IDENTIFICATION OF THE THERMOPHYSICAL PROPERTIES OF ORTHOTROPIC SEMI-TRANSPARENT MATERIALS

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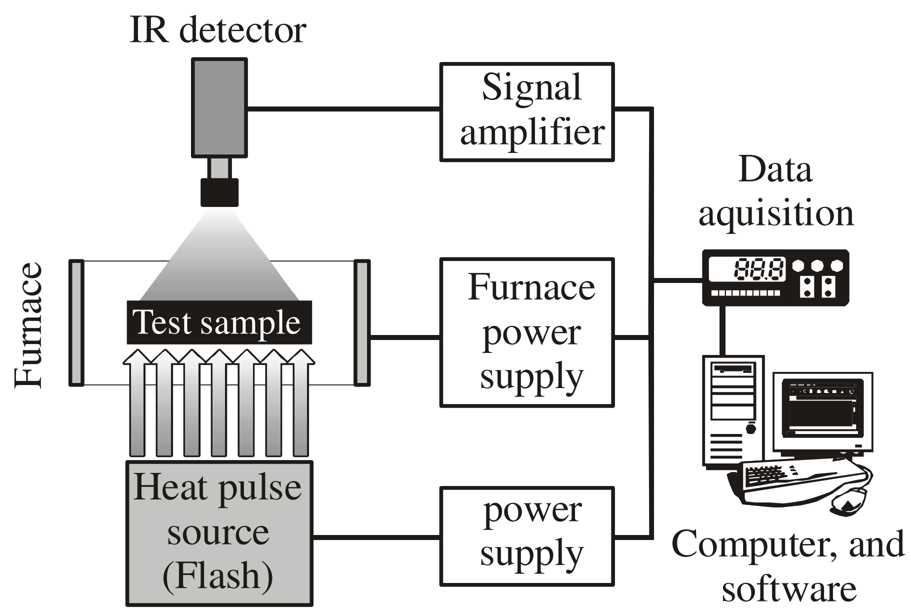
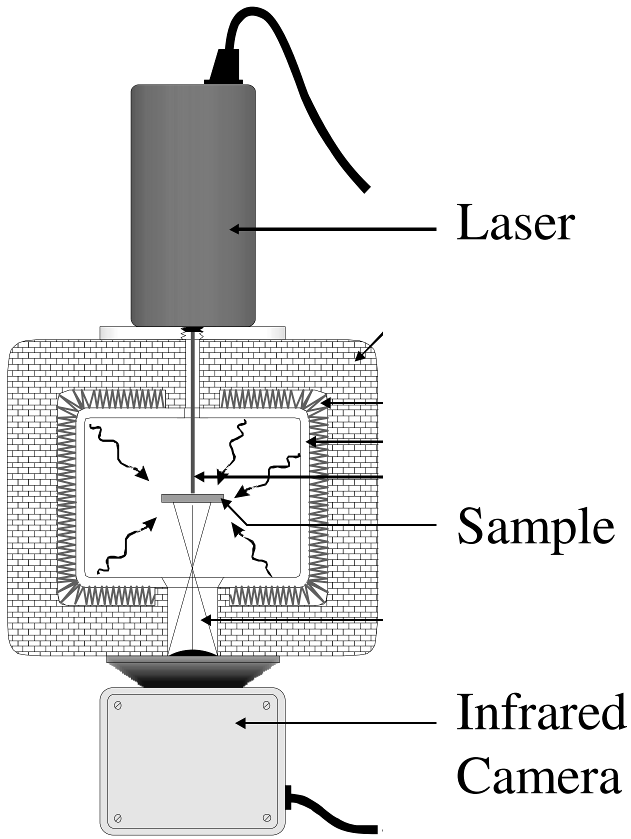
SUMMARY

1. INTRODUCTION

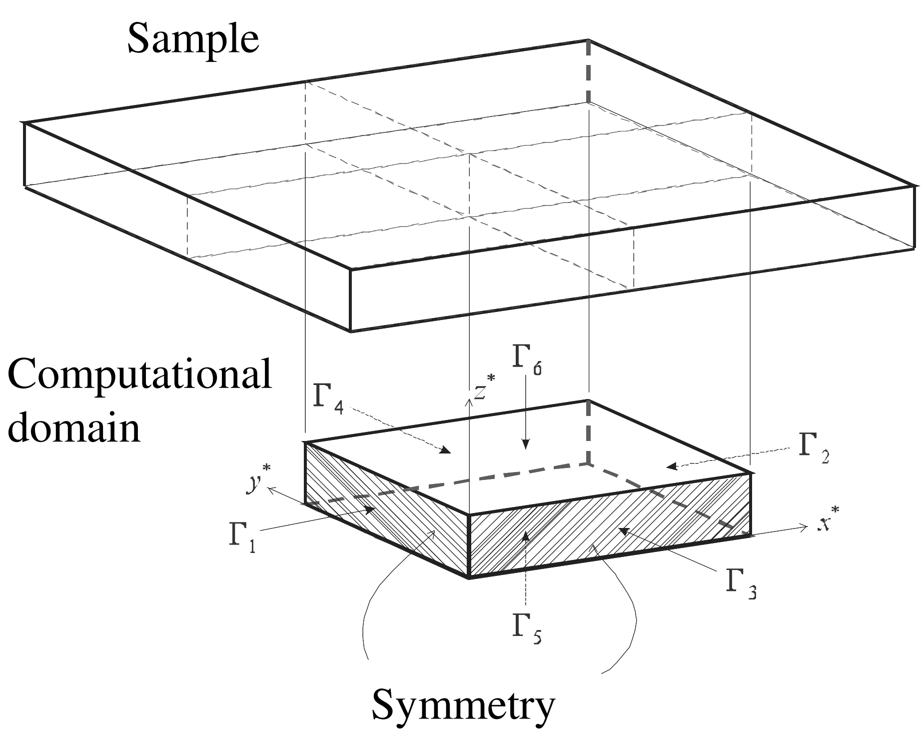
- Identification of thermophysical properties at high temperatures;
- Flash method for thermal diffusivity;
- Semi-transparent materials;
- Coupled conduction-radiation heat transfer.

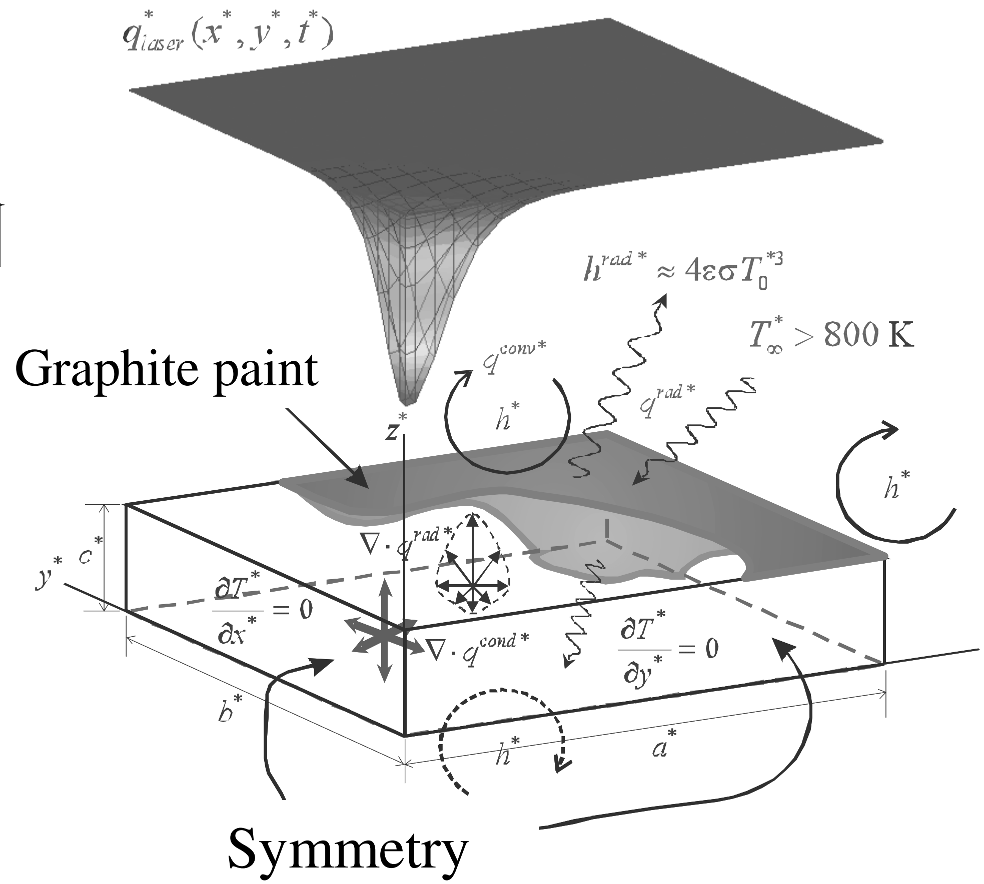


2. PHYSICAL PROBLEM



3. MATHEMATICAL FORMULATION





3. MATHEMATICAL FORMULATION

Equation of Radiative Transfer

$$\xi \frac{\partial I^l}{\partial x} + \eta \frac{\partial I^l}{\partial y} + \mu \frac{\partial I^l}{\partial z} = -\left(\kappa_a + \sigma_s \right) I^l + S^l \quad \text{in } 0 < x < a, \, 0 < y < b, \, 0 < z < c$$

where

$$S^l = \kappa_a n_r^2 I_b \left(T \right) + \frac{\sigma_s}{4\pi} \int_{\Omega' = 4\pi} I^{l'} p\left(\vec{s}\,' \to \vec{s} \right) d\Omega'$$

