions for engineering problems with multiphysics

UNIT Project

Hybrid solutions for engineering problems with multiphysics

- **Environmental Modeling**: Fluid Flow, Mass Transfer, Heat Transfer, Solid Mechanics, Biochemistry, Risk Analysis.
- **Micro-Electro-Mechanical Systems (MEMS):** Fluid Flow, Electrodynamics, Mass Transfer, Heat Transfer, Biochemistry.
- **Nano-structured Materials (Solids and Fluids):** Fluid Flow, Heat Transfer, Mass Transfer, Solid Mechanics, Electromagnetism.
- **Emerging Energy Sources** (Natural Gas, Hydrogen, Solar and Nuclear Energy): Fluid Flow, Thermochemistry, Heat Transfer, Mass Transfer, Particle Transport.
- **Bioengineering Modeling** (Biofluids, Bioheat, and Tissue Engineering): Fluid Flow, Biochemistry, Heat Transfer, Mass Transfer, Biology, Solid Mechanics.

UNIT Project
Environmental Modeling:

- Contaminants dispersion in soils: unsaturaded, heterogeneous or fractured porous media with non-linear sorption effects and/or chain reactions;

- Flow and dispersion of chemicals in rivers and streams with groundwater and porous bed interactions;

- Atmospheric flow and pollutants dispersion simulation with soil deposition modeling;

INB-COPPE Project
Uranium Mining and Milling

Pond 2

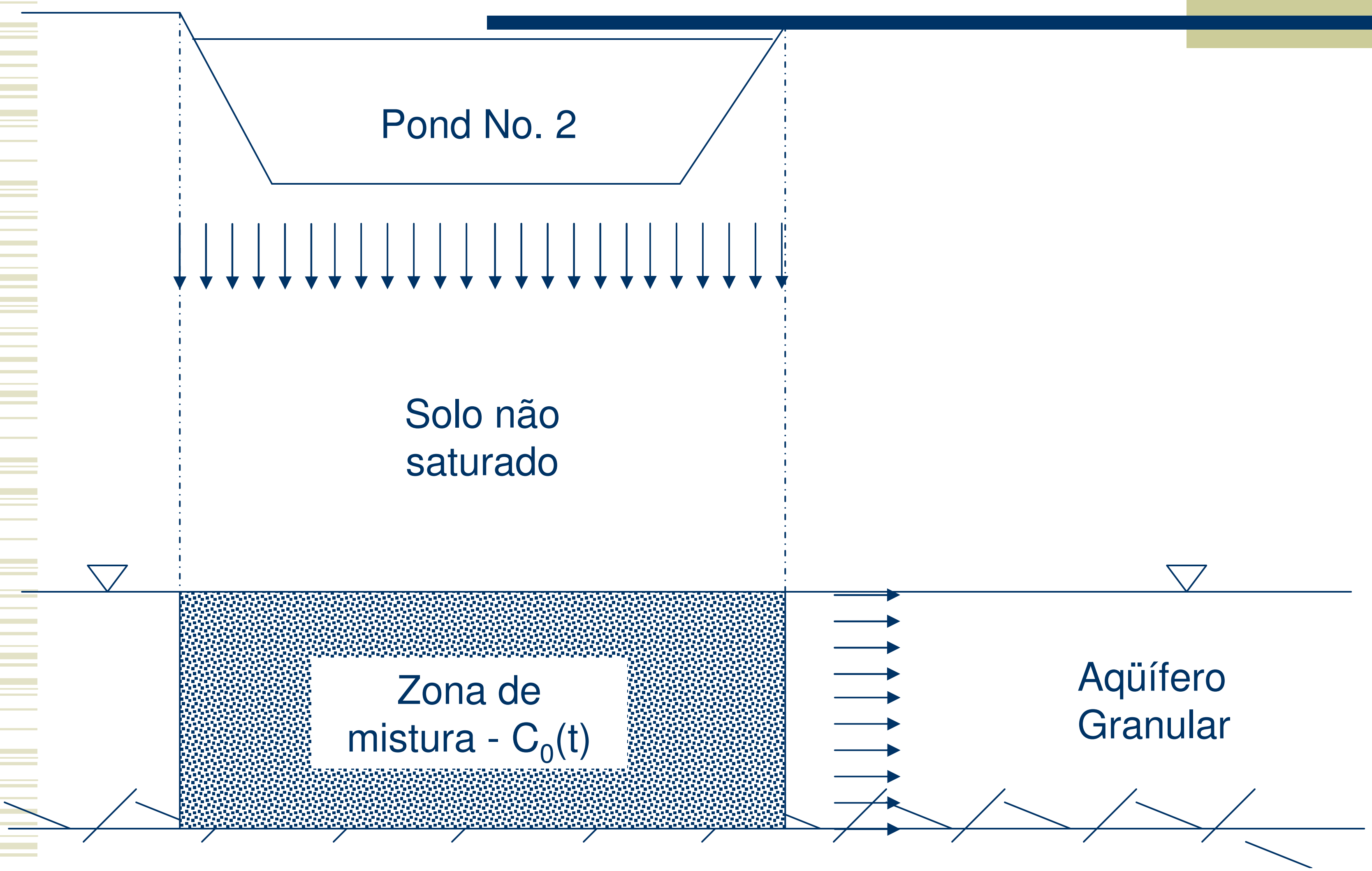
Source Term Characterization: Source Conditions

(c)

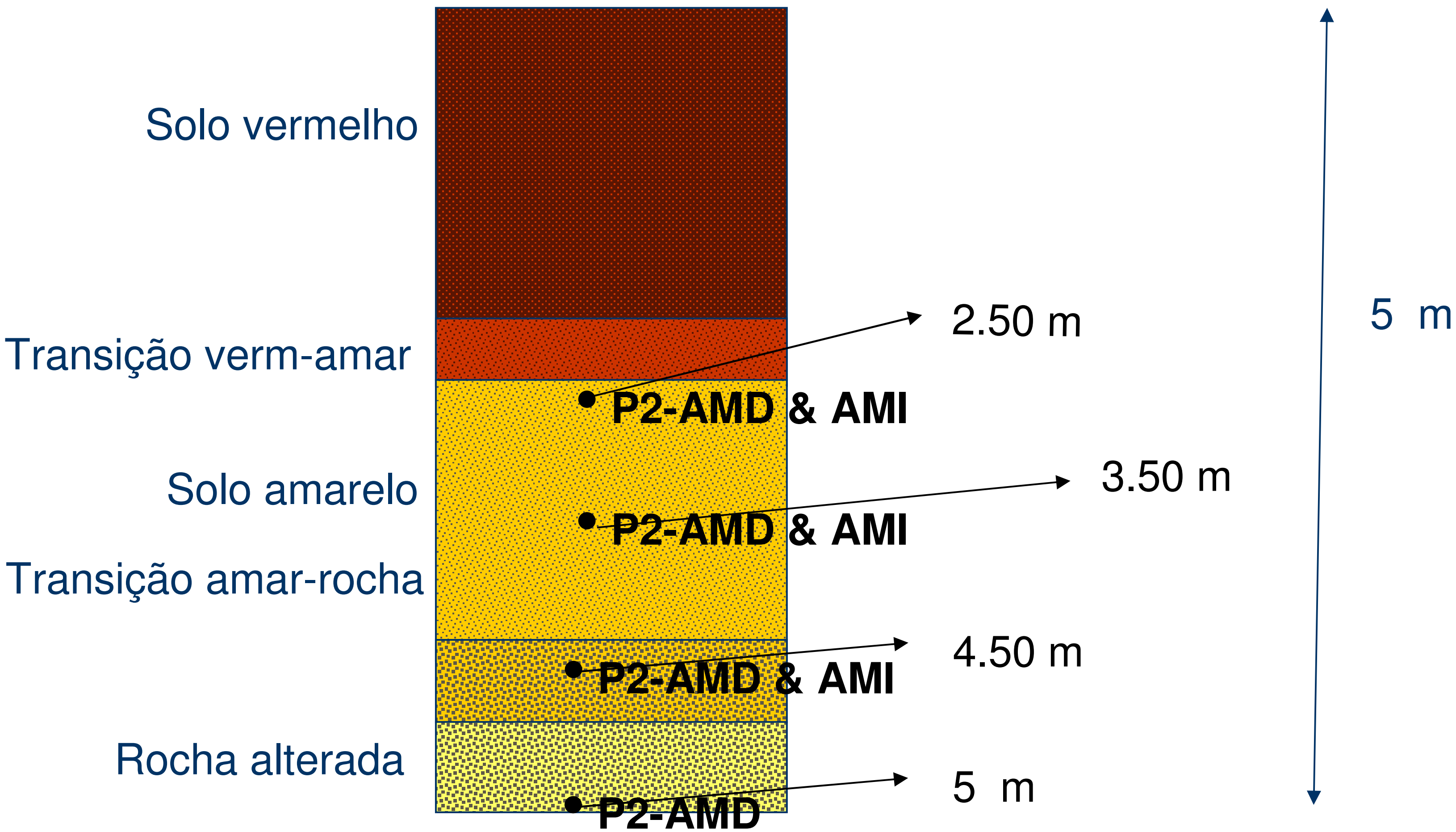
Figuras 8 – Fotografias da fase final de construção do pond no.2 a) escavação b)preparação do fundo c) compactação da camada de argila

Element	Inventory Pond 2	Inventory Pond 1
U	5.34×10^{11} Bq	6.43×10^{11} Bq
Th	1.68×10^{10} Bq	1.66×10^{10} Bq
Ra	5.51×10^{10} Bq	7.16×10^{10} Bq
Pb	6.16×10^{11} Bq	3.46×10^{11} Bq

Coupled Unsaturated Vertical Transport with Saturated Horizontal Transport



Soil Structure
Undisturbed Soil Sampling



Radionuclides Chain Dispersion in Unsaturated/Saturated Porous Media

$$\frac{\partial(\theta R_i C_i)}{\partial t} + \frac{\partial(q C_i)}{\partial x} = \frac{\partial}{\partial x}(\theta D \frac{\partial C_i}{\partial x}) - \mu_i(\theta + \rho K d_i) C_i(x,t)$$

$$+\mu_{i-1}(\theta + \rho K d_{i-1}) C_{i-1}(x,t), \ 0 < x < L, \ t > 0, \ i = 1,..,N_r$$

$$C_i(x,0) = 0 \quad , \quad 0 < x < L \quad , \quad i = 1,..,N_r$$

$$-\alpha \frac{\partial C_i}{\partial x} + C_i = f_i(t) \quad ou \quad C_i = f_i(t) \quad , \quad x = 0 \quad , \quad t > 0 \quad , \quad i = 1,..,N_r$$

$$\alpha \frac{\partial C_i}{\partial x} + h^* C_i = 0 \quad , \quad x = L \quad , \quad t > 0 \quad , \quad i = 1,..,N_r$$

Radionuclides Chain Dispersion: GITT Code validation

21. H.C. Lung, P.L. Chambré, T.H. Pigford, and W.W.L. Lee, **Transport of Radioactive Decay Chains in Finite and Semi-Infinite Porous Media**, Earch Sciences Division, Lawrence Berkeley Laboratory, Report LBL23987, 1987.

U234, Th230, e Ra226

$D = 50\ m^2/ano$, $\varepsilon = 3\ 10^{-3}$, $V = 1$ m/ano, R_1=120, R_2=1500, e R_3=300.

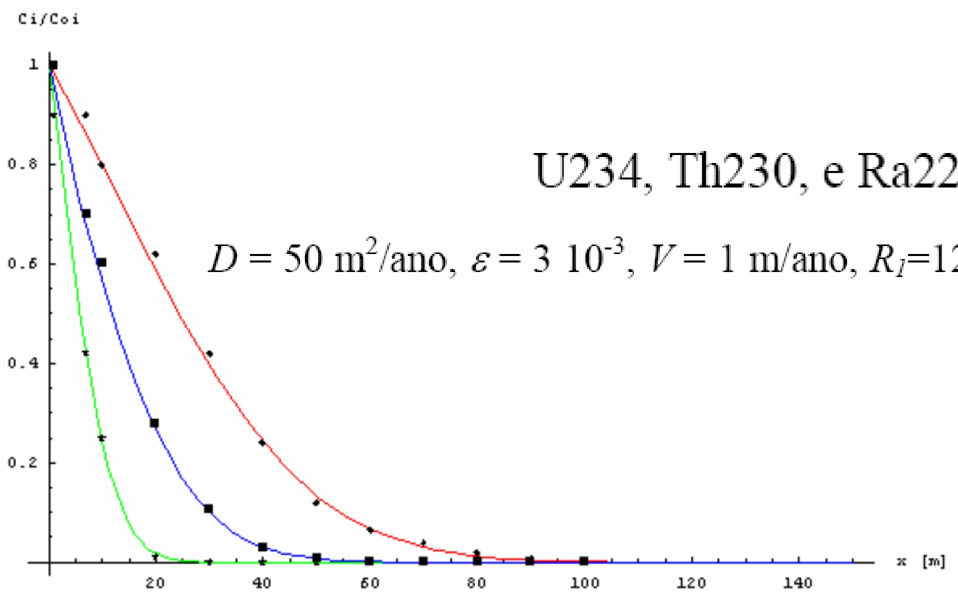


Figura 1.b – Comparação dos campos de concentração dos três radionuclídeos (U234, Th230, Ra226**) obtidos por GITT (linhas sólidas) e por transformada de Laplace, ref.[21] (símbolos), para um tempo de dispersão de 1000 anos.**

Radionuclides Chain Dispersion: GITT Convergence Analysis

Tabela 1.b – Convergência do campo de concentração do U234 obtido por GITT, para um tempo de dispersão de 1000 anos (linhas, posição x [m], colunas, N, ordem de truncamento) : Caso teste da ref.[21] (U234, Th230, Ra226)