Boundary Conditions

$$I(\xi,\eta,\mu) = I(-\xi,\eta,\mu) \qquad \text{at } \Gamma_1 : \begin{cases} x = 0 \\ 0 < y < b \\ 0 < z < c \end{cases}$$

$$I(-\xi,\eta,\mu) = \varepsilon\, n_r^2 I_b + \frac{1-\varepsilon}{\pi}\int_{\xi'>0} I(\xi',\eta',\mu')\ \ \xi' d\Omega' \qquad \text{at } \Gamma_2 : \begin{cases} x = a \\ 0 < y < b \\ 0 < z < c \end{cases}$$

$$I(\xi,\eta,\mu) = I(\xi,-\eta,\mu) \qquad \text{at } \Gamma_3 : \begin{cases} 0 < x < a \\ y = 0 \\ 0 < z < c \end{cases}$$

$$I(\xi,-\eta,\mu) = \varepsilon\, n_r^2 I_b + \frac{1-\varepsilon}{\pi}\int_{\eta'>0} I(\xi',\eta',\mu')\ \ \eta'\ \ d\Omega' \qquad \text{at } \Gamma_4 : \begin{cases} 0 < x < a \\ y = b \\ 0 < z < c \end{cases}$$

$$I(\xi,\eta,\mu) = \varepsilon\, n_r^2 I_b + \frac{1-\varepsilon}{\pi}\int_{\mu'<0} I(\xi',\eta',\mu')\ \ \mu'\ \ d\Omega' \qquad \text{at } \Gamma_5 : \begin{cases} 0 < x < a \\ 0 < y < b \\ z = 0 \end{cases}$$

$$I(\xi,\eta,-\mu) = \varepsilon\, n_r^2 I_b + \frac{1-\varepsilon}{\pi}\int_{\mu'>0} I(\xi',\eta',\mu')\ \ \mu'\ \ d\Omega' \qquad \text{at } \Gamma_6 : \begin{cases} 0 < x < a \\ 0 < y < b \\ z = c \end{cases}$$

3. MATHEMATICAL FORMULATION

Energy Conservation Equation

$$C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)-\nabla\cdot q^{rad}$$

in $0<x<a,\ 0<y<b,\ 0<z<c,$ for $t>0$

where: $$\nabla\cdot q^{rad}=\frac{\kappa_a\tau_0}{N_{\mathrm{pl}}}\left[4\pi n_r^2 I_b-\int_{\Omega=4\pi}I^l\,d\Omega\right]$$

Boundary Conditions

$$\frac{\partial T}{\partial x} = 0 \qquad \text{at}\,\Gamma_1 \ \text{ for } t > 0$$

$$k_x \frac{\partial T}{\partial x} + Bi^{rad} T = \frac{\varepsilon \tau_0}{N_{\mathrm{pl}}} \left[\int_{\xi > 0} I^l .\xi . d\Omega - n_r^2 \pi I_b \right] + Bi^{rad} T_\infty \qquad \text{at } \Gamma_2 \ \text{ for } t > 0$$

$$\frac{\partial T}{\partial y} = 0 \qquad \text{at } \Gamma_3 \ \text{ for } t > 0$$

$$k_y \frac{\partial T}{\partial y} + Bi^{rad} T = \frac{\varepsilon \tau_0}{N_{\mathrm{pl}}} \left[\int_{\eta > 0} I^l .\eta . d\Omega - n_r^2 \pi I_b \right] + Bi^{rad} T_\infty \qquad \text{at } \Gamma_4 \ \text{ for } t > 0$$

$$-k_z \frac{\partial T}{\partial z} + Bi^{rad} T = \frac{\varepsilon \tau_0}{N_{\mathrm{pl}}} \left[\int_{\mu < 0} I^l .\mu . d\Omega - n_r^2 \pi I_b \right] + Bi^{rad} T_\infty \qquad \text{at } \Gamma_5 \ \text{ for } t > 0$$

$$k_z \frac{\partial T}{\partial z} + Bi^{rad} T = \frac{\varepsilon \tau_0}{N_{\mathrm{pl}}} \left[\int_{\mu > 0} I^l .\mu . d\Omega - n_r^2 \pi I_b \right] + Bi^{rad} T_\infty + \varepsilon_{10.6\,\mu\mathrm{m}} q_{laser}(x, y, t) \qquad \text{at } \Gamma_6 \ \text{ for } t > 0$$

Initial Condition

$$T = 0 \qquad \text{in } 0 < x < a,\ 0 < y < b,\ 0 < z < c, \text{ for } \ t = 0$$

Dimensionless Variables

$$k_x = \frac{k_x^*}{k_{ref}^*} \quad , \quad k_y = \frac{k_y^*}{k_{ref}^*} \quad , \quad k_z = \frac{k_z^*}{k_{ref}^*}$$

$$x = \frac{x^*}{d_{ref}^*} \quad , \quad y = \frac{y^*}{d_{ref}^*} \quad , \quad z = \frac{z^*}{d_{ref}^*}$$

$$a = \frac{a^*}{d_{ref}^*} \quad , \quad b = \frac{b^*}{d_{ref}^*} \quad , \quad c = \frac{c^*}{d_{ref}^*}$$

$$C = \frac{C^*}{C_{ref}^*} \quad , \quad q = \frac{q^*}{q_{ref}^*} \quad , \quad T = \frac{T^* - T_0^*}{\Delta T_{\max}^*}$$

$$t = \frac{k_{ref}^* t^*}{C_{ref}^* d_{ref}^{*2}} \quad , \quad \nabla \cdot q^{rad} = \frac{d_{ref}^{*2}}{k_{ref}^* \Delta T_{\max}^*} \nabla \cdot q^{rad*}$$

$$Bi^{rad} = \frac{h^{rad*} d_{ref}^*}{k_{ref}^*} = \frac{\left[h^* + 4\varepsilon\sigma T_\infty^{*3} \right] d_{ref}^*}{k_{ref}^*}$$

$$I_\lambda = \frac{I_\lambda^*}{4\sigma T_0^{*4}} \quad , \quad \tau_{0\lambda} = \beta_\lambda^* d_{ref}^* \quad , \quad \kappa_{a\lambda} = \kappa_{a\lambda}^* d_{ref}^*$$

$$\sigma_{s\lambda} = \sigma_{s\lambda}^* d_{ref}^* \quad , \quad N_{\mathrm{pl}\lambda} = \frac{\beta_\lambda^* k_{ref}^* \Delta T_{\max}^*}{4\sigma T_0^{*4}}$$

4. DIRECT AND INVERSE PROBLEMS

DIRECT PROBLEM

Known:

- Boundary and initial conditions
- $C,\ k_x,\ k_y,\ k_z,\ Bi^{rad},\ \kappa_a$ *and* σ_s

Determine:

- Temperature distribution $T(x,y,z,t)$
- Intensity distribution $I^l(x,y,z,\xi^l,\eta^l,\mu^l,t)$

INVERSE PROBLEM

Known:

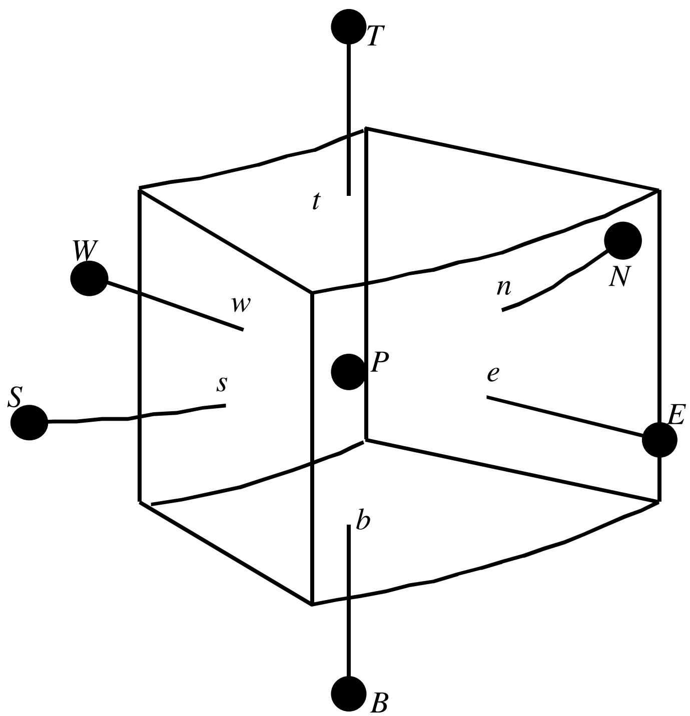
- Boundary and initial conditions
- ***Temperature measurements*** $Y_m(t_i)$ taken at locations $(x_m\ ,\ y_m)$ $m=1,\ldots,M$ at the boundary $z = 0$ and times t_i, $i=1,\ldots,I$

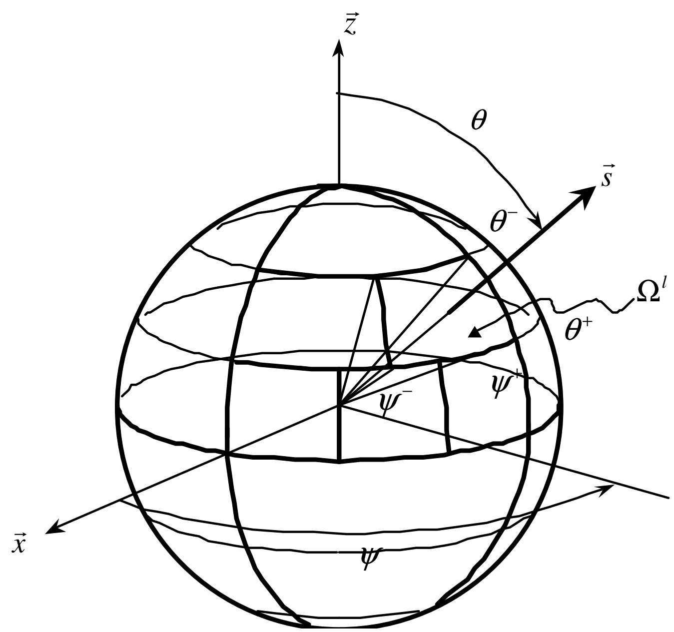
Estimate:

- $C,\ k_x,\ k_y,\ k_z$ *and* Bi^{rad}

5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

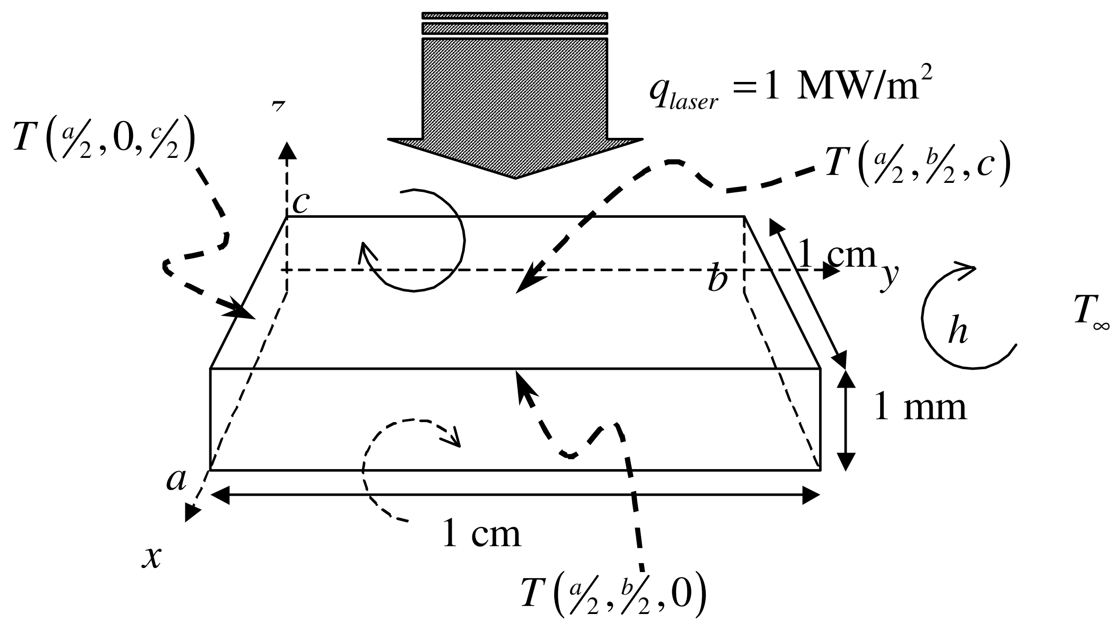
Finite-volumes for the Equation of Radiative Transfer and for the Energy Conservation Equation





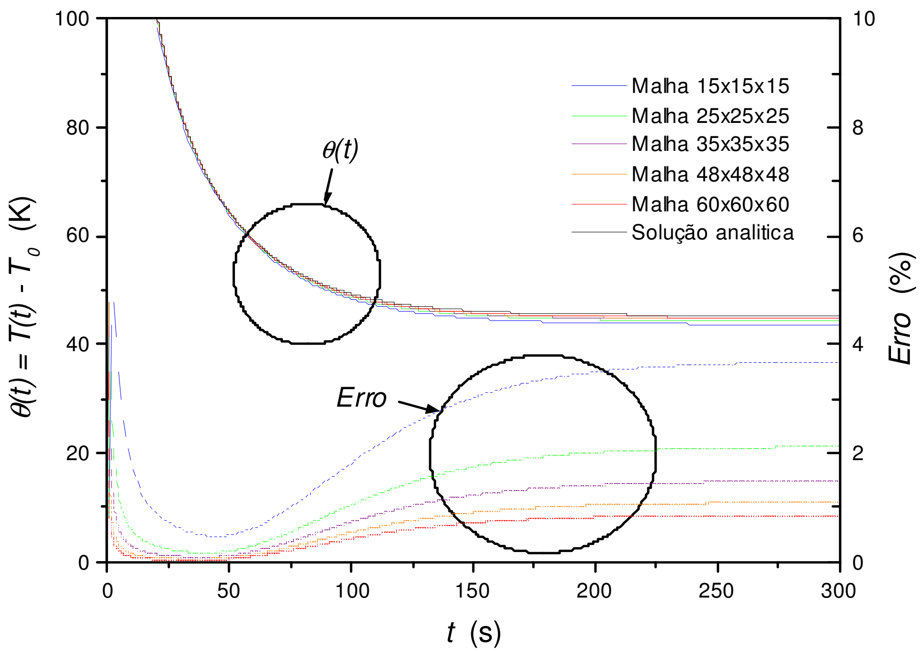
5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