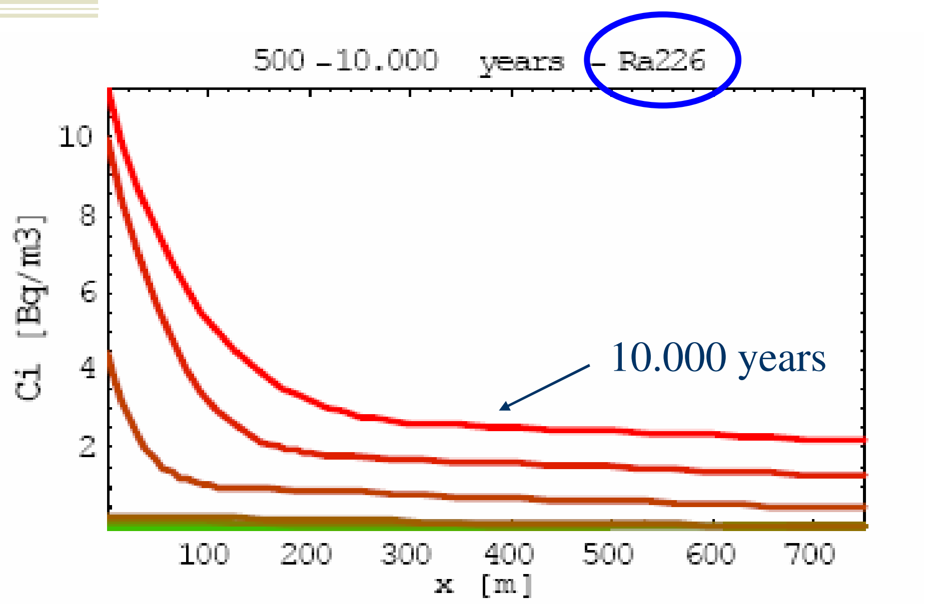
order N →

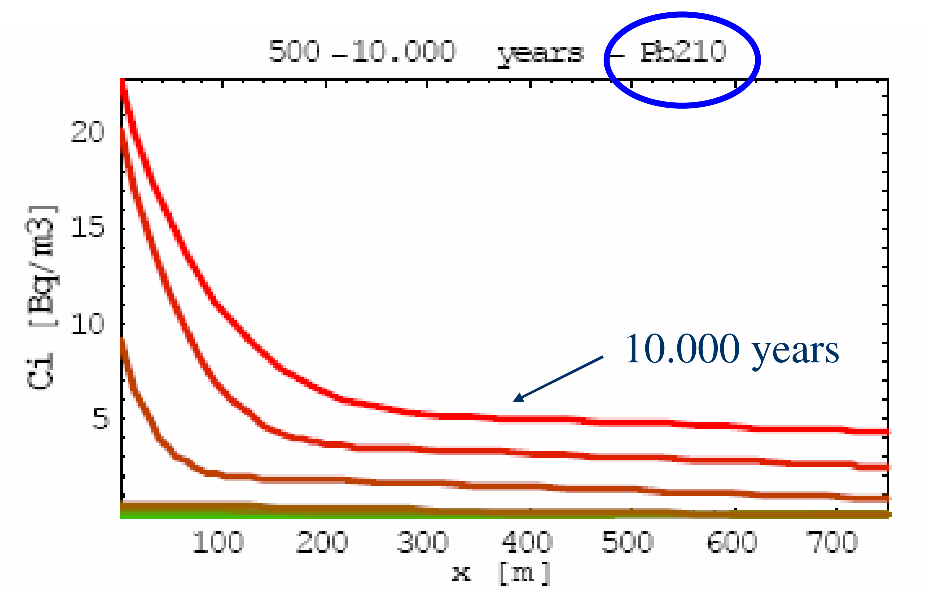
x [m] ↓

List	50	75	100	125	150
1	0.980879	0.980935	0.980955	0.980962	0.980963
7	0.860367	0.86039	0.860389	0.860386	0.860389
10	0.797314	0.797306	0.797301	0.7973	0.7973
20	0.585814	0.585807	0.585811	0.58581	0.58581
30	0.393687	0.393696	0.393693	0.393694	0.393694
40	0.240582	0.240578	0.240579	0.240579	0.240579
50	0.133059	0.133058	0.133057	0.133057	0.133057
60	0.0663583	0.0663609	0.0663615	0.0663618	0.0663619
70	0.0297639	0.0297631	0.0297624	0.0297621	0.0297621
80	0.0119769	0.0119747	0.0119757	0.0119756	0.0119755
90	0.00431287	0.00431632	0.0043152	0.0043157	0.00431543
100	0.00139205	0.00139003	0.00139092	0.00139085	0.00139068

Results – Different Radionuclides Migration Behavior

Pb & Ra Concentrations [Bq/m3] along horizontal layer (t=500 to 10.000 years, from green to red, with vertical layer):

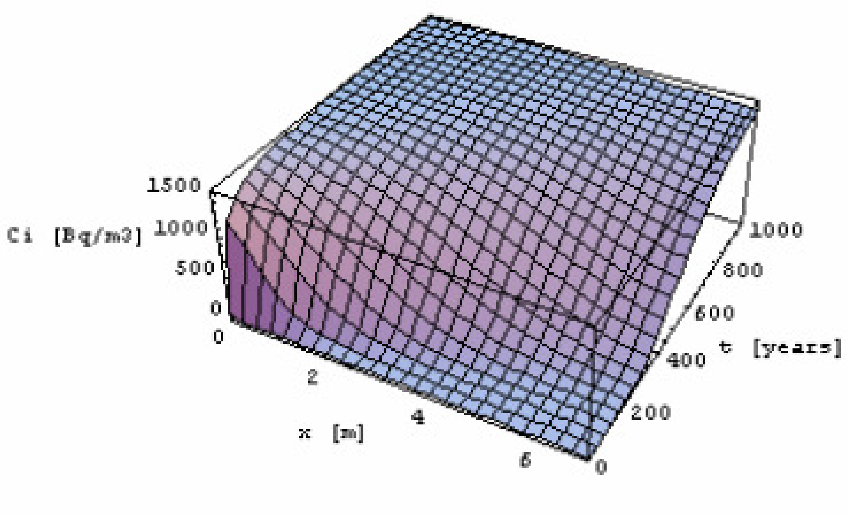




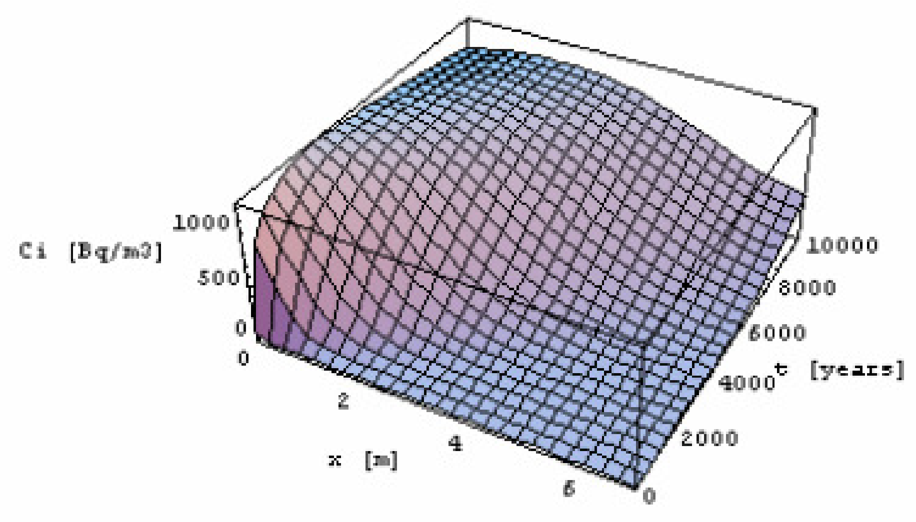
Results – Influence of Transport Properties Identification

Uranium Concentrations [Bq/m3] along vertical layer (different retardation, Kd):

Kd for sandy soil



Kd for clay soil



UNIT Project

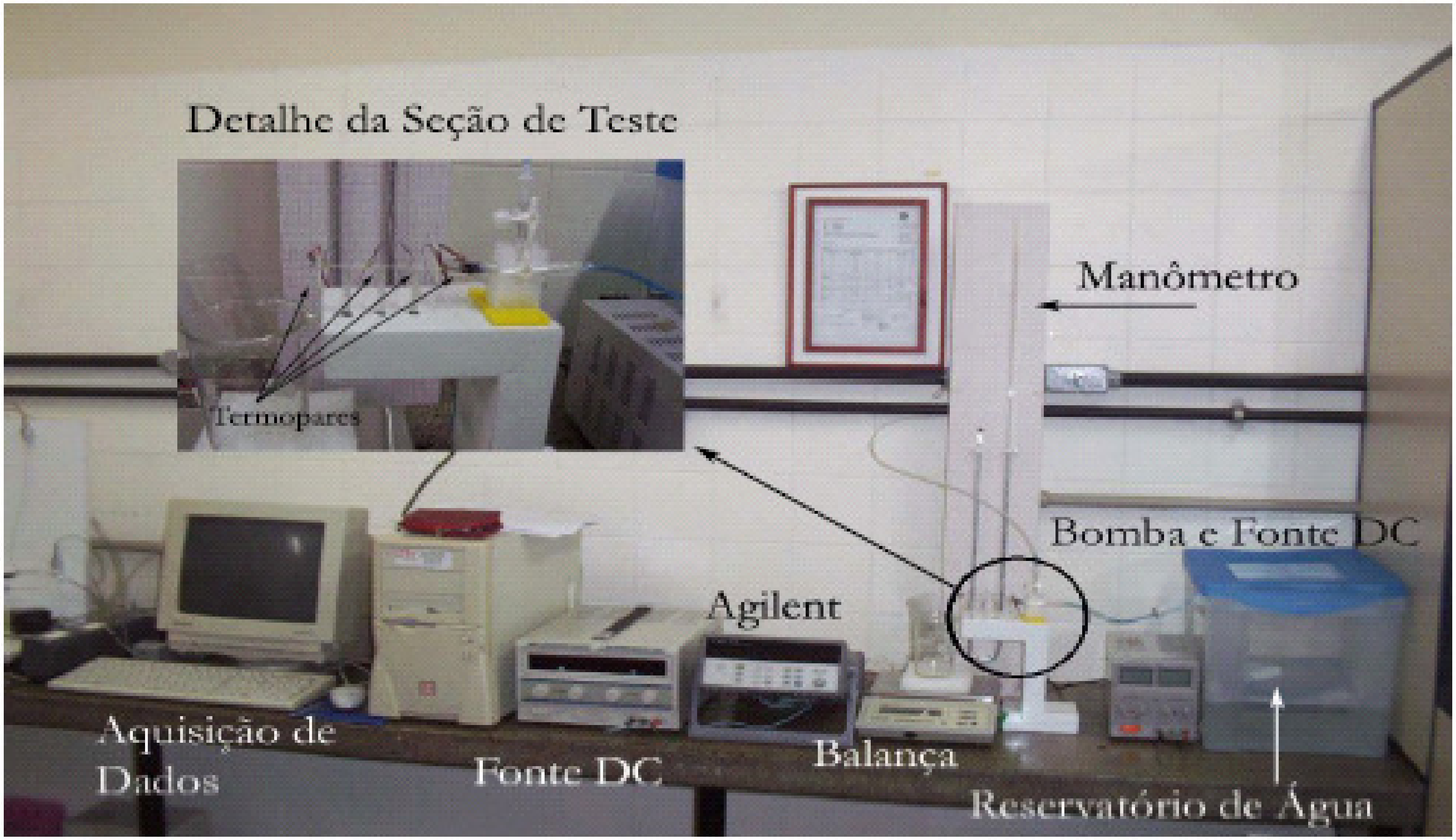
Micro-Electro-Mechanical Systems

- **Roughness effects in heat transfer enhancement in micro-channels;**

- **Analysis of electro-osmotic flows with heat transfer;**

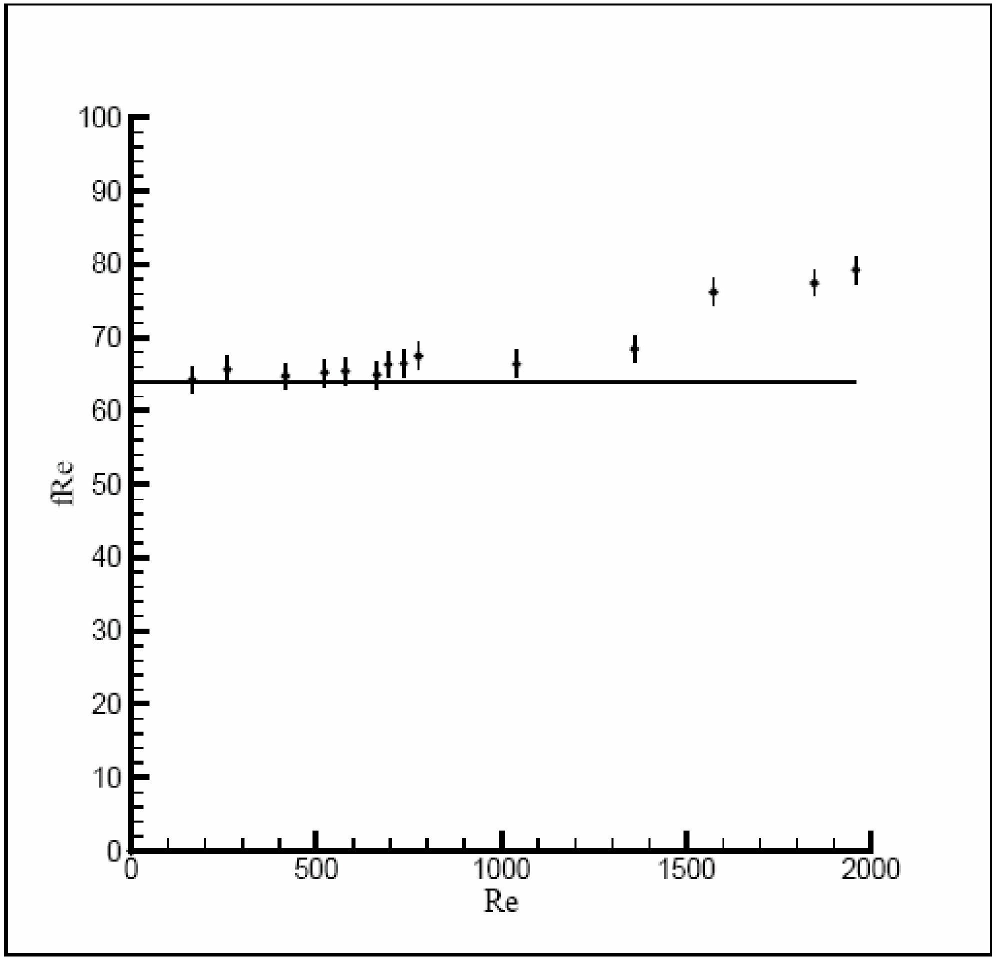
- **Simulation of reactive flow with heat and mass transfer within microchannels;**

Experiments with Microchannels
Experimental Setup

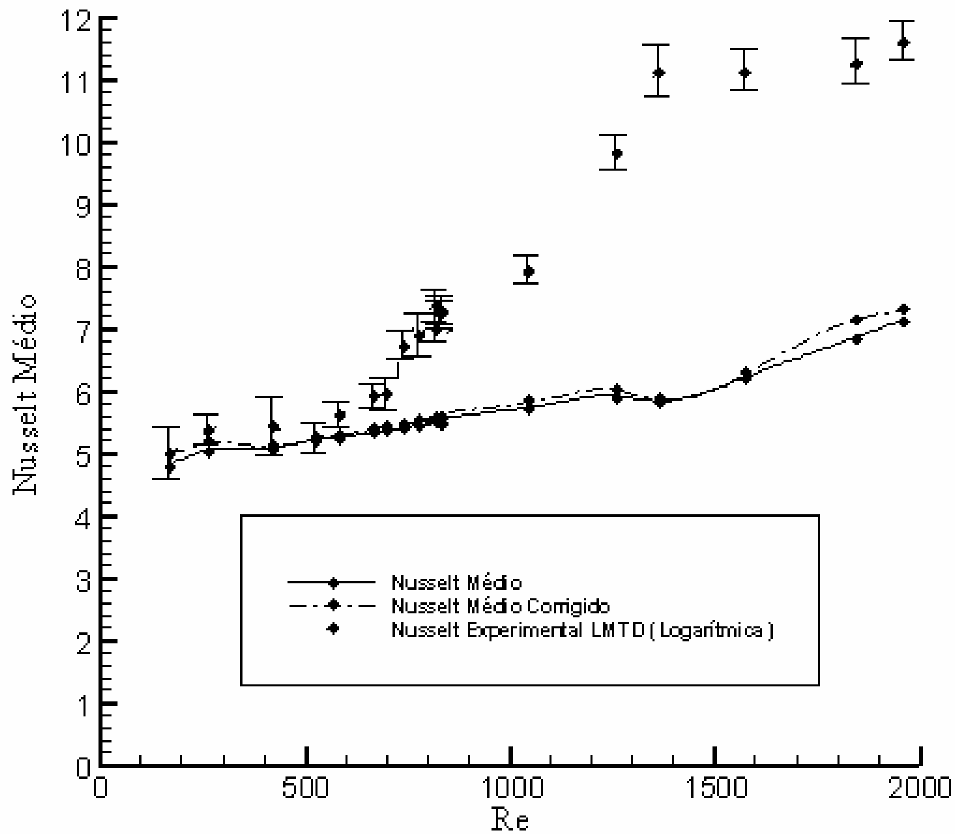


Experiments with Microchannels

Results – MICROCHANNEL (280 μm)