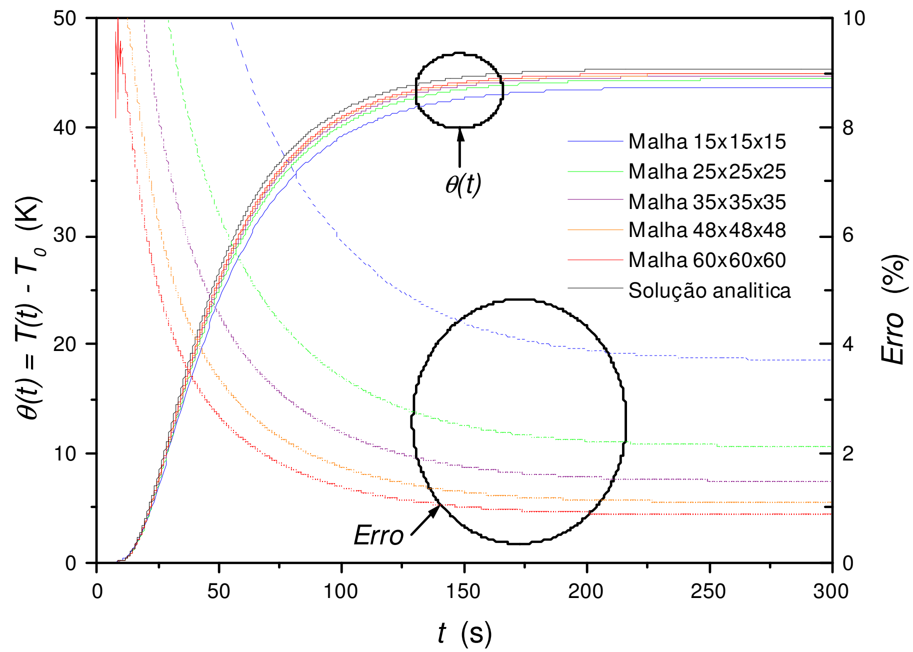
Conduction



5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

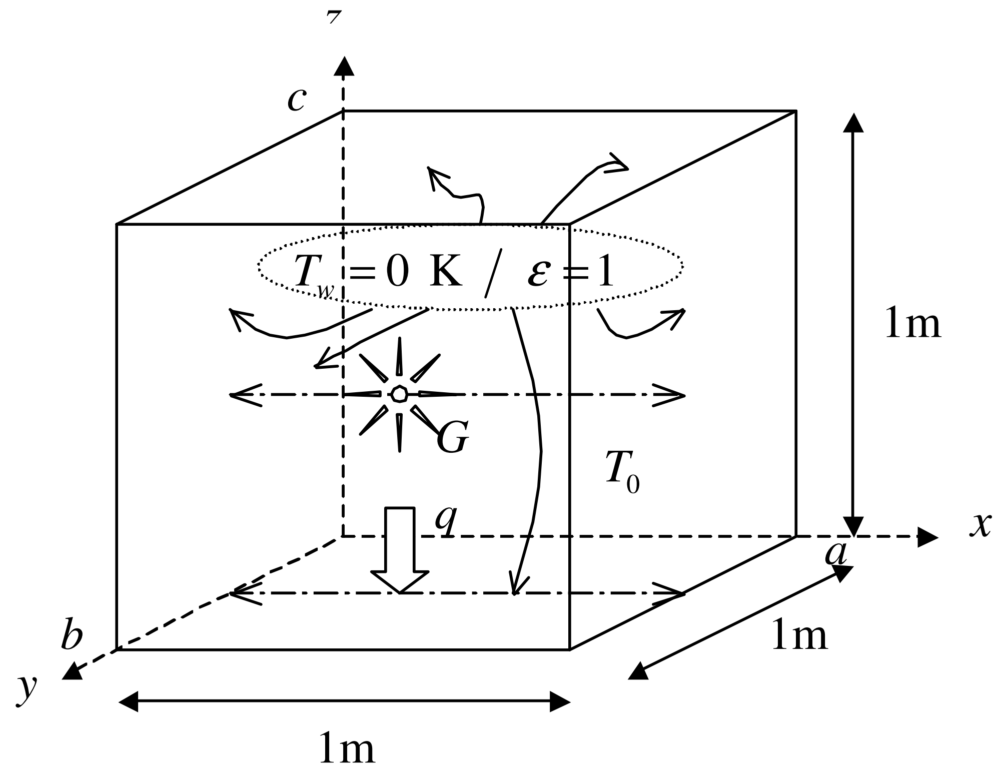
Comparison with analytical solution





5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

Radiation



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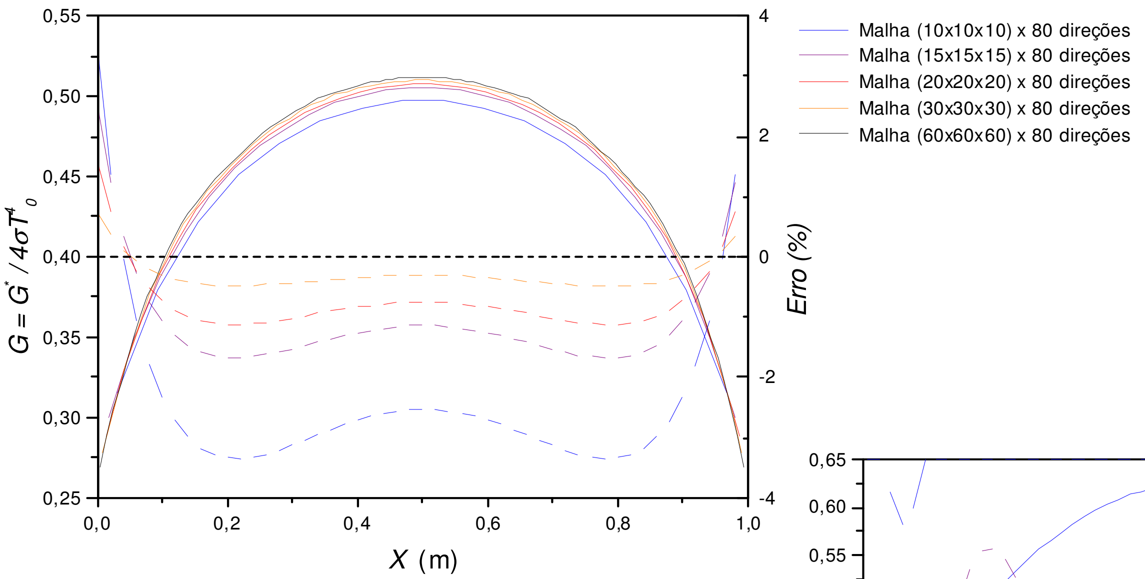
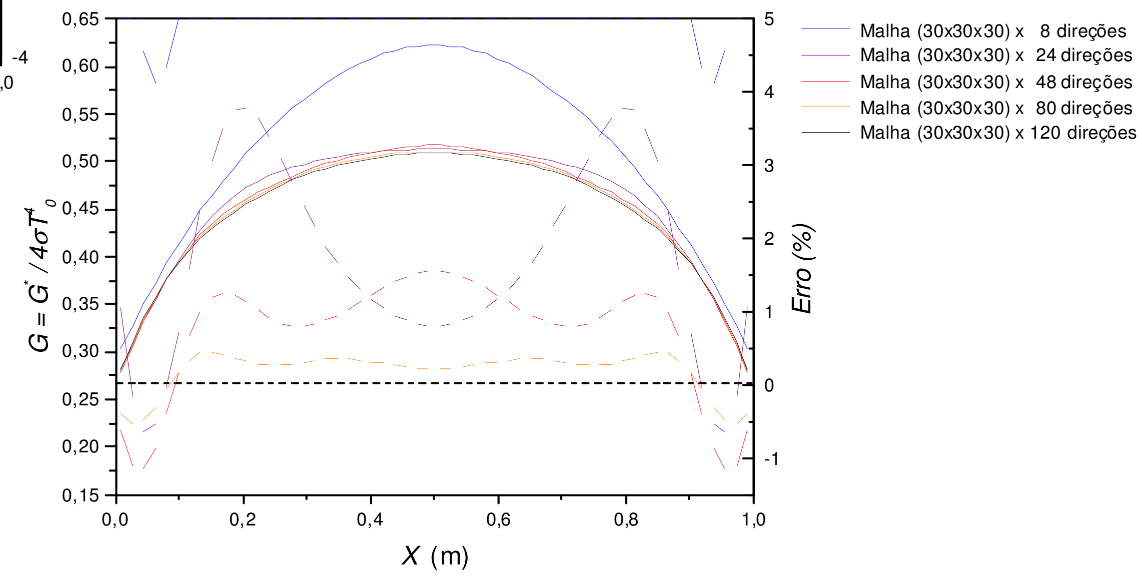
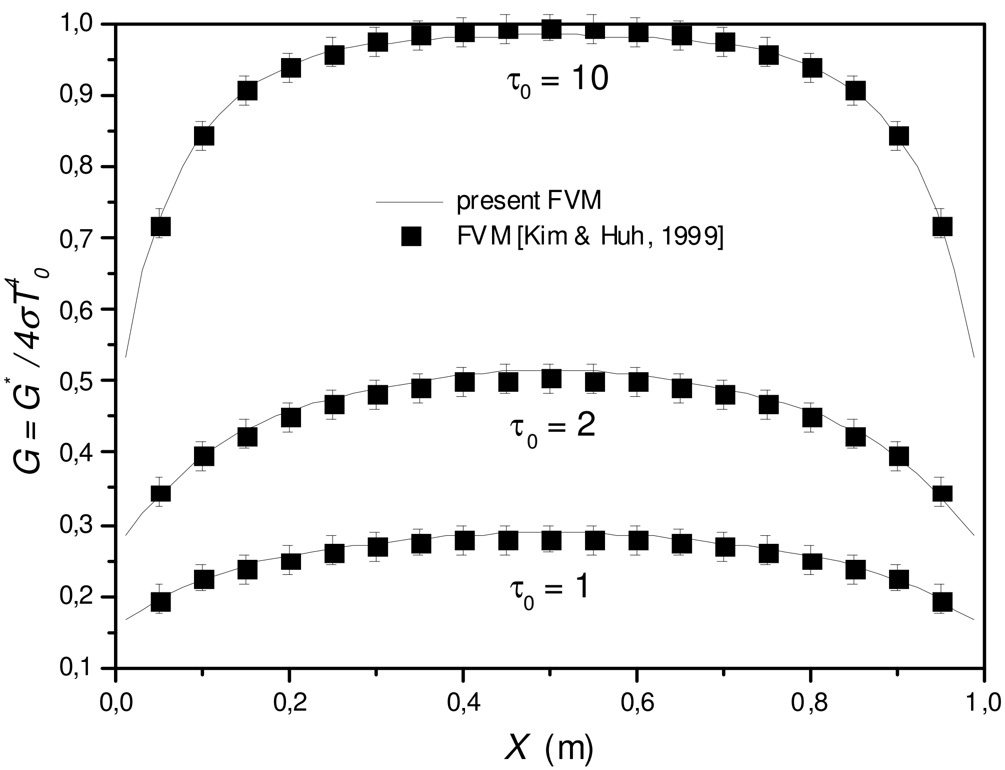


Grid convergence analysis

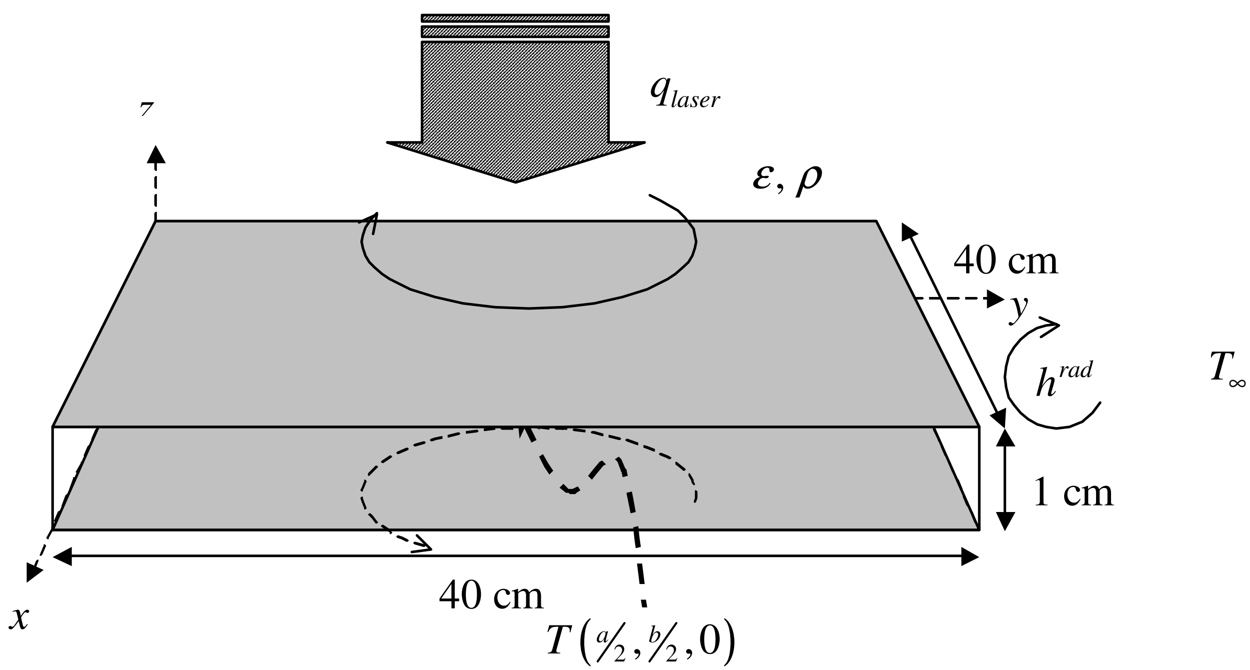


5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

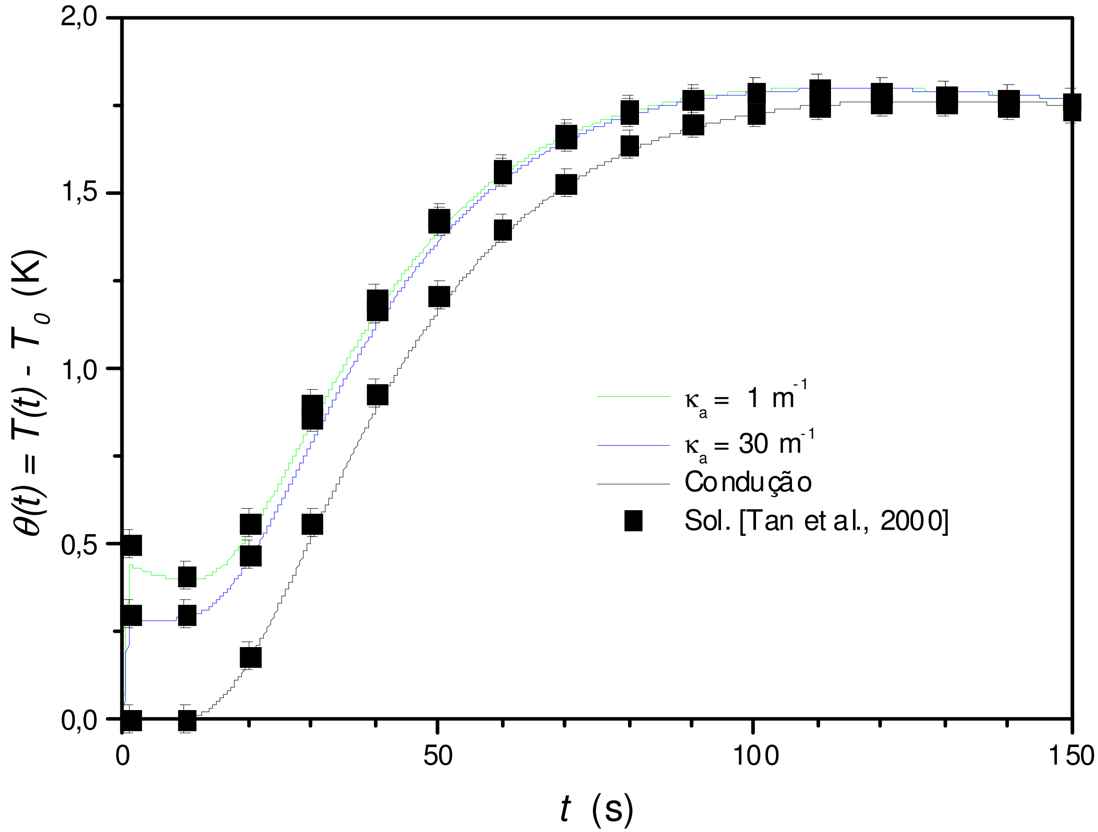


5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

Conduction-Radiation (1D)
Gray medium



5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

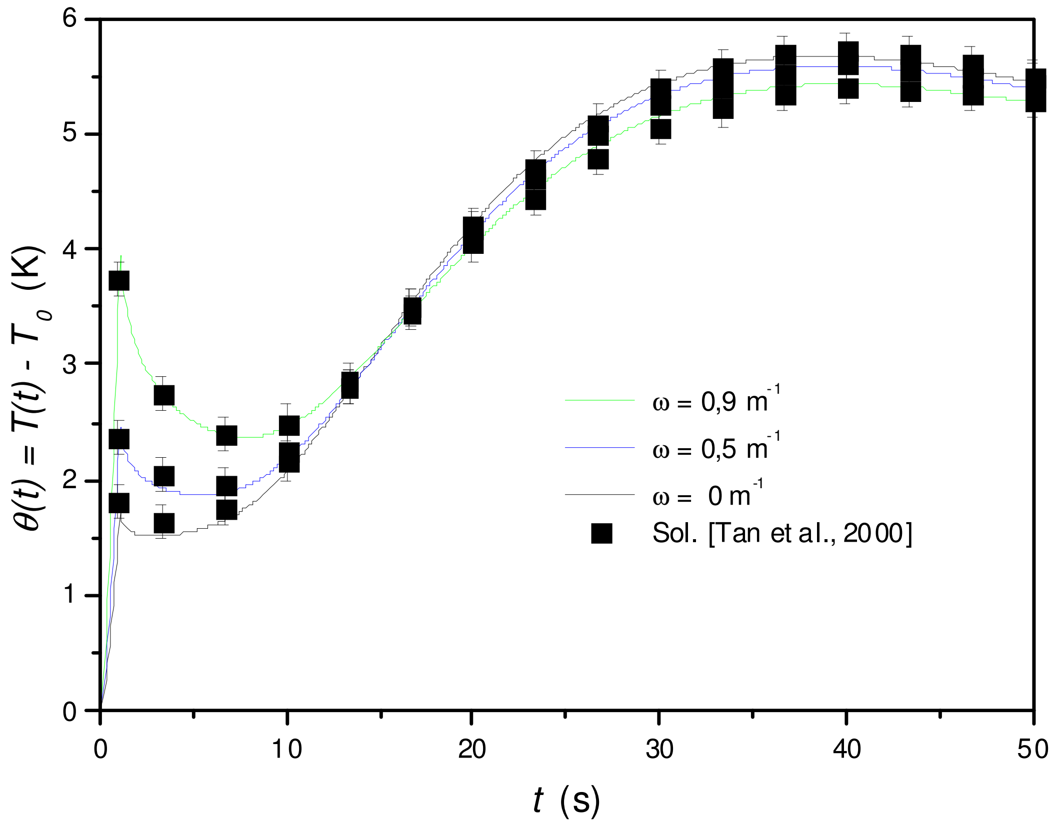


5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

Conduction-Radiation (1D) Non-gray medium

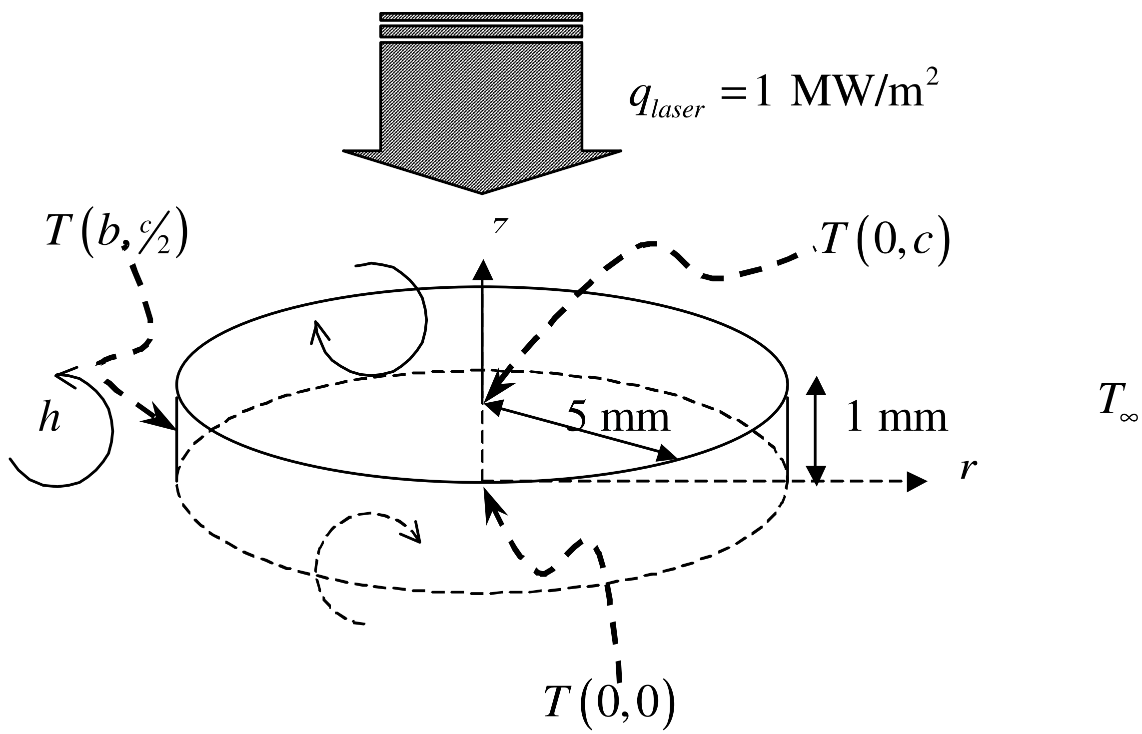
λ (μm)	n_r	ρ_λ	β_λ (m^{-1})
0.5-1.0	1.5	0.04	10
1.0-2.7	1.5	0.04	100
2.7-4.3	1.5	0.04	1000
4.3-10.3	1.5	0.06	10000
10.3-50	1.8	0.15	10000

5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM



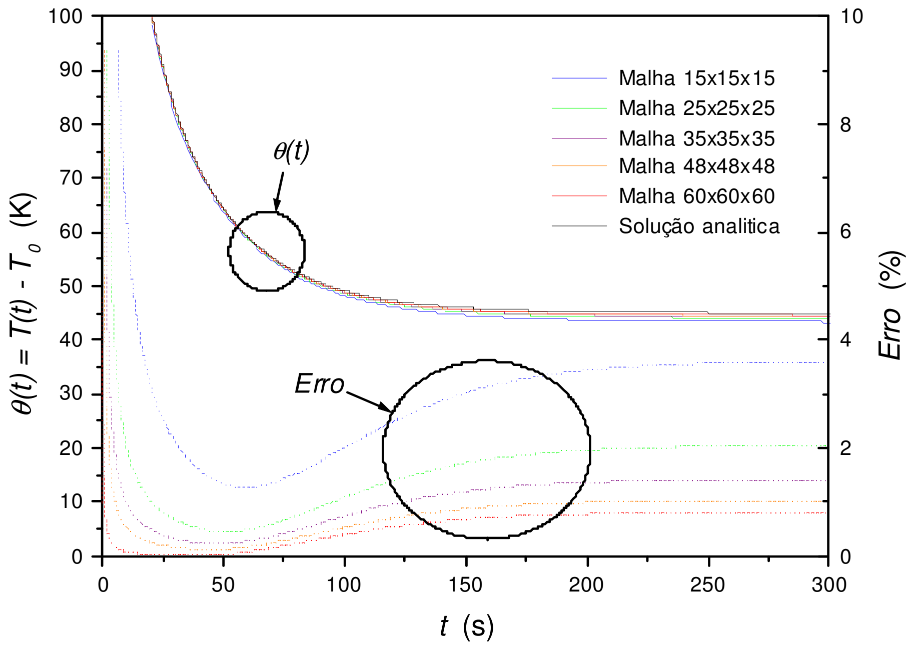
5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