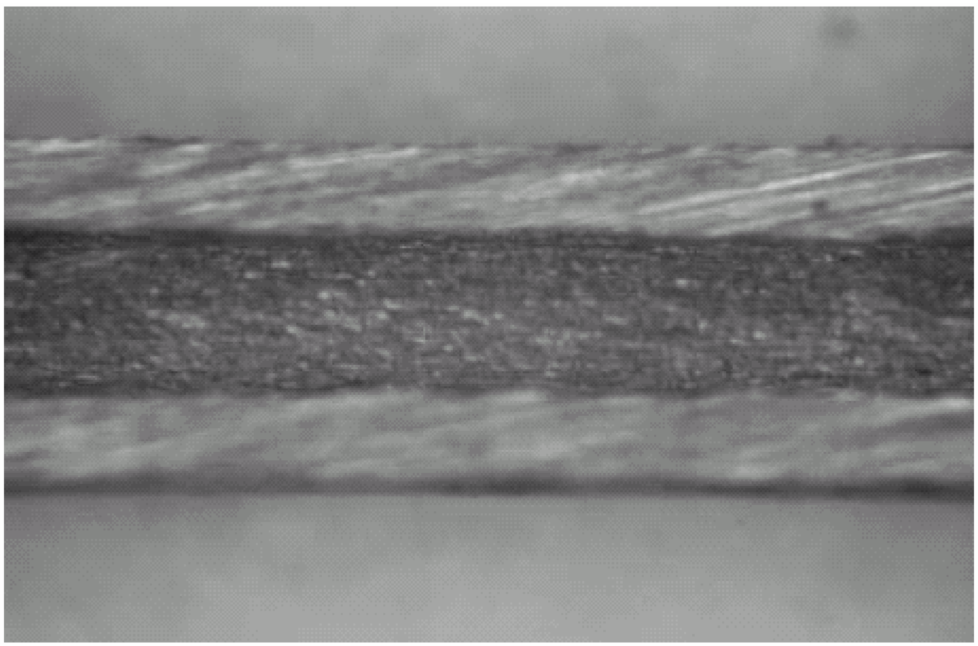
Experiments with Microchannels
Results – MICROCHANNEL (280 μm)

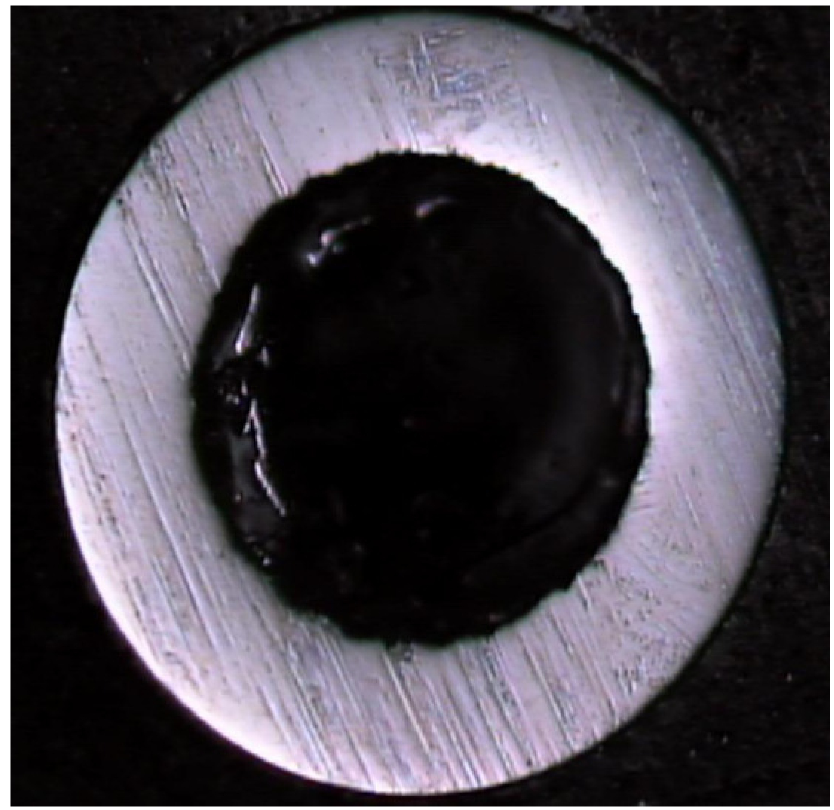


Experiments with Microchannels

Roughness – MICROCHANNEL (280 μm)

;





Experiments with Macrochannels
Roughness Effects – Laminar Flow

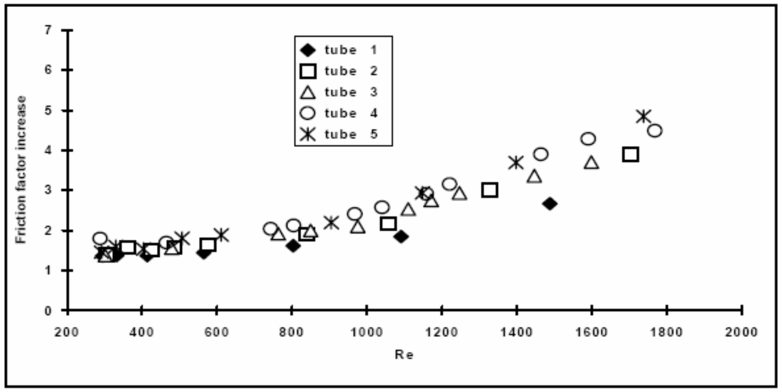


Figure 10- Friction factor increase versus Reynolds number

fRe

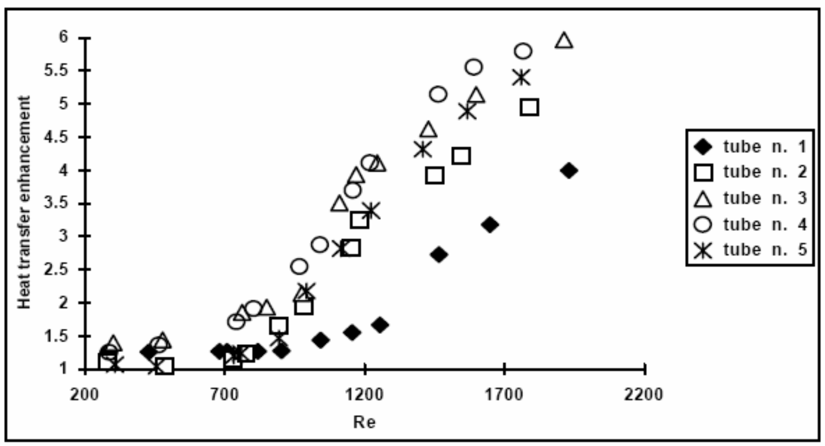


Figure 8- Heat transfer enhancement

Nu

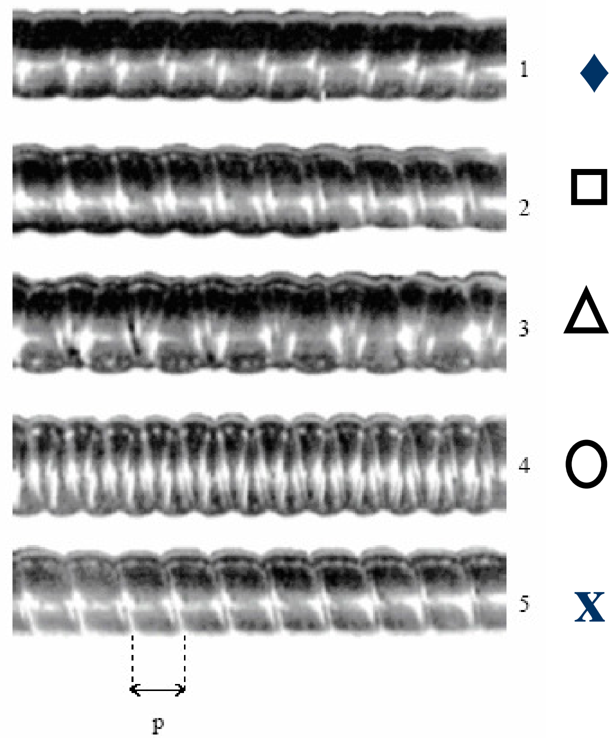


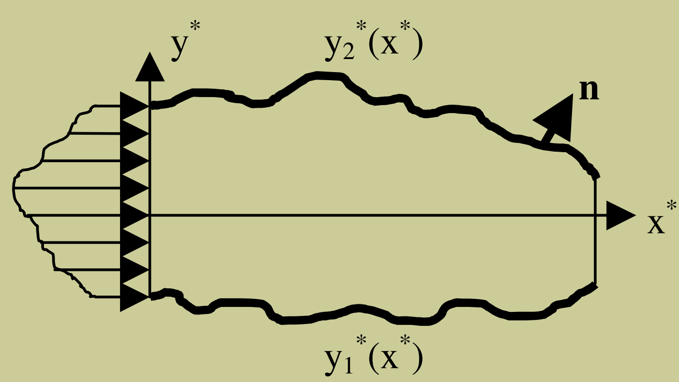
Figure 1- Tubes tested

Streamfunction Formulation
Navier-Stokes Equations

2D steady-state

$$\frac{\partial \psi}{\partial y}\left(\frac{\partial^3\psi}{\partial x^3}+\frac{\partial^3\psi}{\partial x \partial y^2}\right)-\frac{\partial \psi}{\partial x}\left(\frac{\partial^3\psi}{\partial x^2 \partial y}+\frac{\partial^3\psi}{\partial y^3}\right)=\frac{1}{Re}\left(\frac{\partial^4\psi}{\partial x^4}+2\frac{\partial^4\psi}{\partial x^2 \partial y^2}+\frac{\partial^4\psi}{\partial y^4}\right)$$

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$



General irregular geometry and coordinates system for channel flow.

Primitive Variables Formulation

Navier-Stokes Equations

$$\frac{\partial U(X,Y)}{\partial X}+\frac{\partial V(X,Y)}{\partial Y}=0, \qquad X>0,\quad 0<Y<1$$

$$U\frac{\partial U(X,Y)}{\partial X}+V\frac{\partial U(X,Y)}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \quad X>0,\quad 0<Y<1$$

$$\frac{\partial^2 P(X,Y)}{\partial X^2}+\frac{\partial^2 P(X,Y)}{\partial Y^2}=2\left[\frac{\partial U}{\partial X}\frac{\partial V}{\partial Y}-\frac{\partial V}{\partial X}\frac{\partial U}{\partial Y}\right], \quad X>0,\quad 0<Y<1$$

$$\frac{\partial P(X,1)}{\partial Y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial Y^2}\right)_{Y=1}$$

Mixed Formulation
Navier-Stokes and Energy Equations

$$\frac{\partial u_F}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad where,\quad u(x,y)=u_F(x,y)+u_\infty(y)$$

$$u_F\frac{\partial^2 u_F}{\partial x\partial y}+u_\infty\frac{\partial^2 u_F}{\partial x\partial y}+v\frac{\partial^2 u_F}{\partial y^2}+v\frac{d^2 u_\infty}{dy^2}-u_F\frac{\partial^2 v}{\partial x^2}-u_\infty\frac{\partial^2 v}{\partial x^2}-v\frac{\partial^2 v}{\partial x\partial y}$$
$$=\frac{4}{Re}\left(\frac{\partial^3 u_F}{\partial x^2\partial y}+\frac{\partial^3 u_F}{\partial y^3}-\frac{\partial^3 v}{\partial x^3}-\frac{\partial^3 v}{\partial x\partial y^2}\right)$$

$$u_F\frac{\partial T}{\partial x}+u_\infty\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{4}{Re}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

$$u_F(x,y)=\sum_{i=1}^{\infty}\tilde{Y}_i'(y)\bar{u}_i(x) \qquad v(x,y)=-\sum_{i=1}^{\infty}\tilde{Y}_i(y)\frac{d\bar{u}_i(x)}{dx} \qquad T(x,y)=\sum_{i=1}^{\infty}\tilde{\Gamma}_i(y)\bar{T}_i(x)$$

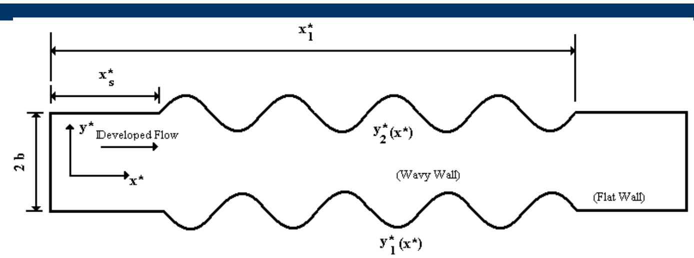
S.F. *x* P.V. *x* Mixed Formulations
Regular Channel

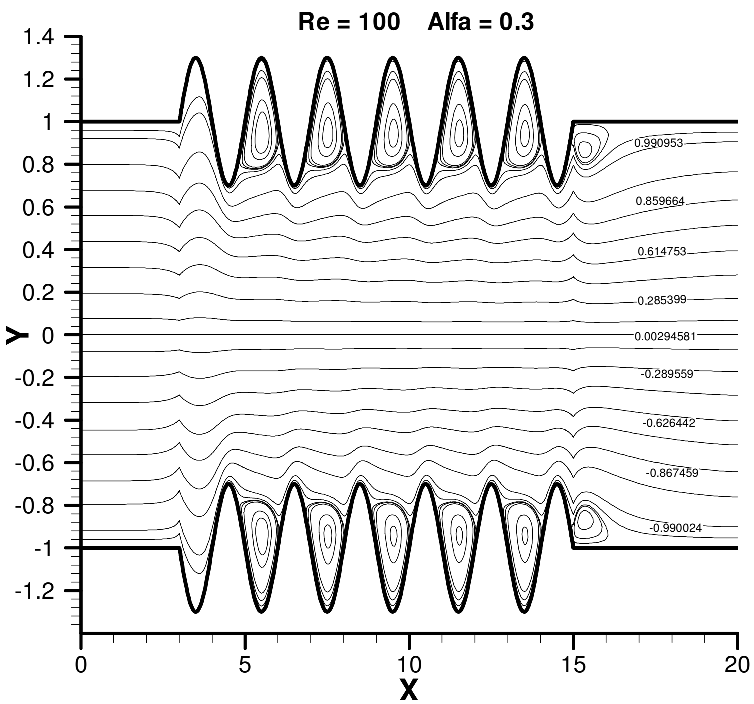
Table 1 – Covalidation of centerline velocity along channel length, *U(X,0)*, between the primitive variables [27] and the streamfunction [18] formulations. Relative error control 10^{-4}.

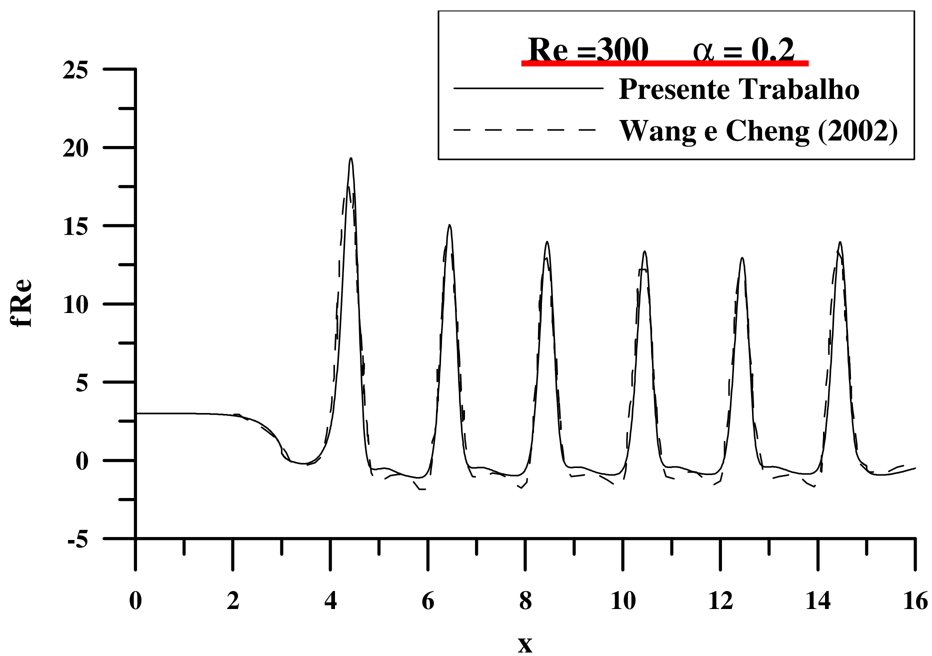
Re	Formulation	x = 0.2083	x = 3.3333	x = 7.5000
300	Primitive Variables	1.050	1.334	1.444
300	Streamfunction	1.052	1.337	1.444
300	Mixed	1.052	1.337	1.444
600	Primitive Variables	1.039	1.243	1.348
600	Streamfunction	1.036	1.242	1.347
600	Mixed	1.036	1.242	1.347
1200	Primitive Variables	1.026	1.173	1.252
1200	Streamfunction	1.024	1.170	1.250
1200	Mixed	1.024	1.170	1.250

Streamfunction Formulation

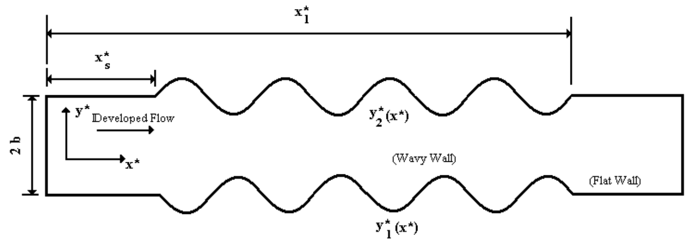
Irregular Wavy Channel

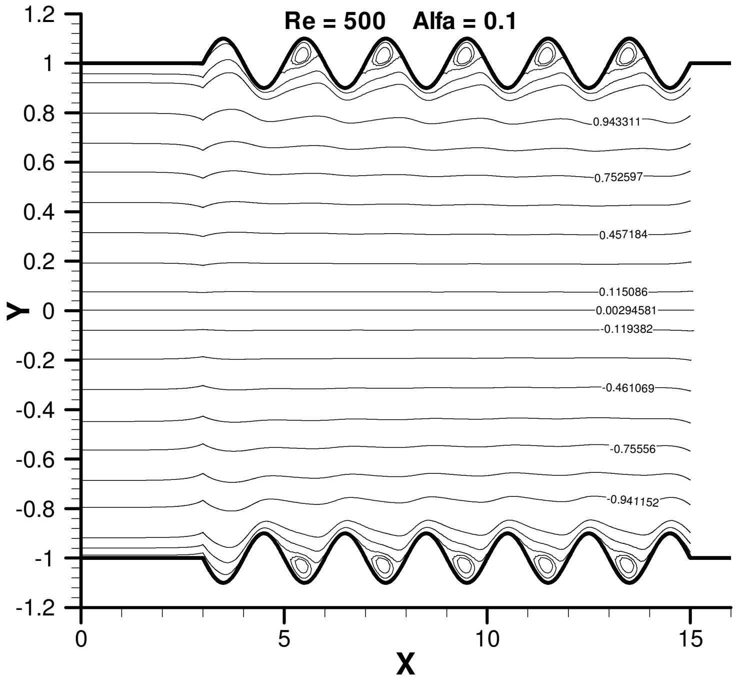


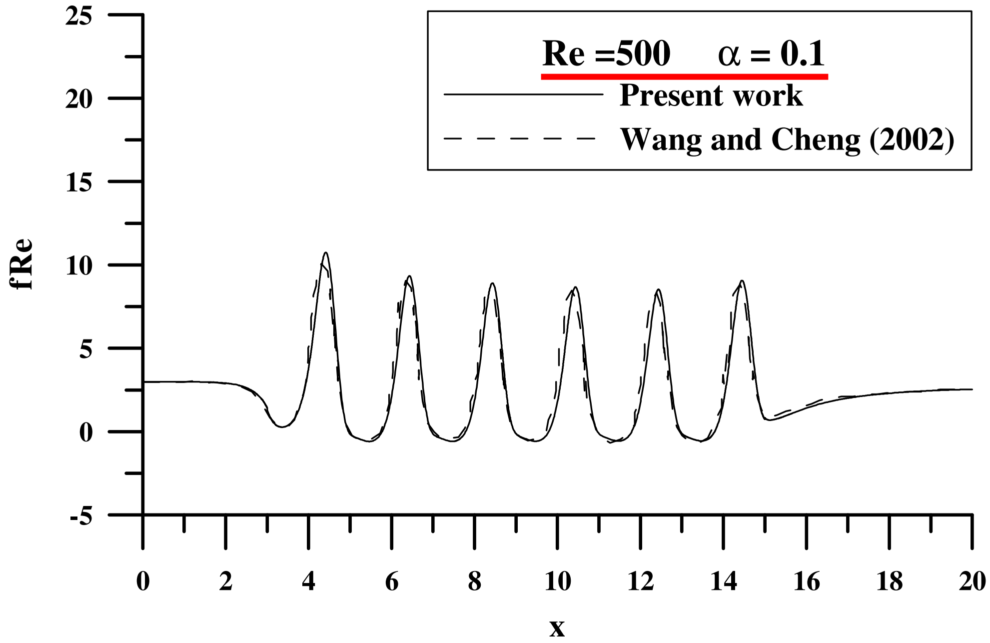




Streamfunction Formulation
Irregular Wavy Channel







Streamfunction Formulation

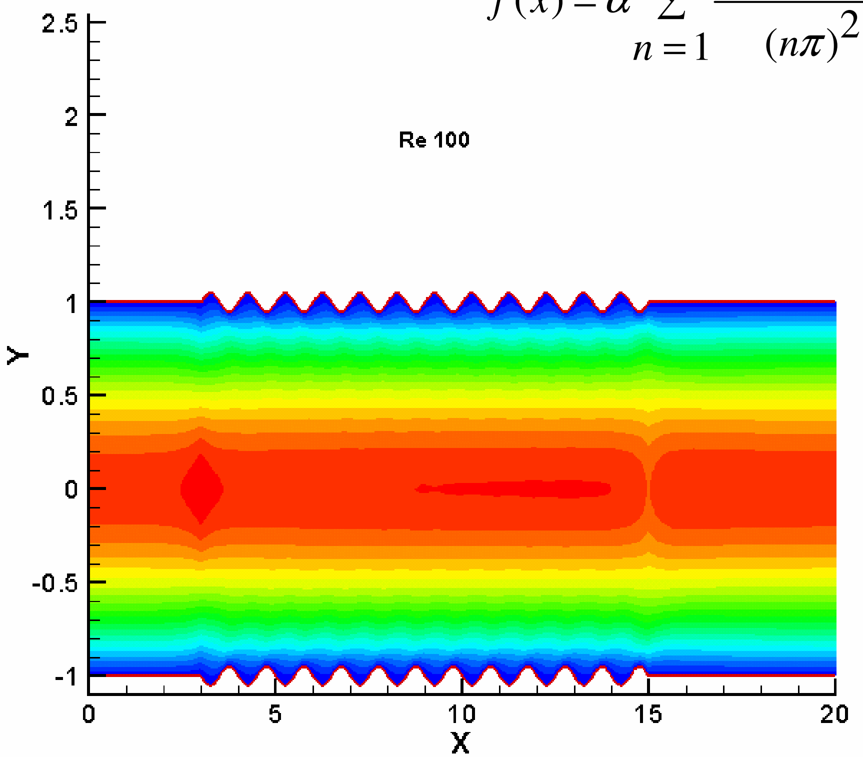
Convergence Behavior of the Streamfunction at $y = 0.5$ for $Re = 100$ and $\alpha = 0.2$.

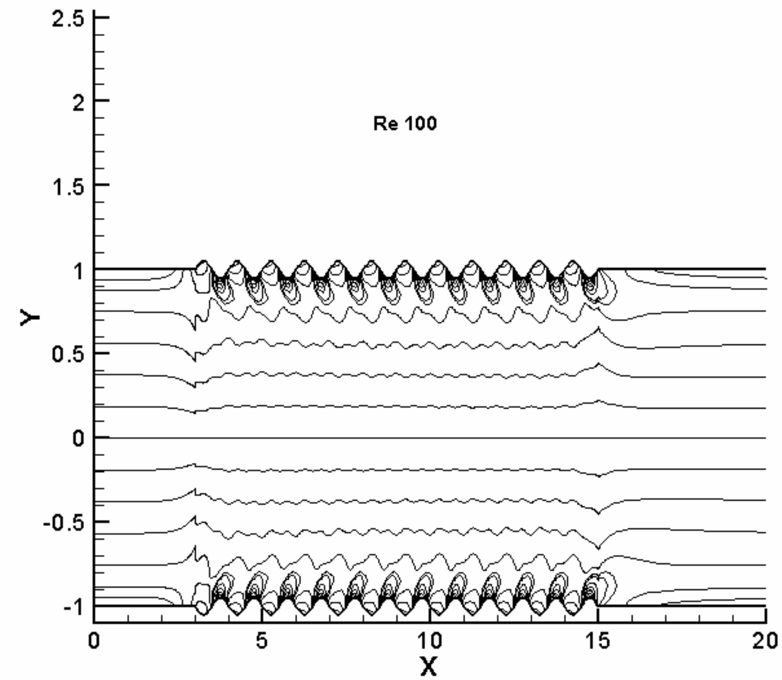
x	N →				
	6	10	14	18	30
3.5	0.6757	0.6761	0.6762	0.6763	0.6763
5.5	0.7197	0.7268	0.7275	0.7276	0.7276
7.5	0.7341	0.7394	0.7401	0.7401	0.7401
9.5	0.7408	0.7457	0.7463	0.7463	0.7463
11.5	0.7445	0.7493	0.7498	0.7499	0.7499
15	0.7167	0.7204	0.7211	0.7212	0.7212
20	0.7147	0.7157	0.7159	0.7159	0.7159

Streamfunction Formulation

Rough Micro-Channel Simulation: Streamfunction
at $y = 0.5$ for $Re = 100$, $\alpha = 5\%$ and $\omega = 2\pi$

$$f(x) = \alpha \sum_{n=1}^{5} \frac{8\,\text{sen}(n\pi/2)}{(n\pi)^2} \text{sen}[n\omega(x-3)]$$





Streamfunction Formulation

Rough Micro-Channel Simulation: Streamfunction at $y = 0.5$ for $Re = 100$, $\alpha = 5\%$ and $\omega = 2\pi$

N	x = 0	x = 3	x = 9	x = 15	x = 18
6	.68750	.71648	.69453	.66659	.69096
10	.68750	.71677	.69537	.66710	.69107
14	.68750	.71681	.69568	.66733	.69112
18	.68750	.71681	.69570	.66736	.69114

UNIT Project

Nano-structured Materials (Solids and Fluids):

- Modeling, characterization and simulation of nanofluids for energy, petroleum and natural gas sectors;

- Thermal and structural modeling and characterization of nano-structured composites for aerospace thermal protection systems;

Nanofluids Project – UFRJ/Petrobras

Nanofluids for Energy Efficiency in the Natural Gas & Petroleum Sector

- COPPE/UFRJ – Lab. of Transmission & Technology of Heat

- CENPES - Petrobras Research Center

- INMETRO – National Institute of Metrology

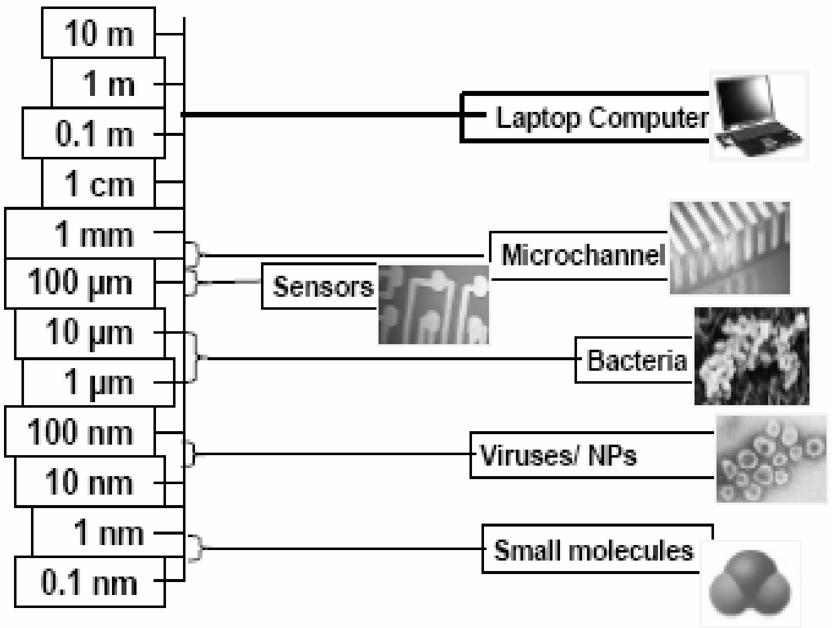
PETROBRAS

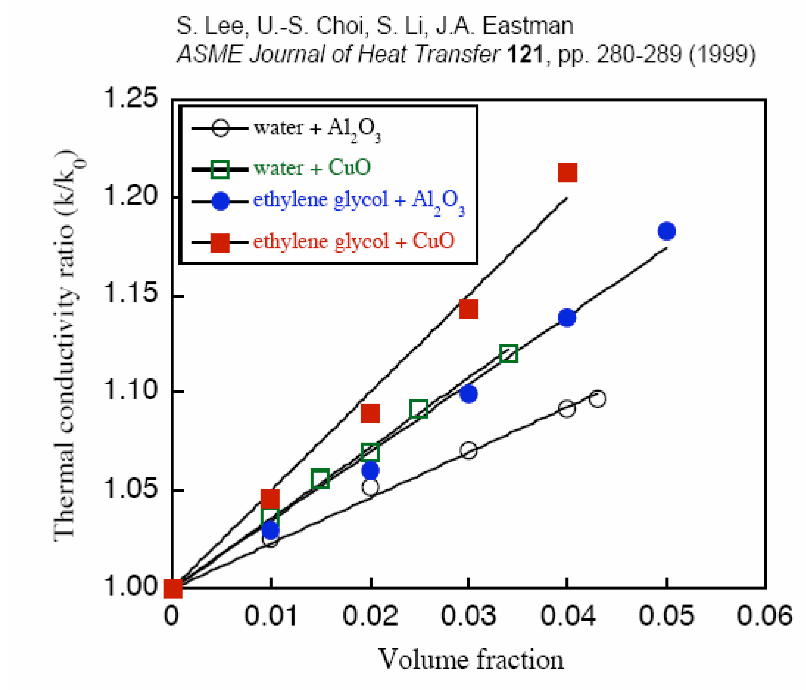
LABORATÓRIO DE TRANSMISSÃO E TECNOLOGIA DO CALOR

INMETRO

Concept

The term nanofluid has been created by S. Choi, Argonne National Lab, USA, to describe the two-phase mixture (solid-liquid) in which the disperse phase are nanoparticles of metals or metalic oxides, in general smaller than 100 nm.