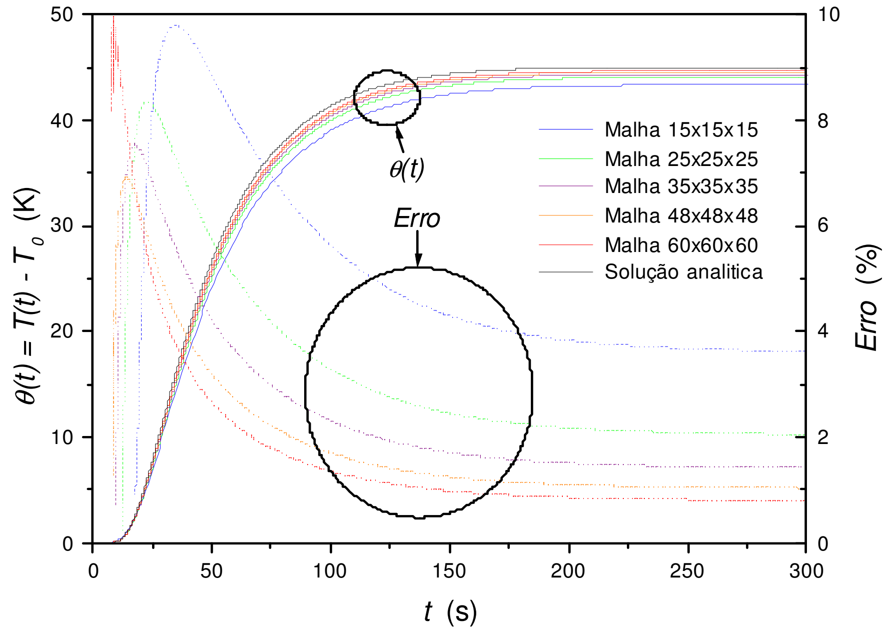
Conduction – Cylindrical coordinates



5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

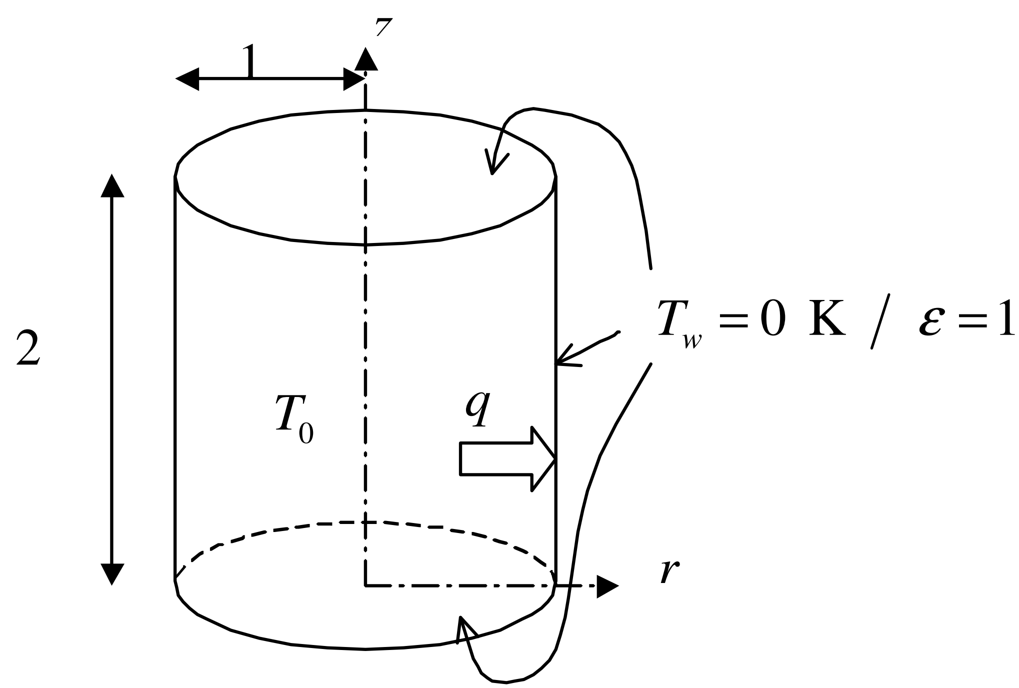
Comparison with analytical solution



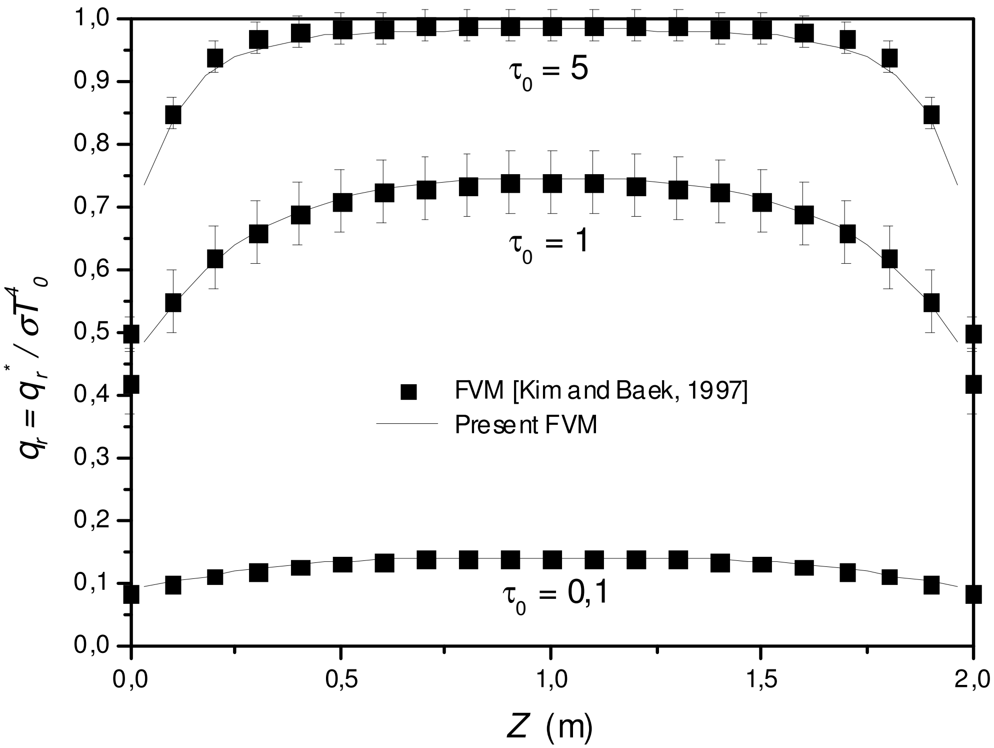


5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

Radiation – Cylindrical coordinates

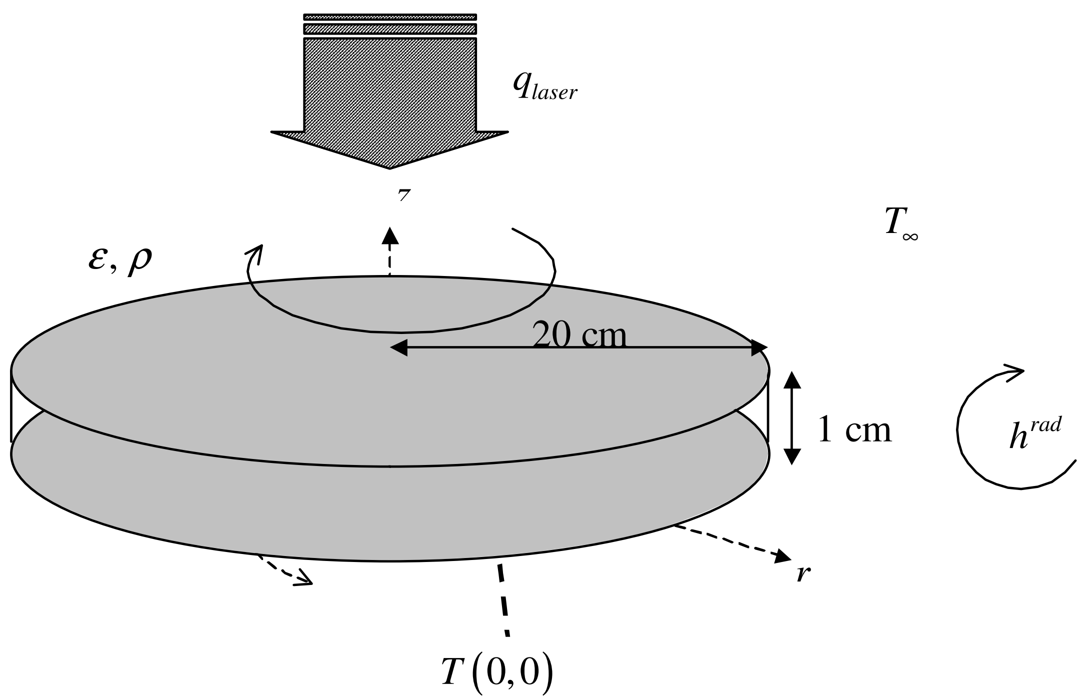


5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

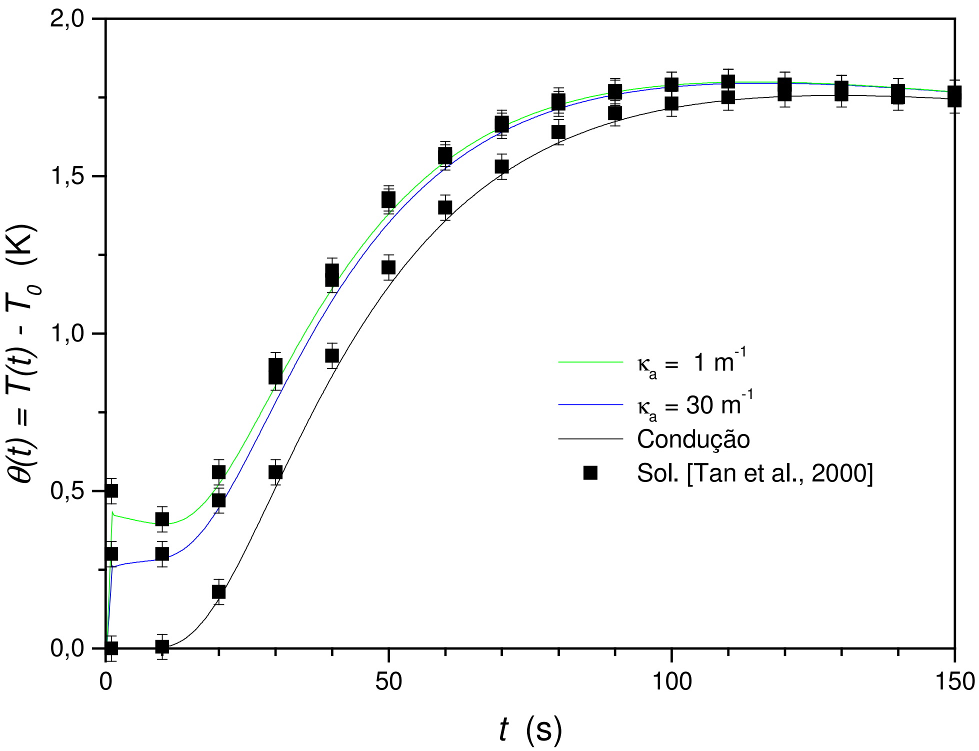


5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

Conduction-Radiation (1D)
Gray medium – Cylindrical coordinates

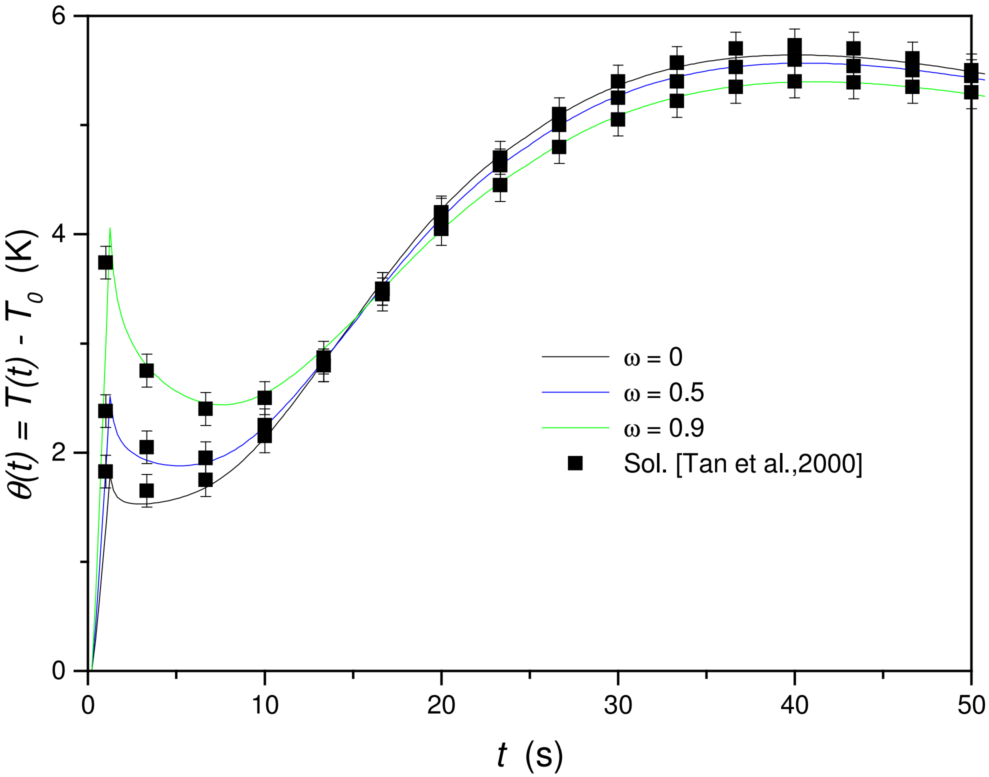


5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM



5. VALIDATION OF THE SOLUTION OF THE DIRECT PROBLEM

Non-gray medium – Cylindrical coordinates

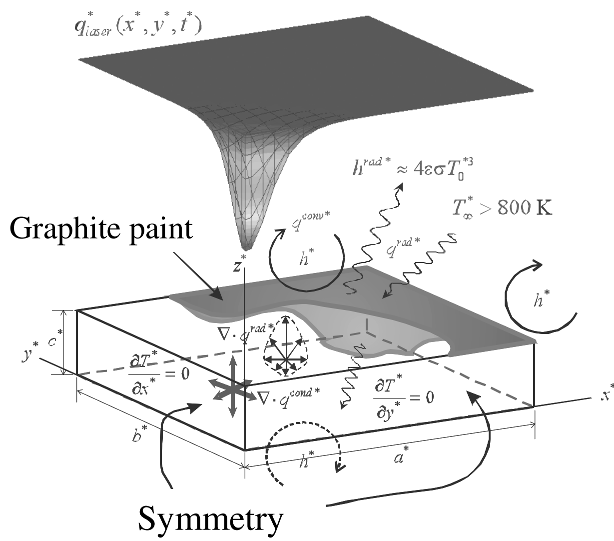


6. INVERSE PROBLEM

The inverse problem of interest is concerned with the estimation of the vector of unknown parameters

$$\mathbf{P} = [k_x, k_y, k_z, C, Bi^{rad}]$$

by using transient temperature measurements taken at the non-heated surface Γ_5 at $z = 0$.



6. INVERSE PROBLEM

For the solution of the present parameter estimation problem, different minimization techniques were used:

- the Levenberg-Marquardt method applied to the minimization of the *ordinary least squares norm (OLS)*,

$$S_{OLS}(\mathbf{P}) = [\mathbf{Y} - \mathbf{T}(\mathbf{P})]^T [\mathbf{Y} - \mathbf{T}(\mathbf{P})]$$

- the Gauss method applied to the minimization of the *maximum a posteriori objective function (MAP),*

$$S_{MAP}(\mathbf{P}) = \left[\mathbf{Y} - \mathbf{T}(\mathbf{P})\right]^T \mathbf{W} \left[\mathbf{Y} - \mathbf{T}(\mathbf{P})\right] + (\boldsymbol{\mu} - \mathbf{P})^T \mathbf{V}^{-1} (\boldsymbol{\mu} - \mathbf{P})$$

- and the Hybrid method applied to the minimization of the *ordinary least squares norm (OLS)*, which combines deterministic (BFGS method) and evolutionary/stochastic methods (Particle Swarm and Differential Evolution methods).

6. INVERSE PROBLEM

MAXIMUM LIKELIHOOD OBJECTIVE FUNCTION

$$S_{ML}(\mathbf{P}) = [\mathbf{Y} - \mathbf{T}(\mathbf{P})]^T \mathbf{W} [\mathbf{Y} - \mathbf{T}(\mathbf{P})]$$

where $\mathbf{P}$ = vector of unknown parameters
$\mathbf{Y}$ = vector of measured temperatures
$\mathbf{T}(\mathbf{P})$ = vector of estimated temperatures

6. INVERSE PROBLEM

Hypotheses:

- The errors are additive, with zero mean and normally distributed.
- The statistical parameters describing the errors are known.
- There are no errors in the independent variables.
- There is no prior information about **P**.

For uncorrelated measurements:

$$\mathbf{W} = \begin{bmatrix} 1/\sigma_1^2 & & & 0 \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_I^2 \end{bmatrix}$$

6. INVERSE PROBLEM

THE LEVENBERG-MARQUARDT METHOD

$$\mathbf{P}^{k+1} = \mathbf{P}^{k} + [\mathbf{J}^{T}\mathbf{W}\mathbf{J} + \lambda^{k}\boldsymbol{\Omega}^{k}]^{-1}\mathbf{J}^{T}\mathbf{W}[\mathbf{Y} - \mathbf{T}(\mathbf{P}^{k})]$$

where λ^k is the *damping parameter* and $\boldsymbol{\Omega}^k$ is a *diagonal matrix.*

- The Levenberg-Marquardt Method is related to ***Tikhonov's regularization*** approach.
- Compromise between steepest-descent method and Gauss' method.
- Simple, powerful and straightforward iterative procedure.
- Capable of treating complex physical situations.