Fabrication - Cooperation with DIMAT/INMETRO

Two steps method

Nanoparticles

Formulation

Nanoparticles: Al2O3 e CuO
Base-fluids: Water (miliQ) and Ethilene-glicol
Method: Ultrasonic vibration + Dispersant

Processing (ultra-sound)

Caratherization and tests

Fabrication - Cooperation with DIMAT/INMETRO
Materials employed

Nanoparticles: Al_2O_3 and CuO

	Granulometria (nm)	**Área superficial (m^2/g)**	**Pureza**	**Densidade**	**Produtor**
γ-Al_2O_3	20 – 30	180	99,97%	3,97 g/cm^3	*Nanostructured & Amorphous Materials Houston, USA*
α-Al_2O_3	30 – 40	/	99 %	3,97	*Nanostructured & Amorphous Materials Houston, USA*
CuO	30 – 50	131	99,97%	6.4	*Nanostructured & Amorphous Materials Houston, USA*

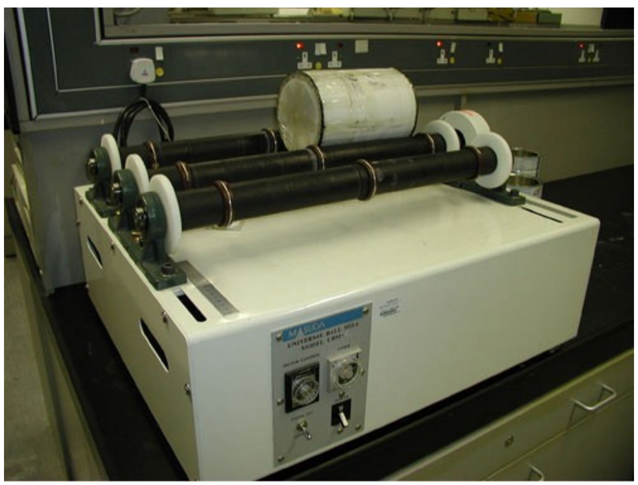
Especificações declaradas pelo próprio fabricante

Base-fluids: Water (miliQ) and Ethilene-glicol

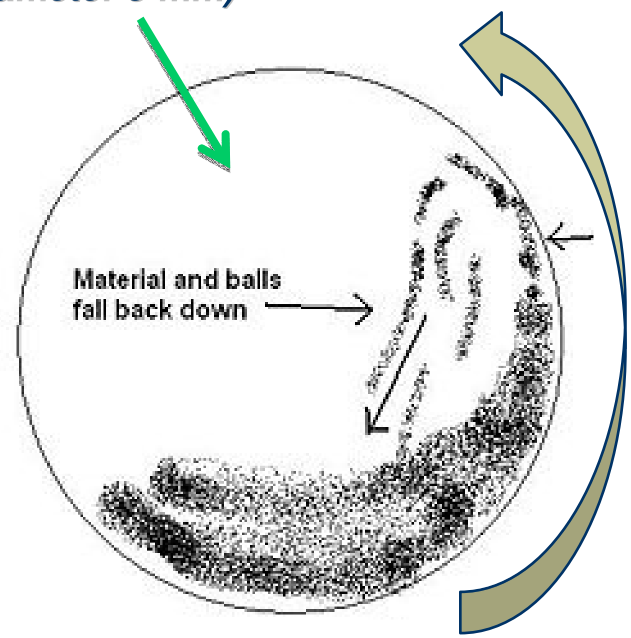
	Pureza		**Produtor**
H_2O Milli-Q	99,99%		*Milli-Pore*
Etileno Glicol	99,5 %		*VETEC QUÍMICA FINA LTDA*

Fabrication - Cooperation with DIMAT/INMETRO
Preparation of Nanofluids for Convection Experiment

Ball Milling

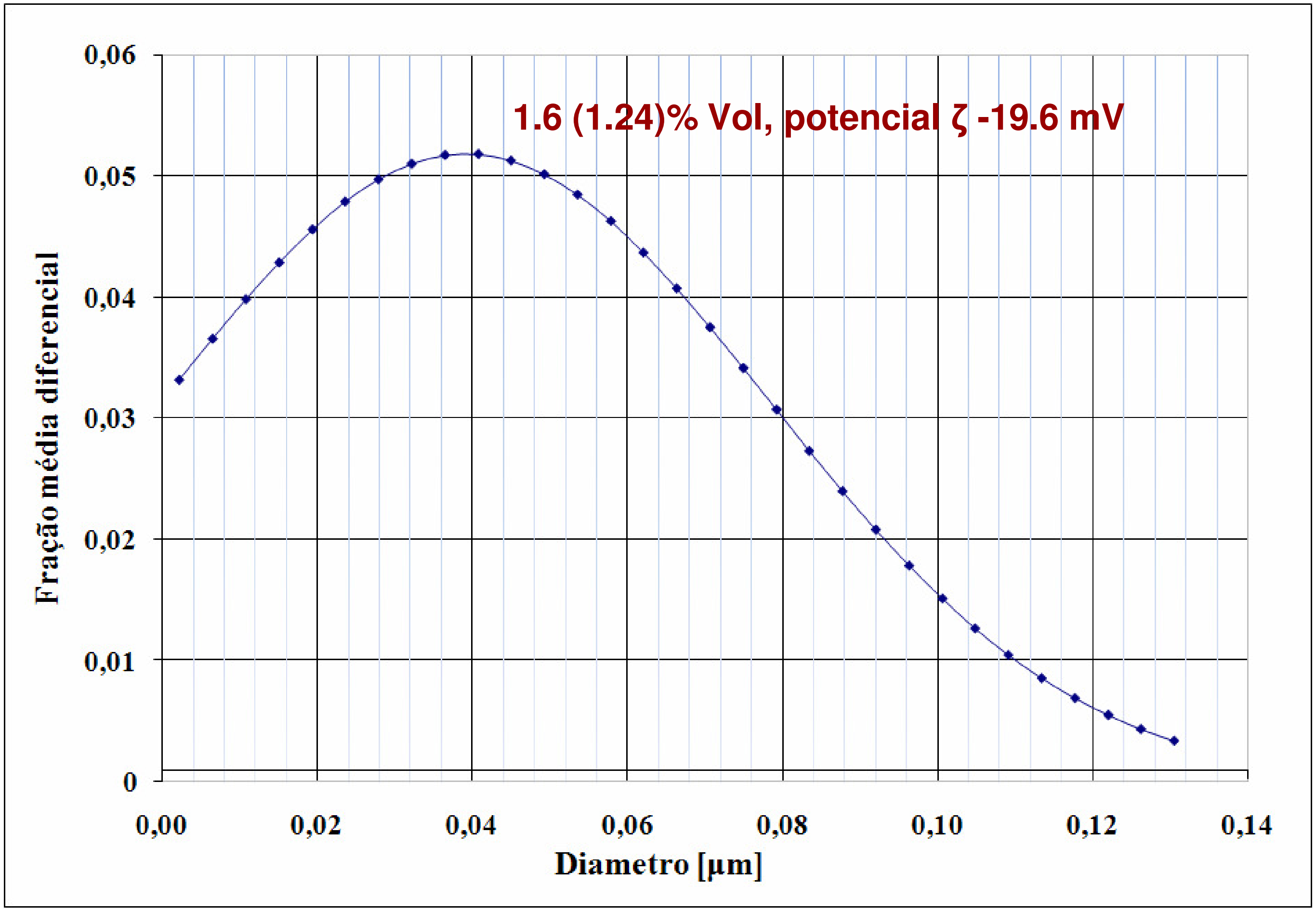


Powder with dispersant and spheres (diameter 5 mm)



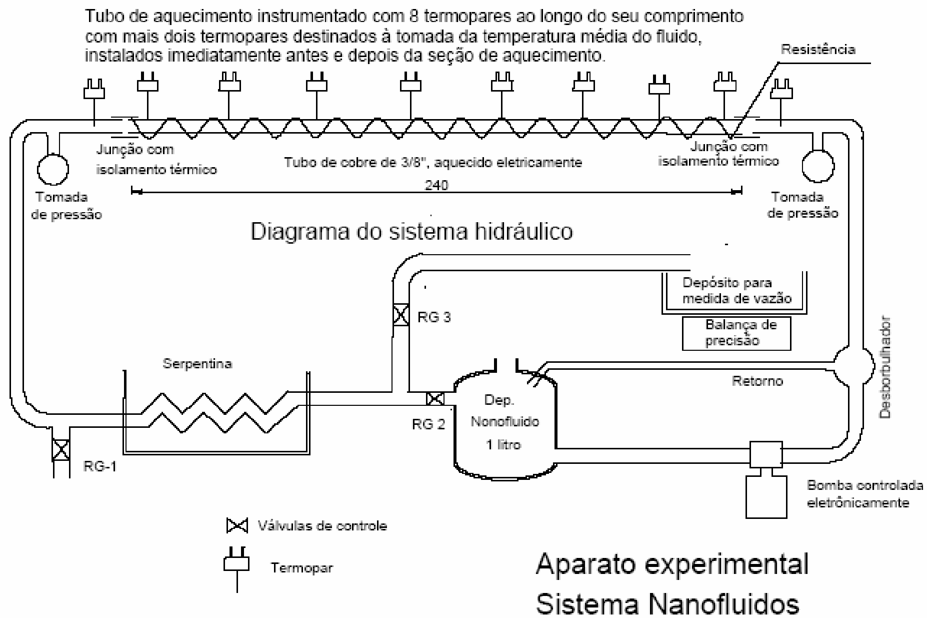
Nanofluids in Forced Convection

Zeta Potential and Particle Size Distribution (Al2O3-water)



Nanofluids in Forced Convection

Experimental Setup



Nanofluids in Forced Convection

Problem Formulation – Models

Model 1 – Effective Thermophysical Properties

Model 2 – Thermal Dispersion Effect

Model 3 – Migration of Dispersed Phase

$$u_{fd}(r).\frac{\partial \phi(r,z)}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(rD_B(r,z)\frac{\partial \phi(r,z)}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{D_T(r,z)}{T(r,z)}\frac{\partial T(r,z)}{\partial r}\right),$$

$$\rho_{nf}(r,z)c_{pnf}(r,z)u_{fd}(r).\frac{\partial T(r,z)}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(rk_{nf}(r,z)\frac{\partial T(r,z)}{\partial r}\right),\quad 0<r<R;\ z>0$$

$$\phi(r,0)=\phi_0,\qquad \left.\frac{\partial \phi(r,z)}{\partial r}\right|_{r=0}=0,\qquad \left.\frac{\partial \phi(r,z)}{\partial r}\right|_{r=R}=0$$

$$T(r,0)=T_0,\qquad \left.\frac{\partial T(r,z)}{\partial r}\right|_{r=0}=0,\qquad k_{nf}(R,z)\left.\frac{\partial T(r,z)}{\partial r}\right|_{r=R}=q_w$$

Nanofluids in Forced Convection

Problem Formulation – Nonlinear Effective Fluid Properties

$$\rho(T)c_p(T)\left[u(r,z,T)\frac{\partial T(r,z)}{\partial z}+v(r,z,T)\frac{\partial T(r,z)}{\partial r}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left[rk(T)\frac{\partial T(r,z)}{\partial r}\right],\ 0<r<r_w,\ z>0$$

$$T(r,0)=T_0,\qquad 0\le r\le r_w$$

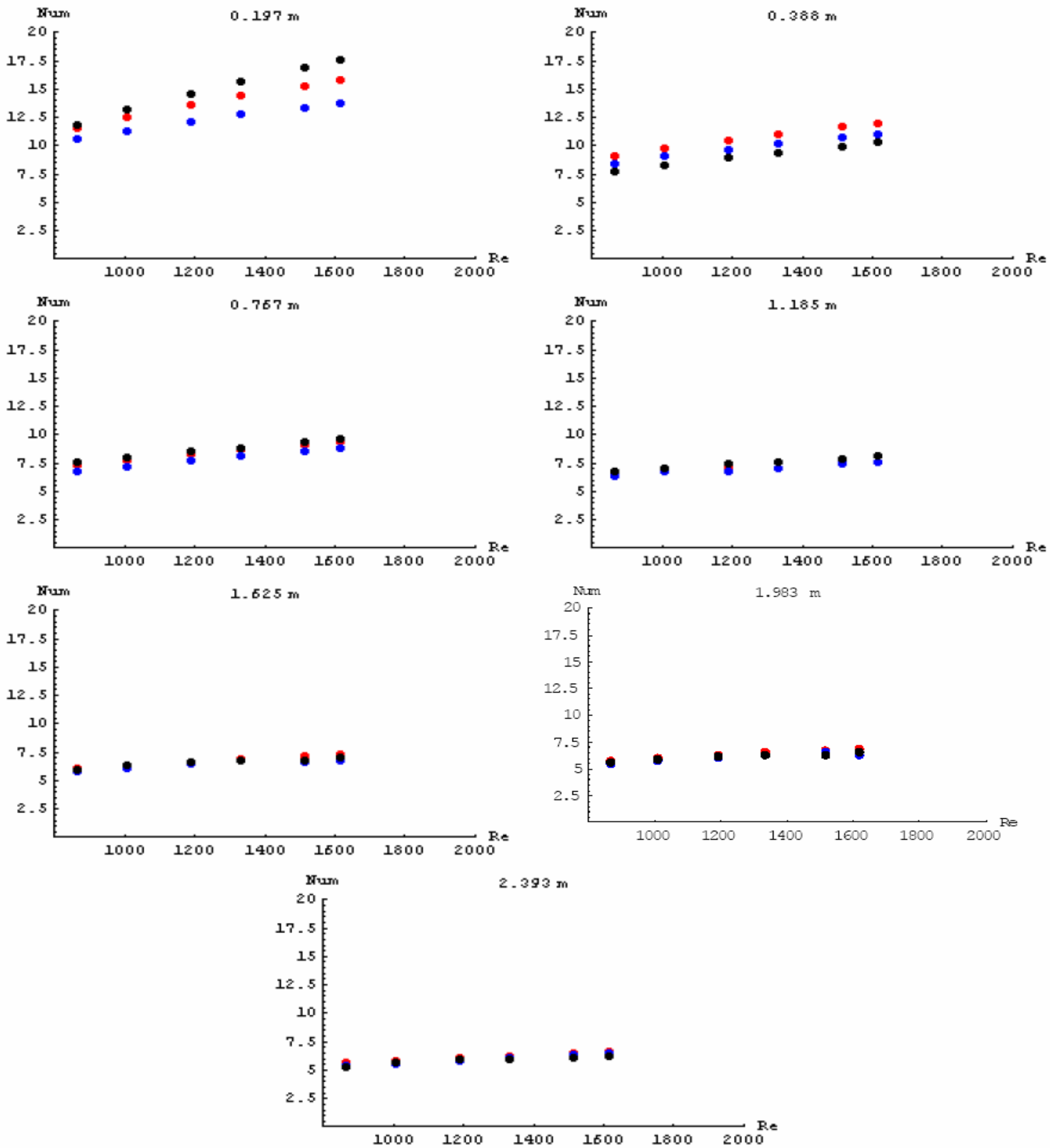
$$\frac{\partial T(r,z)}{\partial r}=0,\quad r=0;\qquad -k(T)\frac{\partial T(r,z)}{\partial r}=-q_w,\quad r=r_w,\ z>0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu(T)\frac{\partial u(r,z)}{\partial r}\right]=\frac{dp(z)}{dz},\qquad 0<r<r_w,\ z>0$$

$$\frac{\partial u(r,z)}{\partial r}=0,\qquad r=0;\quad u(r,z)=0,\quad r=r_w,\qquad z>0$$

NANOFLUIDS FORCED CONVECTION

Validation with Nusselt numbers from empirical and theoretical correlations



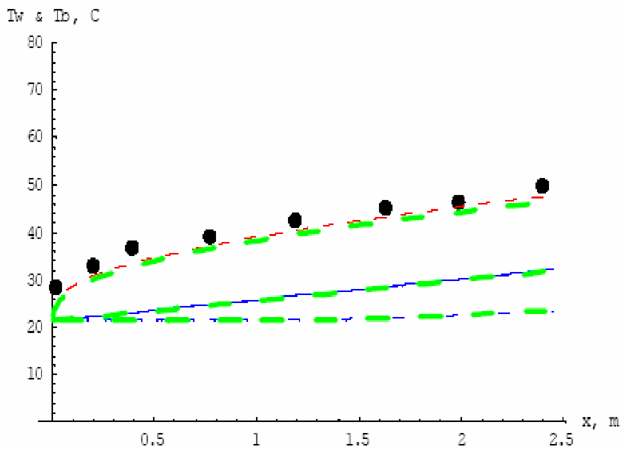
Figuras IV.7.10 – Resultados experimentais para o número de Nusselt médio no escoamento laminar de nanofluido a 1.20% e dispersante Orotan comparados com correlações: Shah [47], em pontos azuis, e Churchill e Ozoe [62], em pontos vermelhos.

Experiments

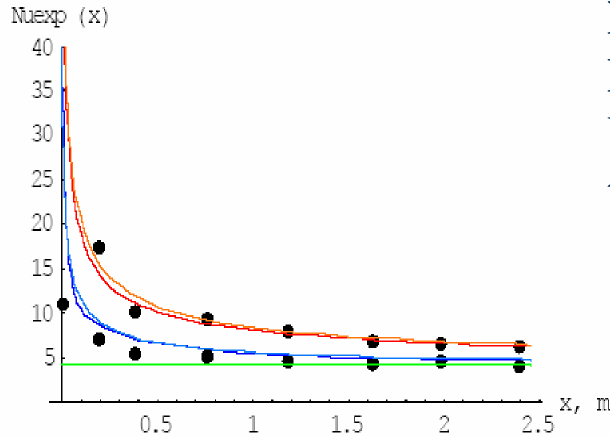
Shah

Churchill-Ozoe

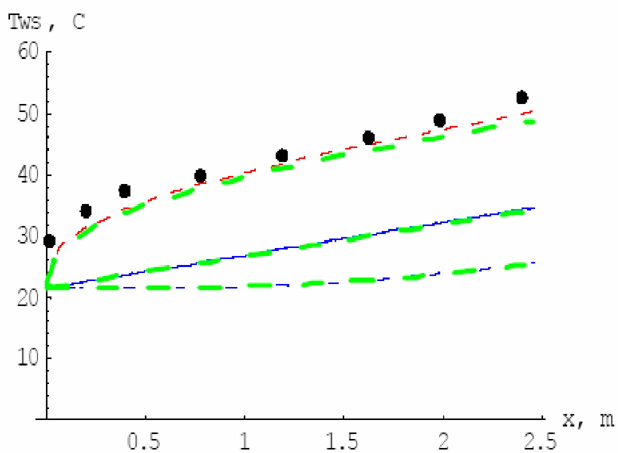
NANOFLUIDS
FORCED CONVECTION
Validation of experimental and theorethical results (GITT)



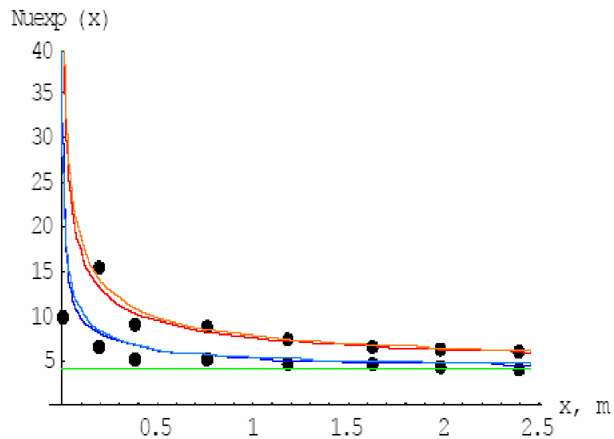
(a) Temperaturas (parede, média, centro) – Re=1616



(b) Números de Nusselt locais (azul) e médios (vermelho) – Re=1616



(c) Temperaturas (parede, média, centro) – Re=1335



(d) Números de Nusselt locais (azul) e médios (vermelho) – Re=1335

Figuras IV.7.11 – Resultados experimentais e teóricos para temperaturas na parede e números de Nusselt locais e médios para nanofluido a 1.20% e dispersante Orotan.

Experiments

GITT-Nux

GITT - Num

Nanofluids in Forced Convection
Heat Transfer Enhancement (nanofluid x water)

Tabela IV.7.20 – Comparação do coeficiente de transferência de calor médio para escoamento laminar de nanofluido água-alumina a 1.2% (Re=1616) e água (Re=1604 e Re=1632).

x (m)	hm Nano	hm Agua	Dif.%	hm Agua	Dif.%
0.197	1713.15	1421.61	20.5072	1487.24	15.1897
0.388	999.175	849.628	17.6014	895.838	11.5353
0.767	932.572	831.318	12.1799	850.539	9.64484
1.185	787.502	721.634	9.12761	737.006	6.8515
1.625	689.169	649.042	6.18239	662.112	4.08646
1.983	648.306	615.296	5.36492	629.51	2.98582
2.393	615.041	583.651	5.37832	597.478	2.93965

Tabela IV.7.21 – Comparação do coeficiente de transferência de calor médio para escoamento laminar de nanofluido água-alumina a 1.2% (Re=1515) e água (Re=1453 e Re=1588).

x (m)	hm Nano	hm Agua	Dif.%	hm Agua	Dif.%	Dif.Int.%
0.197	1648.94	1371.62	20.2187	1417.06	16.3638	18.4513
0.388	968.117	844.699	14.6108	844.093	14.6931	14.6486
0.767	908.912	811.431	12.0135	824.781	10.2005	11.1822
1.185	768.321	709.533	8.28557	714.152	7.58517	7.96445
1.625	667.215	638.641	4.47417	641.299	4.04117	4.27564
1.983	629.054	606.274	3.75738	607.335	3.57622	3.67432
2.393	595.456	573.986	3.74046	574.693	3.61283	3.68194

Nanofluids in Forced Convection

Heat Transfer Enhancement (nanofluid x water)

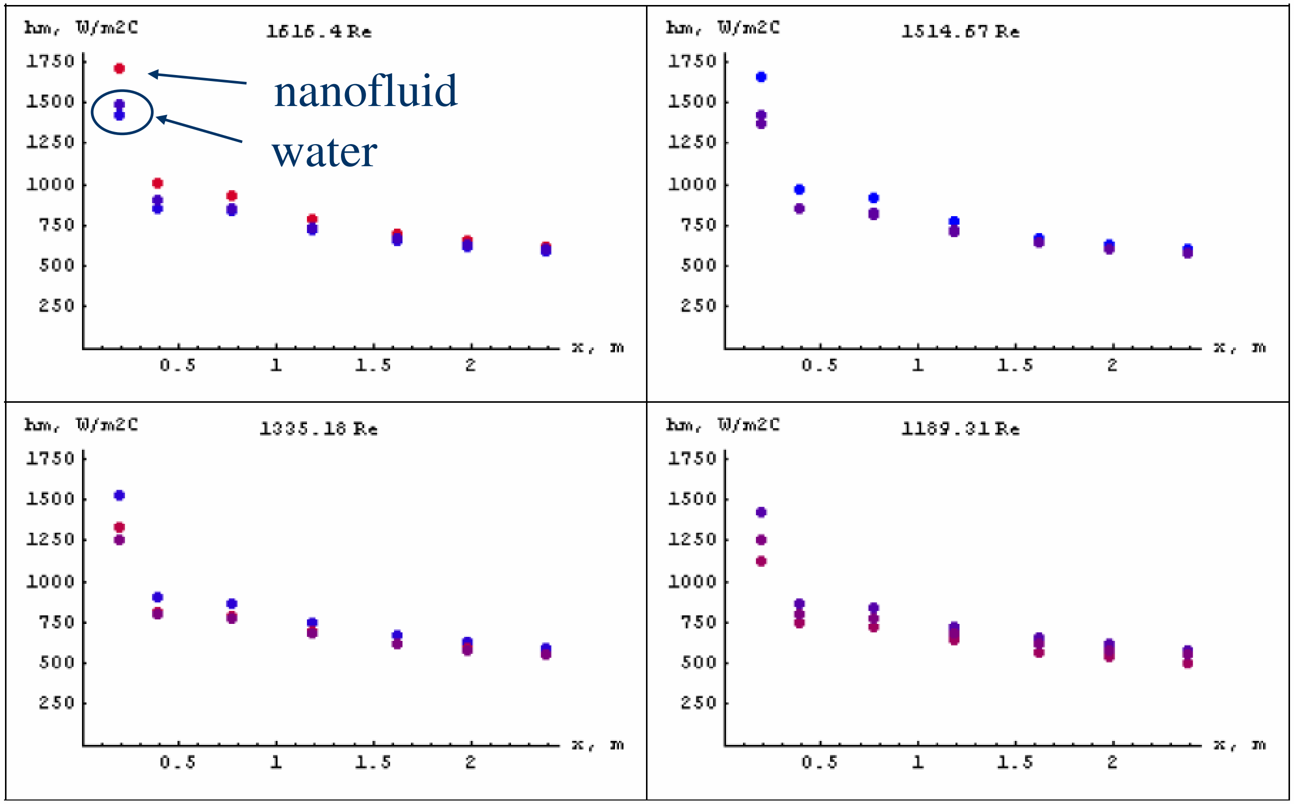
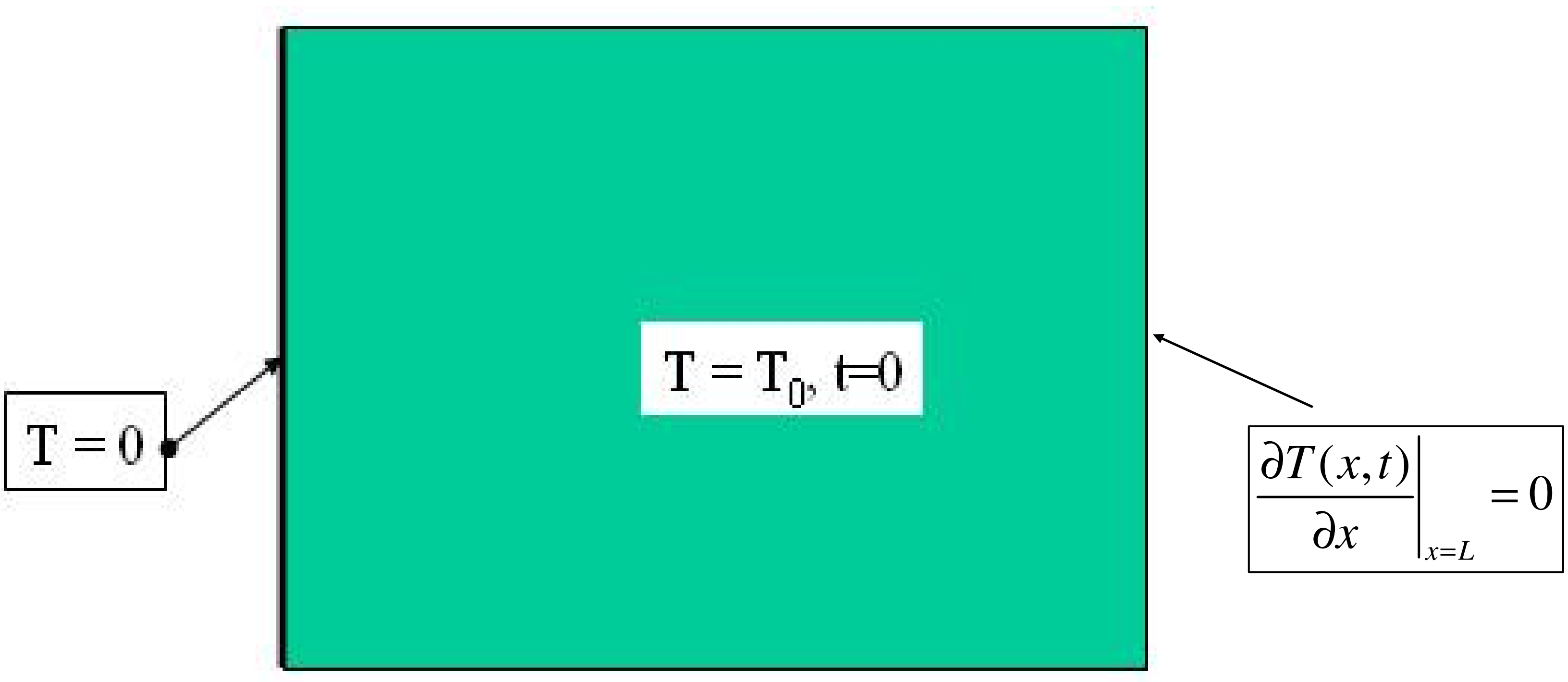


Figura IV.7.9 – Intensificação térmica obtida com o nanofluido água-alumina a 1.20% de fração volumétrica com dispersante Orotan em comparação com água (plotada para dois números de Reynolds abaixo e acima do valor referente ao nanofluido).

Part II

- Tutorial on the Generalized Integral Transform Technique (GITT)

Separation of Variables
Heat Conduction in a slab



Problem Formulation

Separation of Variables

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2}, \qquad 0 < x < L, \quad t > 0$$

$$T(x,0) = T0, \qquad 0 \le x \le L$$

$$T(0,t) = 0, \qquad t > 0 \qquad \left.\frac{\partial T(x,t)}{\partial x}\right|_{x=L} = 0, \qquad t > 0$$

Separation of Variables $\longrightarrow$ $T(x,t) \overset{?}{=} \psi(x)\Gamma(t)$

Problem Formulation

Separation of Variables

$$\psi(x)\frac{\partial\Gamma(t)}{\partial t} = \alpha\,\Gamma(t)\frac{\partial^2\psi(x)}{\partial x^2} \qquad \div\left(\alpha\,\psi(x)\,\Gamma(t)\right)$$

$$\frac{1}{\alpha\Gamma(t)}\frac{\partial\Gamma(t)}{\partial t} = \frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} = -\mu^2$$

t

$$\frac{1}{\alpha\Gamma(t)}\frac{\partial\Gamma(t)}{\partial t} = -\mu^2$$

$$\Gamma(t) = C\,Exp(-\alpha\,\mu^2\,t)$$

?

Initial Condition

x

$$\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} = -\mu^2$$

$$\psi(x) = C_1\cos(\mu x) + C_2\,sin(\mu x)$$

? ?

Boundary Condition

Problem Formulation

Separation of Variables

$$\psi(x) = C_1 \cos(\mu x) + C_2\, sin(\mu x)$$

$$\Downarrow$$

Boundary Conditions：

$$bc1: \quad \psi(x=0)=0 \quad \rightarrow \quad C_1 = 0 \quad and \quad C_2 = 1$$

$$bc2: \quad \left.\frac{\partial \psi(x)}{\partial x}\right|_{x=L} = 0 \quad \rightarrow \quad \cos(\mu L) = 0 \quad \rightarrow \quad \mu_i = \frac{(2i-1)}{2L}\pi, \qquad i = 1, 2, \ldots$$

$$\Downarrow$$

$$\psi_i(x) = sin(\mu_i x)$$

and

$$\Gamma_i(t) = C_i Exp(-\alpha\, \mu_i^2\, t)$$

? Initial Condition

$$T(x,t) = \sum_{i=1}^{\infty} \psi_i(x) \Gamma_i(t)$$

Separation of Variables

Orthogonality Property

$$T(x,t)=\sum_{i=1}^{\infty}\psi_i(x)\Gamma_i(t)=\sum_{i=1}^{\infty} sin(\mu_i x)\ C_i Exp(-\alpha\,\mu_i^2\ t)$$

Orthogonality Property : $\int_0^L \psi_i(x)\psi_j(x)\ dx=\delta_{ij}N_i \qquad \therefore \quad N_i=\int_0^L \psi_i^2\ dx=\frac{L}{2}$

$$\int_0^L \psi_i(x)\_\_\_\_\_\ dx \qquad T(x,0)=T0$$

$$\int_0^L \psi_i(x)T(x,0)\,dx=\sum_{j=1}^{\infty}C_j\int_0^L \psi_i(x)\psi_j(x)\,dx=C_iN_i \qquad \therefore \quad C_i=\frac{1}{N_i}T0\int_0^L \psi_i(x)\,dx$$

Separation of Variables

Eigenfunction Expansion

$$T(x,t) = \sum_{i=1}^{\infty} \frac{\bar{f}_i}{N_i} sin(\mu_i x)\, Exp(-\alpha\, \mu_i^2\, t) \qquad \therefore \quad \bar{f}_i = T0 \int_0^L \psi_i(x)\, dx$$

$$T(x,t) = \sum_{i=1}^{\infty} A_i(t)\psi_i(x) \qquad \therefore \qquad A_i(t) = \frac{\bar{f}_i}{N_i} Exp(-\alpha\, \mu_i^2\, t)$$

Eigenfunction Expansion

Formal Solution
(General Problem Formulation)