- Easy to program.
- Stable and converges fast.

6. INVERSE PROBLEM

Remark: With the statistical hypotheses described above, the minimization of the least-squares norm yields *maximum likelihood* estimates, that is, the values estimated for the unknown parameters **P** are those most likely to produce the measured data **Y**.

Remark: Although very popular and useful in many situations, the minimization of the least-squares norm is a non-Bayesian estimator. A Bayesian estimator is basically concerned with the analysis of the *posterior probability density*, which is the conditional probability of the parameters **P** given the measurements **Y.**

<u>6. INVERSE PROBLEM</u>

The **<u>statistical inversion approach</u>** is based on the following principles:

1. All variables included in the model are modeled as random variables.
2. The randomness describes our degree of information concerning their realizations.
3. The degree of information concerning these values is coded in the probability distributions.
4. The solution of the inverse problem is the posterior probability distribution.

Jari P. Kaipio and Erkki Somersalo, *Computational and Statistical Methods for Inverse Problems*, Springer, 2004.

6. INVERSE PROBLEM

BAYES' FORMULA

$$\pi_{posterior}(\mathbf{P}) = \pi(\mathbf{P}\,|\,\mathbf{Y}) = \frac{\pi_{prior}(\mathbf{P})\pi(\mathbf{Y}\,|\,\mathbf{P})}{\pi(\mathbf{Y})}$$

Where: $\pi_{posterior}(\mathbf{P})$ = posterior probability density (conditional probability of the parameters **P** given the measurements **Y**)

$\pi_{prior}(\mathbf{P})$ = prior density (information about the parameters prior to the measurements)

$\pi(\mathbf{Y}|\mathbf{P})$ = likelihood function (expresses the likelihood of different measurement outcomes **Y** with **P** given)

$\pi(\mathbf{Y})$ = probability density of the measurements (normalizing constant)

$$posterior \propto prior \times likelihood$$

6. INVERSE PROBLEM

Maximum a Posteriori Objective Function

$$S_{MAP}(\mathbf{P}) = [\mathbf{Y} - \mathbf{T}(\mathbf{P})]^T \mathbf{W} [\mathbf{Y} - \mathbf{T}(\mathbf{P})] + (\boldsymbol{\mu} - \mathbf{P})^T \mathbf{V}^{-1} (\boldsymbol{\mu} - \mathbf{P})$$

Hypotheses:

- The errors are additive, with zero mean and normally distributed.
- The statistical parameters describing the errors are known.
- There are no errors in the independent variables.
- **P** is a random vector with known mean **μ** and known covariance matrix **V**.
- **P** is distributed normally and is independent of **Y**.

6. INVERSE PROBLEM

For uncorrelated measurements:

$$\mathbf{W} = \begin{bmatrix} 1/\sigma_1^2 & & & 0 \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_I^2 \end{bmatrix}$$

For the minimization of $S_{MAP}(\mathbf{P})$:

$$\frac{\partial S_{MAP}(\mathbf{P})}{\partial P_1} = \frac{\partial S_{MAP}(\mathbf{P})}{\partial P_2} = \cdots = \frac{\partial S_{MAP}(\mathbf{P})}{\partial P_N} = 0$$

$$-2\,\mathbf{J}^T\,\mathbf{W}[\mathbf{Y}-\mathbf{T}(\mathbf{P})] - 2\mathbf{V}^{-1}[\mu - \mathbf{P}] = 0$$

where **J** is the *Sensitivity Matrix*.

6. INVERSE PROBLEM

$$-2\mathbf{J}^T\mathbf{W}[\mathbf{Y}-\mathbf{T}(\mathbf{P})]-2\mathbf{V}^{-1}[\mu-\mathbf{P}]=0$$

Linear Problems: $\mathbf{J}$ does not depend on $\mathbf{P}$ ⇨ $\mathbf{T}(\mathbf{P})=\mathbf{J}\,\mathbf{P}$