Nonlinear convection-diffusion (M potentials)

$$w_k^*(\mathbf{x},t,T_l)\frac{\partial T_k(\mathbf{x},t)}{\partial t}+\mathbf{u}(\mathbf{x},t,T_l).\nabla T_k(\mathbf{x},t)=\nabla.k_k^*(\mathbf{x},t,T_l)\nabla T_k(\mathbf{x},t)-$$
$$-d_k^*(\mathbf{x},t,T_l)T_k(\mathbf{x},t)+P_k^*(\mathbf{x},t,T_l),\ \ \mathbf{x}\in V,\ t>0,\ \ k,l=1,2,...M$$

Initial and Boundary conditions

$$T_k(\mathbf{x},0)=f_k(\mathbf{x}),\quad \mathbf{x}\in V$$
$$\alpha_k^*(\mathbf{x},t,T_l)T_k(\mathbf{x},t)+\beta_k^*(\mathbf{x},t,T_l)k_k^*(\mathbf{x},t,T_l)\frac{\partial T_k(\mathbf{x},t)}{\partial \mathbf{n}}$$
$$=\phi_k^*(\mathbf{x},t,T_l),\quad \mathbf{x}\in S$$

Formal Solution
(Eigenfunction Expansion)

Proposed eigenfunction expansion

$$T_k(\mathbf{x},t) = \sum_{i=1}^{\infty} A_{k,i}(t)\psi_{k,i}(\mathbf{x})$$

Eigenvalue Problem

$$\nabla . k_k(\mathbf{x})\nabla \psi_{k,i}(\mathbf{x}) + (\mu_{k,i}^2 w_k(\mathbf{x}) - d_k(\mathbf{x}))\psi_{k,i}(\mathbf{x}) = 0, \mathbf{x} \in V$$

$$\alpha_k(\mathbf{x}_l)\psi_{k,i}(\mathbf{x}) + \beta_k(\mathbf{x})k_k(\mathbf{x})\frac{\partial \psi_{k,i}(\mathbf{x})}{\partial \mathbf{n}} = 0, \quad \mathbf{x} \in S$$

Formal Solution

(Transform-Inverse pair)

Orthogonality Property

$$\int_V w_k(\mathbf{x})\psi_{k,i}(\mathbf{x})\psi_{k,j}(\mathbf{x})dv = \delta_{i,j}N_{k,i}$$

$$N_{k,i} = \int_V w_k(\mathbf{x})\psi_{k,i}^2(\mathbf{x})dv$$

$$A_{k,j}(t) = \frac{1}{N_{k,j}}\int_V w_k(\mathbf{x})\psi_{k,j}(\mathbf{x})T_k(\mathbf{x},t)dv$$

$$\tilde{\psi}_{k,i}(\mathbf{x}) = \frac{\psi_{k,i}(\mathbf{x})}{\sqrt{N_{k,i}}}$$

Integral Transform Pair

$$T_k(\mathbf{x},t) = \sum_{i=1}^{\infty}\tilde{\psi}_{k,i}(\mathbf{x})\overline{T}_{k,i}(t), \qquad \textit{inverse}$$

$$\overline{T}_{k,i}(t) = \int_V w_k(\mathbf{x})\tilde{\psi}_{k,i}(\mathbf{x})T_k(\mathbf{x},t)dv, \qquad \textit{transform}$$

Formal Solution
(Explicit System)

Transient term coefficient

$$w_k^*(\mathbf{x},t,T_l) = w_k(\mathbf{x})\frac{w_k^*(\mathbf{x},t,T_l)}{w_k(\mathbf{x})} = w_k(\mathbf{x})C_k^{-1}(\mathbf{x},t,T_l)$$

Explicit PDE System

$$w_k(\mathbf{x})\frac{\partial T_k(\mathbf{x},t)}{\partial t} = H_k(\mathbf{x},t,T_l), \quad t>0,\, l,k=1,2,\ldots,M$$

where,

$$H_k(\mathbf{x},t,T_l) = C_k(\mathbf{x},t,T_l)[\nabla . k_k^*(\mathbf{x},t,T_l)\nabla T_k(\mathbf{x},t)$$

$$-d_k^*(\mathbf{x},t,T_l)T_k(\mathbf{x},t) - \mathbf{u}(\mathbf{x},t,T_l).\nabla T_k(\mathbf{x},t) + P_k^*(\mathbf{x},t,T_l)]$$

$$\alpha_k(\mathbf{x})T_k(\mathbf{x},t) + \beta_k(\mathbf{x})k_k(\mathbf{x})\frac{\partial T_k(\mathbf{x},t)}{\partial \mathbf{n}} = \phi_k(\mathbf{x},t,T_l), \quad \mathbf{x}\in S$$

where,

$$\phi_k(\mathbf{x},t,T_l) = \phi_k^*(\mathbf{x},t,T_l) + [\alpha_k(\mathbf{x}) - \alpha_k^*(\mathbf{x},t,T_l)]T_k(\mathbf{x},t)$$

$$+[\beta_k(\mathbf{x})k_k(\mathbf{x}) - \beta_k^*(\mathbf{x},t,T_l)k_k^*(\mathbf{x},t,T_l)]\frac{\partial T_k(\mathbf{x},t)}{\partial \mathbf{n}}$$

Formal Solution
(Transformed System)

Integral Transformation $\int_V \tilde{\psi}_{k,i}(\mathbf{x}) - dv$

Transformed ODE System

$$\frac{d\overline{T}_{k,i}(t)}{dt} = \int_V \tilde{\psi}_{k,i}(\mathbf{x}) H_k(\mathbf{x},t,T_l)dv, \qquad t>0, i=1,2,\ldots$$

$$\frac{d\overline{T}_{k,i}(t)}{dt} = \int_V \tilde{\psi}_{k,i}(\mathbf{x}) C_k(\mathbf{x},t,T_l)\nabla.k_k^*(\mathbf{x},t,T_l)\nabla T_k(\mathbf{x},t)dv + \int_V \tilde{\psi}_{k,i}(\mathbf{x}) C_k(\mathbf{x},t,T_l)[P_k^*(\mathbf{x},t,T_l) - d_k^*(\mathbf{x},t,T_l)T_k(\mathbf{x},t) - \mathbf{u}(\mathbf{x},t,T_l).\nabla T_k(\mathbf{x},t)]dv$$

Formal Solution
(Nonhomogeneous Conditions)

Nonhomogeneous BC´s

$$\frac{d\overline{T}_{k,i}(t)}{dt} = \int_V \tilde{\psi}_{k,i}(\mathbf{x})[C_k(\mathbf{x},t,T_l)-1]\nabla.k_k^*(\mathbf{x},t,T_l)\nabla T_k(\mathbf{x},t)dv +$$
$$\int_V \tilde{\psi}_{k,i}(\mathbf{x})\nabla.k_k^*(\mathbf{x},t,T_l)\nabla T_k(\mathbf{x},t)dv +$$
$$\int_V \tilde{\psi}_{k,i}(\mathbf{x})C_k(\mathbf{x},t,T_l)[P_k^*(\mathbf{x},t,T_l)-d_k^*(\mathbf{x},t,T_l)T_k(\mathbf{x},t)$$
$$-\mathbf{u}(\mathbf{x},t,T_l).\nabla T_k(\mathbf{x},t)]dv$$

2nd Green´s Formula

$$\frac{d\overline{T}_{k,i}(t)}{dt} = \int_V \tilde{\psi}_{k,i}(\mathbf{x})[C_k(\mathbf{x},t,T_l)-1]\nabla.k_k^*(\mathbf{x},t,T_l)\nabla T_k(\mathbf{x},t)dv +$$
$$\int_V T_k(\mathbf{x},t)\nabla.k_k^*(\mathbf{x},t,T_l)\nabla \tilde{\psi}_{k,i}(\mathbf{x})dv +$$
$$\int_S k_k^*(\mathbf{x},t,T_l)[\tilde{\psi}_{k,i}(\mathbf{x})\frac{\partial T_k(\mathbf{x},t)}{\partial \mathbf{n}} - T_k(\mathbf{x},t)\frac{\partial \tilde{\psi}_{k,i}(\mathbf{x})}{\partial \mathbf{n}}]ds +$$
$$\int_V \tilde{\psi}_{k,i}(\mathbf{x})C_k(\mathbf{x},t,T_l)[P_k^*(\mathbf{x},t,T_l)-d_k^*(\mathbf{x},t,T_l)T_k(\mathbf{x},t)$$
$$-\mathbf{u}(\mathbf{x},t,T_l).\nabla T_k(\mathbf{x},t)]dv$$

Formal Solution
(Transformed System)

Transformed System

$$\frac{d\bar{T}_{k,i}(t)}{dt} = \hat{\bar{h}}_{k,i}(t, \bar{T}_{l,j}), \qquad t > 0,\ k = 1, 2, \ldots, M,\ i = 1, 2, \ldots$$

Transformed initial conditions

$$\bar{T}_{k,i}(0) = \int_V w_k(\mathbf{x}) \tilde{\psi}_{k,i}(\mathbf{x}) f_k(\mathbf{x}) dv$$

Formal Solution
(Filtering)

Filtering solution

$$T_k(\mathbf{x},t) = T_k^*(\mathbf{x},t) + T_{k,f}(\mathbf{x};t)$$

Filtered problem

$$w_k(\mathbf{x})\frac{\partial T_k^*(\mathbf{x},t)}{\partial t} =$$
$$= C_k(\mathbf{x},t,T_l)[\nabla . k_k^*(\mathbf{x},t,T_l)\nabla T_k^*(\mathbf{x},t) - d_k^*(\mathbf{x},t,T_l)T_k^*(\mathbf{x},t)$$
$$-\mathbf{u}(\mathbf{x},t,T_l).\nabla T_k^*(\mathbf{x},t) + P_{k,f}(\mathbf{x},t,T_l)],\ \mathbf{x}\in V,\ t>0$$

$$P_{k,f}(\mathbf{x},t,T_l) = P_k^*(\mathbf{x},t,T_l) + \nabla . k_k^*(\mathbf{x},t,T_l)\nabla T_{k,F}(\mathbf{x},t)$$
$$-d_k^*(\mathbf{x},t,T_l)T_{k,f}(\mathbf{x},t) - \mathbf{u}(\mathbf{x},t,T_l).\nabla T_{k,f}(\mathbf{x},t) - \frac{w_k(\mathbf{x})}{C_k}\frac{\partial T_{k,f}(\mathbf{x},t)}{\partial t}$$

$$T_k^*(\mathbf{x},0) = f_k^*(\mathbf{x}) = f_k(\mathbf{x}) - T_{k,f}(\mathbf{x},0),\quad \mathbf{x}\in V$$

$$\alpha_k(\mathbf{x})T_k^*(\mathbf{x},t) + \beta_k(\mathbf{x})k_k(\mathbf{x})\frac{\partial T_k(\mathbf{x},t)}{\partial \mathbf{n}} = \phi_{k,f}(\mathbf{x},t,T_l),\ \mathbf{x}\in S$$

$$\phi_{k,f}(\mathbf{x},t,T_l) = \phi_k(\mathbf{x},t,T_l) - \alpha_k(\mathbf{x})T_{k,f}(\mathbf{x},t) - \beta_k(\mathbf{x})k_k(\mathbf{x})\frac{\partial T_{k,f}(\mathbf{x},t)}{\partial \mathbf{n}}$$

Formal Solution
(Convergence Testing)

Filtering solution

$$T_k(\mathbf{x},t) = T_k^*(\mathbf{x},t) + T_{k,f}(\mathbf{x};t)$$

Convergence test

$$\varepsilon = \max_{\boldsymbol{x}\in V} \left| \frac{\sum_{i=N^*}^{N} \tilde{\psi}_{ki}(\boldsymbol{x})\overline{T}_{k,i}(t)}{T_{f,k}(\boldsymbol{x};t) + \sum_{i=1}^{N} \tilde{\psi}_{ki}(\boldsymbol{x})\overline{T}_{k,i}(t)} \right|$$

Application
(Convection with Nanofluids)

Convection Equation – Temperature Dependent Properties

$$\rho(T)c_p(T)u(r,T)\frac{\partial T(r,z)}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[rk(T)\frac{\partial T(r,z)}{\partial r}\right],\ 0 < r < r_w,\ z > 0$$

Inlet and Boundary conditions

$$T(r,0) = T_0, \quad 0 \le r \le r_w$$

$$\frac{\partial T(r,z)}{\partial r} = 0,\ \ r = 0; \quad -k(T)\frac{\partial T(r,z)}{\partial r} = -q_w,\ \ r = r_w, z > 0$$

Application
(Convection with Nanofluids)

Convection Equation – Temperature Dependent Properties

$$\rho(T)c_p(T)u(r,T)\frac{\partial T(r,z)}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[rk(T)\frac{\partial T(r,z)}{\partial r}\right],\ 0<r<r_w,\ z>0$$

$$T(r,0) = T_0, \quad 0 \le r \le r_w$$

$$\frac{\partial T(r,z)}{\partial r} = 0,\ \ r=0; \quad -k(T)\frac{\partial T(r,z)}{\partial r} = -q_w,\ \ r = r_w, z>0$$

Temperature Dependent Velocity Field

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu(T)\frac{\partial u(r,z)}{\partial r}\right] = \frac{dp(z)}{dz}, \quad 0<r<r_w, z>0$$

$$\frac{\partial u(r,z)}{\partial r} = 0, \quad r=0; \quad u(r,z)=0,\ \ r=r_w, \quad z>0$$

Application
(Convection with Nanofluids)

Dimensionless groups

$$R = \frac{r}{r_w}, \quad Z = \frac{\alpha_0 \, z}{u_0 \, r_w^2}, \quad U(R,Z) = \frac{u(r,z)}{u_0},$$

$$U_{fd}(R) = \frac{u_{fd}(r)}{u_0} = 2(1-R^2), \quad \gamma(\theta) = \frac{k(T)}{k_0}, \quad \alpha_0 = \frac{k_0}{\rho_0 c_{p,0}},$$

$$C(\theta) = \frac{\rho_0 c_{p,0} u_{fd}(r)}{\rho(T) c_p(T) u(r,T)}, \quad \theta(R,Z) = \frac{T(r,z)-T_0}{q_w r_w / k_0}$$

Dimensionless problem

$$R U_{fd}(R) \frac{\partial \theta(R,Z)}{\partial Z} = C(\theta) \frac{\partial}{\partial R}\left[R\gamma(\theta) \frac{\partial \theta(R,Z)}{\partial R} \right], \quad 0 < R < 1, \; Z > 0$$

$$\theta(R,0) = 0, \quad 0 \le R \le 1$$

$$\frac{\partial \theta(R,Z)}{\partial R} = 0, \; R = 0; \quad \gamma(\theta) \frac{\partial \theta(R,Z)}{\partial R} = 1, \; R = 1, \quad Z > 0$$

Application
(Convection with Nanofluids)

Filtering solution

$$\theta_f(R) = \frac{R^2}{2}$$

with

$$\theta(R,Z) = \theta^*(R,Z) + \theta_f(R)$$

Filtered problem

$$RU_{fd}(R)\frac{\partial \theta^*(R,Z)}{\partial Z} = C(\theta)\frac{\partial}{\partial R}\left[R\gamma(\theta)\frac{\partial \theta^*}{\partial R}\right] + P_f(\theta^*),\ 0 < R < 1,\ Z > 0$$

$$\theta^*(R,0) = -\frac{R^2}{2}, \qquad 0 \le R \le 1$$

$$P_f(\theta^*) = C(\theta)\left[2R\gamma(\theta) + R^2\frac{\partial \gamma}{\partial \theta}\left(\frac{\partial \theta^*}{\partial R} + R\right)\right]$$

$$\frac{\partial \theta^*(R,Z)}{\partial R} = 0,\ R = 0; \quad \frac{\partial \theta^*(R,Z)}{\partial R} = \left(\frac{1}{\gamma(\theta)} - 1\right),\ R = 1, \quad Z > 0$$

Application
(Convection with Nanofluids)

Eigenvalue problem

$$\frac{d}{dR}\left[R\frac{d\psi_i(R)}{dR}\right]+\mu_i^2 R\psi_i(R)=0, \qquad 0<R<1$$

$$\frac{d\psi_i(R)}{dR}=0, \qquad R=0; \qquad \frac{d\psi_i(R)}{dR}=0, \quad R=1$$

Eigenquantities

$$\psi_i(R)=J_0(\mu_i R)$$

$$N_i=\frac{1}{2}J_0^2(\mu_i)$$

$$J_1(\mu_i)=0, \quad i=0,1,2,\ldots$$

$$\tilde{\psi}_i(R)=\sqrt{2}\,\frac{J_0(\mu_i R)}{J_0(\mu_i)}$$

Application
(Convection with Nanofluids)

Integral Transform Pair

$$\theta^*(R,Z) = \sum_{i=0}^{\infty} \tilde{\psi}_i(R)\overline{\theta}_i(Z), \qquad \textit{inverse}$$

$$\overline{\theta}_i(Z) = \int_0^1 R\tilde{\psi}_i(R)\theta^*(R,Z)dR, \qquad \textit{transform}$$

Transformed System

$$\sum_{j=1}^{\infty} a_{i,j} \frac{d\overline{\theta}_j(Z)}{dZ} = \hat{\overline{h}}_i(Z,\overline{\theta}_l), \quad Z > 0,\ i,j,l = 0,1,2,\ldots$$

$$\overline{\theta}_i(0) = \overline{f}_i$$

$$a_{i,j} = \int_0^1 RU_{fd}(R)\tilde{\psi}_i(R)\tilde{\psi}_j(R)dR$$

$$\overline{f}_i = -\frac{1}{2}\int_0^1 R^3\tilde{\psi}_i(R)dR$$

Results and Discussion

(Application - Convection with Nanofluids)

Input Data

$r_w = 0.00315\ m;\quad q_w = 6891.3\ W/m^2;\quad L = 2.45\,m;$

$u_0 = 0.159\ m/s;\quad T_0 = 21.9\ ^\circ C;\quad k_0 = 0.6\,W/m\,^\circ C;$

$\alpha_0 = 1.436\,x10^{-7}\,m^2/s;\quad \nu_0 = 9.584\,x10^{-7}\,m^2/s$

Convergence behavior

Table 1- Convergence of dimensionless duct wall temperature at different axial positions, Z (N<10, NI=38 segments).

Z N	2	4	6	8	10	Num.*
0.0013	0.1366	0.0968	0.0936	0.0951	0.0958	0.0838
0.0179	0.2629	0.2749	0.2782	0.2783	0.2782	0.2823
0.0353	0.3453	0.3633	0.3642	0.3640	0.3639	0.3662
0.0699	0.4686	0.4837	0.4836	0.4832	0.4830	0.4848
0.1080	0.5762	0.5866	0.5860	0.5855	0.5852	0.5874
0.1480	0.6733	0.6800	0.6792	0.6786	0.6782	0.6812
0.1807	0.7452	0.7499	0.7490	0.7483	0.7479	0.7516
0.2180	0.8229	0.8261	0.8250	0.8242	0.8237	0.8286

(*) NDSolve routine – Method of Lines (linearized velocity field) [31]

Results and Discussion

(Application - Convection with Nanofluids)

Linear x Nonlinear

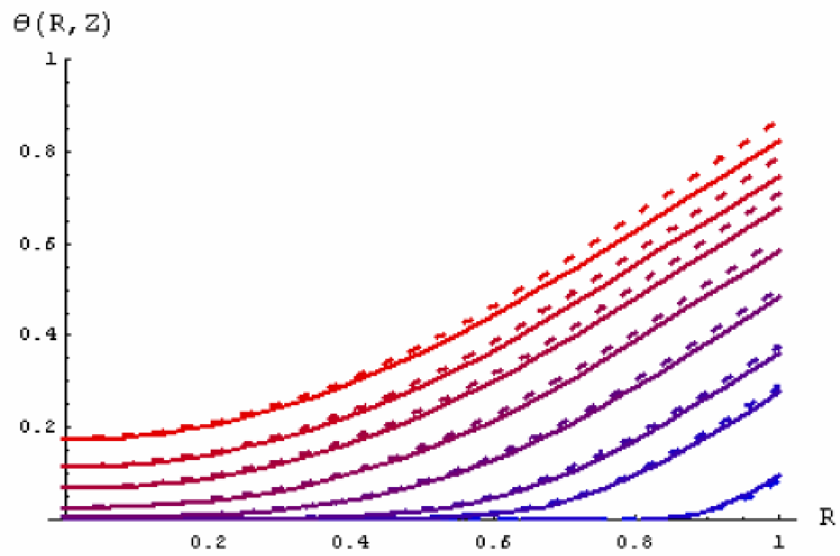
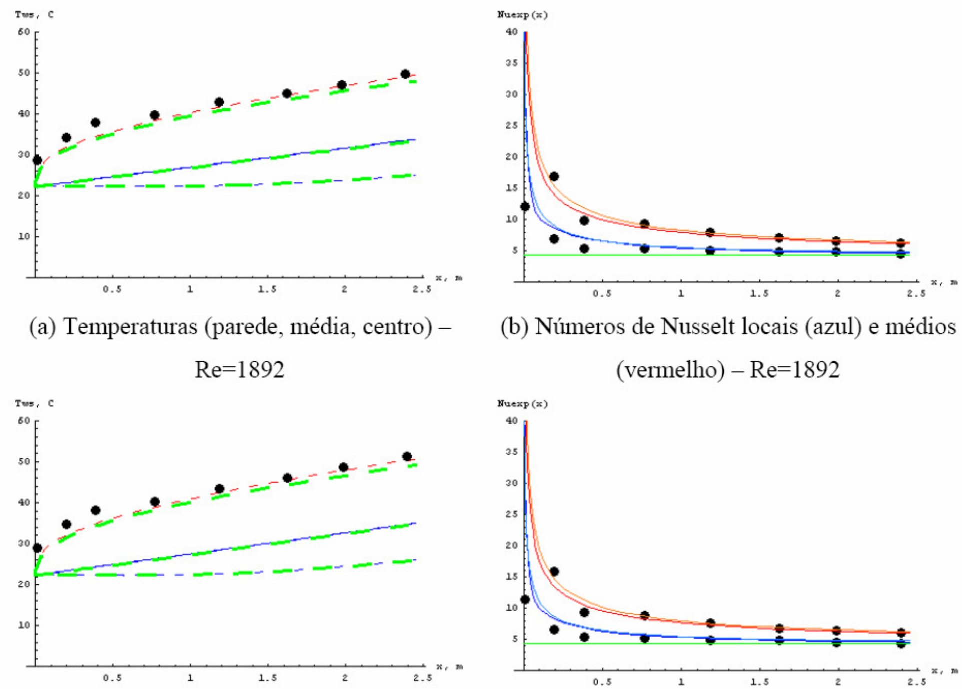


Figure 1- Dimensionless radial temperature distributions for linear (dashed lines) and nonlinear (solid lines) formulations and axial positions increasing from blue to red (Z=0.0013, 0.0179, 0.0353, 0.0699, 0.1080, 0.1480, 0.1807, 0.2180).

Results and Discussion

(Application - Convection with Nanofluids)

Comparisons with Experimental Results



(a) Temperaturas (parede, média, centro) – Re=1892

(b) Números de Nusselt locais (azul) e médios (vermelho) – Re=1892

(c) Temperaturas (parede, média, centro) – Re=1722

(d) Números de Nusselt locais (azul) e médios (vermelho) – Re=1722

Figuras IV.6.4 – Resultados experimentais e teóricos para temperaturas na parede e números de Nusselt locais e médios para água com bomba centrífuga.

Other initiatives LTTC/COPPE/UFRJ

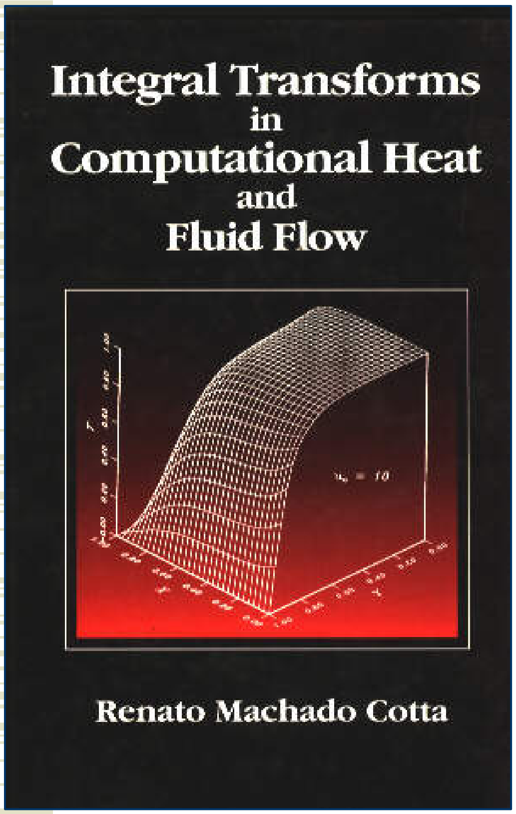
- ***Mathematica* Technical Center –**

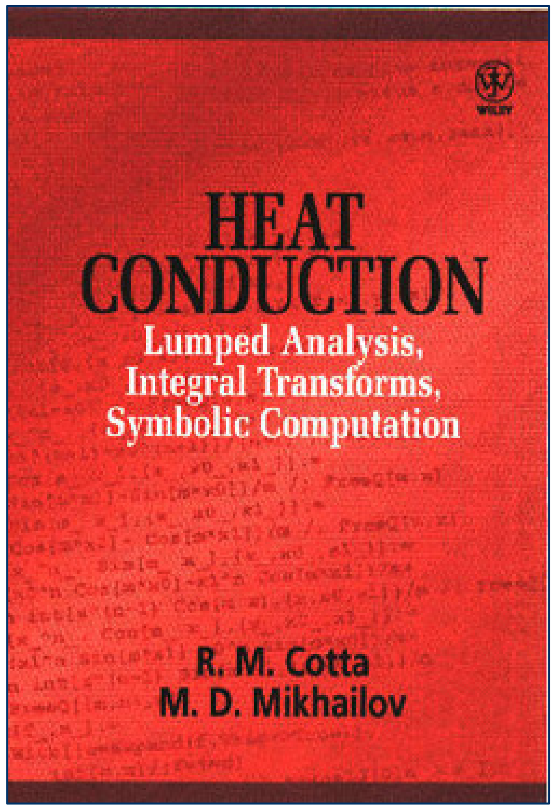
PEM/COPPE & Wolfram Research Inc., USA

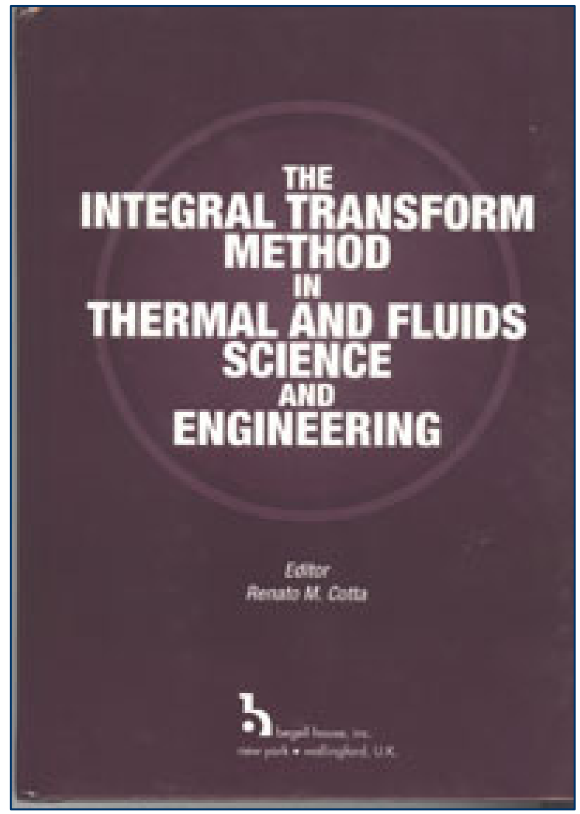
MTC is responsible for basic and advanced training and consulting in mixed symbolic-numerical computation with the platform *Mathematica*.

Sources

Books and Journal

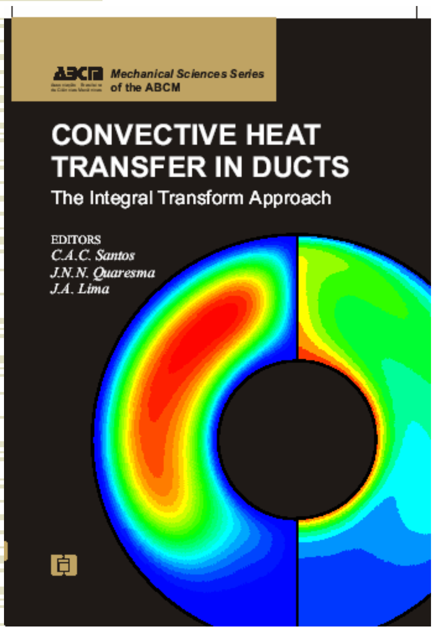


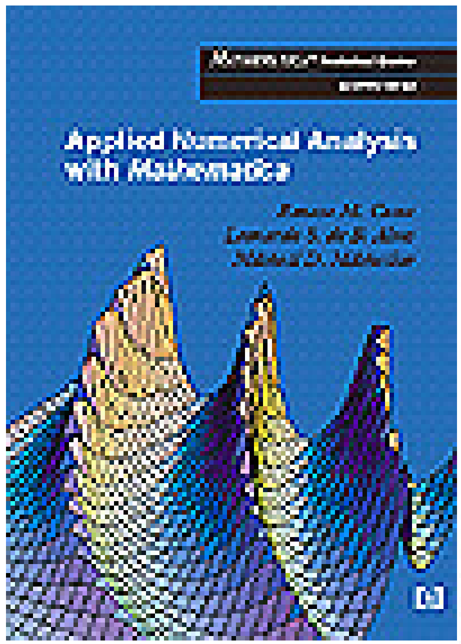


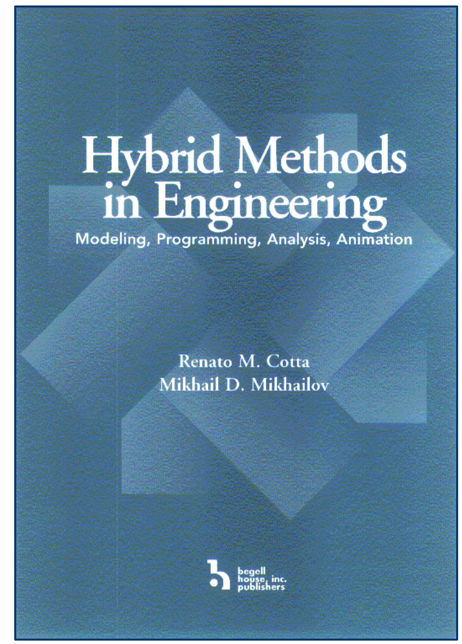


Sources

Books and Journal







New Book CASEE/COPPE/UFRJ

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Muchas Gracias !

- **Profa. Gloria Frontini,** ***Facultad de Ingeniería y el Instituto de Investigaciones en Ciencia y Tecnología de Materiales Mar del Plata, Argentina***

- **Prof. Helcio Orlande, PROSUL Coordinator, COPPE/UFRJ, Brasil.**

- **Sponsoring agencies (CENPES/Petrobras, INB, CNPq & FAPERJ)**

Muchissimas Gracias...

- Familia Zuccardi
- Luigi Bosca
- Humberto Canale
- Finca Flichmann
- Navarro Correas
- Bodega López
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- Finca del Fin del Mundo
- Rodas
- Benjamin Nieto
- Salentein, Etc......

A nuestros hermanos argentinos, chilenos y franceses !

"En el presente intento ser lo más simple posible,
siendo complejo pero de una manera secreta y modesta,
de una manera no evidente."

Jorge Luis Borges