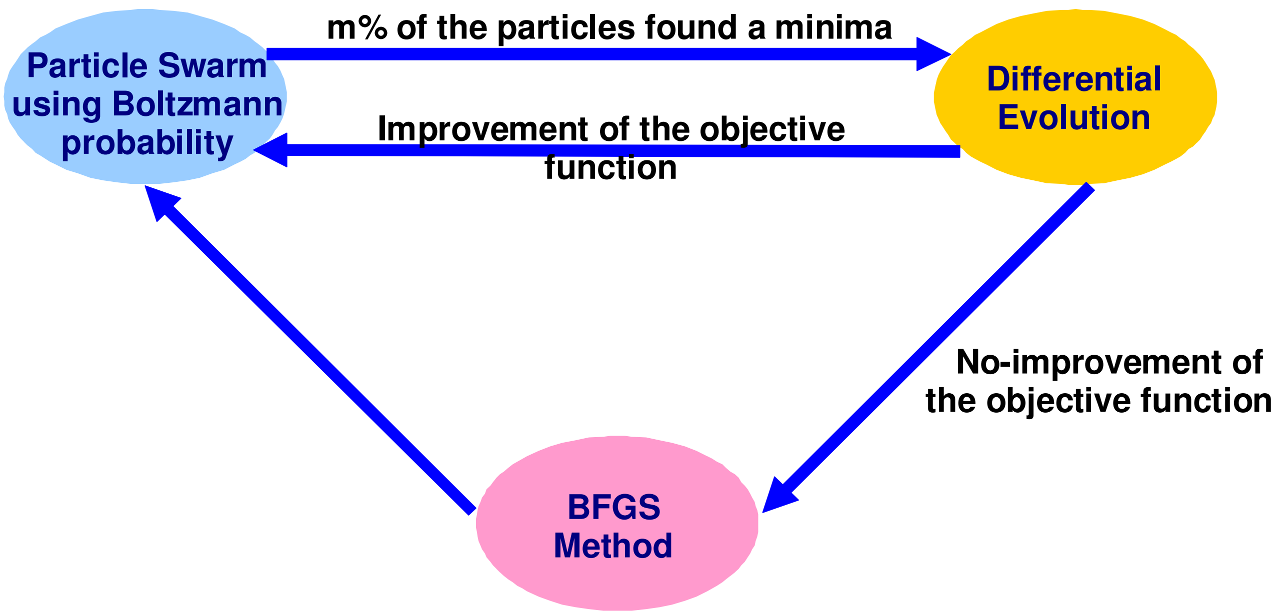
$$\mathbf{P}=[\mathbf{J}^T\mathbf{W}\,\mathbf{J}+\mathbf{V}^{-1}]^{-1}[\mathbf{J}^T\mathbf{W}\,\mathbf{Y}+\mathbf{V}^{-1}\mu]$$

Nonlinear Problems: $\mathbf{J}\equiv\mathbf{J}(\mathbf{P})$ ⇨ $\mathbf{T}(\mathbf{P})=\mathbf{T}(\mathbf{P}^k)+\mathbf{J}^k(\mathbf{P}-\mathbf{P}^k)$

$$\mathbf{P}^{k+1}=\mathbf{P}^k+[\mathbf{J}^T\mathbf{W}\mathbf{J}+\mathbf{V}^{-1}]^{-1}\{\mathbf{J}^T\mathbf{W}[\mathbf{Y}-\mathbf{T}(\mathbf{P}^k)]+\mathbf{V}^{-1}(\mu-\mathbf{P}^k)\}$$

6. INVERSE PROBLEM

Hybrid Method – Minimization of OLS



- **DE Method:**
  - Alternative to the Genetic Algorithm method.
  - Proposed in 1995 by Kenneth Price and Rainer Storn from Berkeley.
- The method initializes with a random generated random matrix **P** which contains N vector parameters **x**
- From the initial population matrix, generations are created until the best generation (optimum) is found.

- The next generation is created as:

$$\mathbf{x}_i^{k+1} = \delta_1 \mathbf{x}_i^k + \delta_2 \left[\boldsymbol{\alpha} + F\left(\boldsymbol{\beta} - \boldsymbol{\gamma}\right) \right]$$

1st parent **2nd parent Mutation included**

where

α, β and γ are three randomly chosen members of the population matrix **P**.

F is a weighting function which defines the mutation ($0.5 < F < 1$).

k is the generation counter.

δ_1 and δ_2 are delta Dirac functions that defines the crossover.

If $f(\mathbf{x}^{k+1}) < f(\mathbf{x}^k)$ → $\mathbf{x}^{k+1}$ replaces $\mathbf{x}^k$ in the population matrix **P**

If $f(\mathbf{x}^{k+1}) > f(\mathbf{x}^k)$ → $\mathbf{x}^k$ is kept in the population matrix **P** and $\mathbf{x}^{k+1}$ is discarded

- The crossover is obtained as: $\mathbf{x}_i^{k+1} = \delta_1 \mathbf{x}_i^k + \delta_2 [\boldsymbol{\alpha} + F(\boldsymbol{\beta} - \boldsymbol{\gamma})]$

$$\delta_1 = \Rightarrow \begin{cases} 0, \text{ if R} < \text{CR} \\ 1, \text{ if R} > \text{CR} \end{cases} \qquad \delta_2 = \Rightarrow \begin{cases} 1, \text{ if R} < \text{CR} \\ 0, \text{ if R} > \text{CR} \end{cases}$$

- R is a random number with uniform distribution between 0 and 1
- CR is the crossover factor ($0.5 < CR < 1$)

- **PS (Particle Swarm) method**:
  - Created in 1995 by an Electric Engineer (Russel Eberhart) and a Social-Psychologist (James Kennedy) as an alternative to Genetic Algorithm.
  - Based on the social behavior of various species (including humans).
  - Balances the individuality and sociability of individuals in order to find a optimum.

↑ Individuality

↑ Chances to find alternatives places

↓ Convergence

↑ Sociability

↑ Learning process among the individuals

↓ Chances to find alternatives places. Individuals can find a local minima

- **PS method:**
- Update process

$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1}$$

$$\mathbf{v}_i^{k+1} = \alpha \mathbf{v}_i^k + \beta \mathbf{r}_{1i}\left(\mathbf{p}_i - \mathbf{x}_i^k\right) + \beta \mathbf{r}_{2i}\left(\mathbf{p}_g - \mathbf{x}_i^k\right)$$

Individuality Sociability

where

$\mathbf{x}_i$ is i-th individual of the vector of parameters

$\mathbf{r}_{1i}$ and $\mathbf{r}_{2i}$ are are random numbers with uniform distribution between 0 and 1

$\mathbf{p}_i$ is the best value found for the vector $\mathbf{x}_i$

$\mathbf{p}_g$ is the vest value found for the entire population

$0 < \alpha < 1; \; 1 < \beta < 2$

6. INVERSE PROBLEM

INITIAL GUESS

↓

1st STEP:

- Surrogate model
- Surrogate gradient

Find the region for the global minimum with a fast solution by using the Hybrid Method.

↓

2nd STEP:

- Coupled conduction-radiation for the computation of the system response
- Surrogate model for the computation of the gradient

Find the global minimum with the complete model, but with a fast gradient.

↓

SOLUTION

6. INVERSE PROBLEM

Simulated measurements (σ = 0.8K)

C^* = 2.5x10^6 Jm^{-3}K^{-1}

k_x^* = 5 Wm^{-1}K^{-1}

k_y^* = 5 Wm^{-1}K^{-1}

k_z^* = 5 Wm^{-1}K^{-1}

h_{rad}^*= 1372 Wm^{-2}K^{-1} .

κ_a^* = 10 m^{-1}

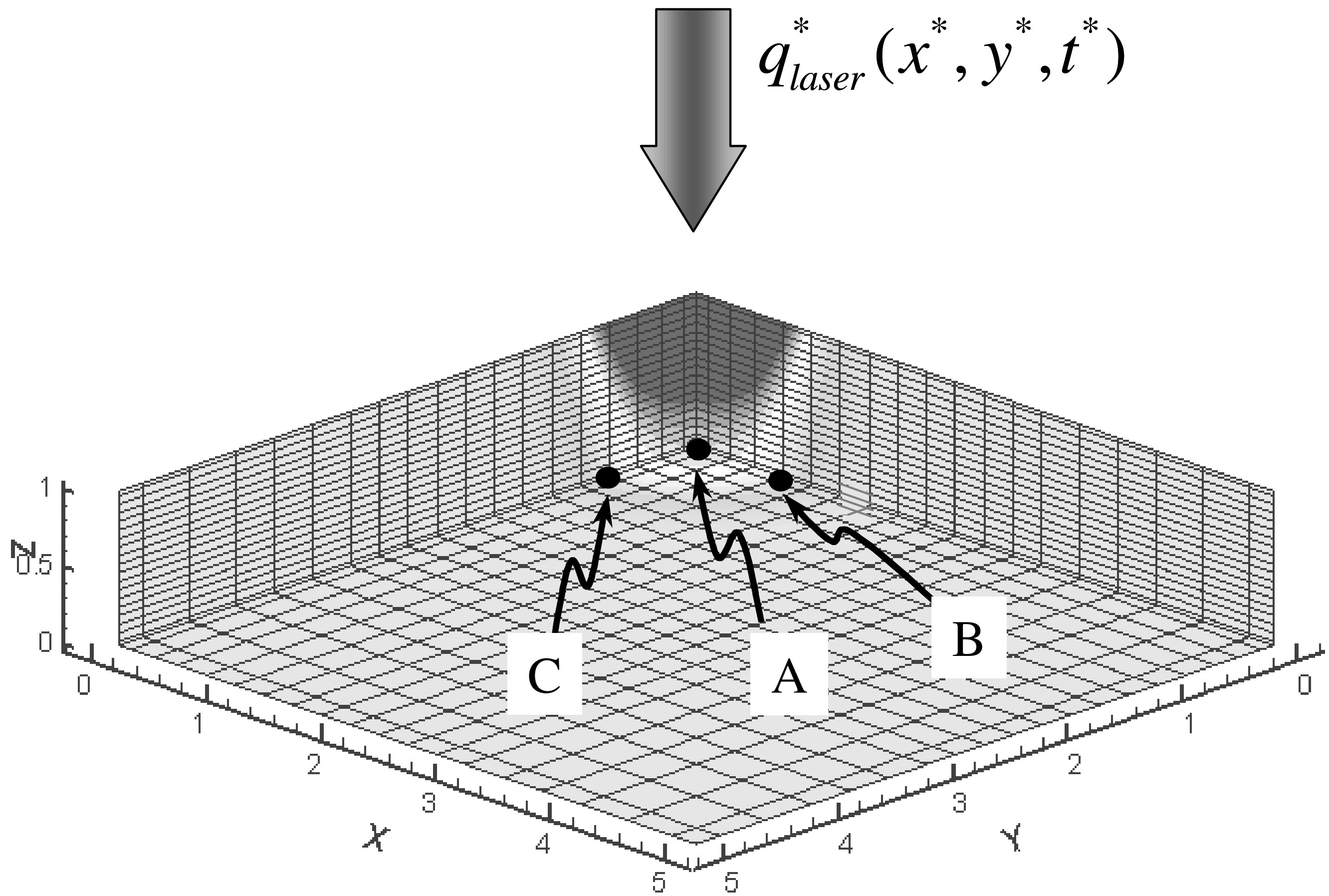
σ_s^* = 10^4 m^{-1}

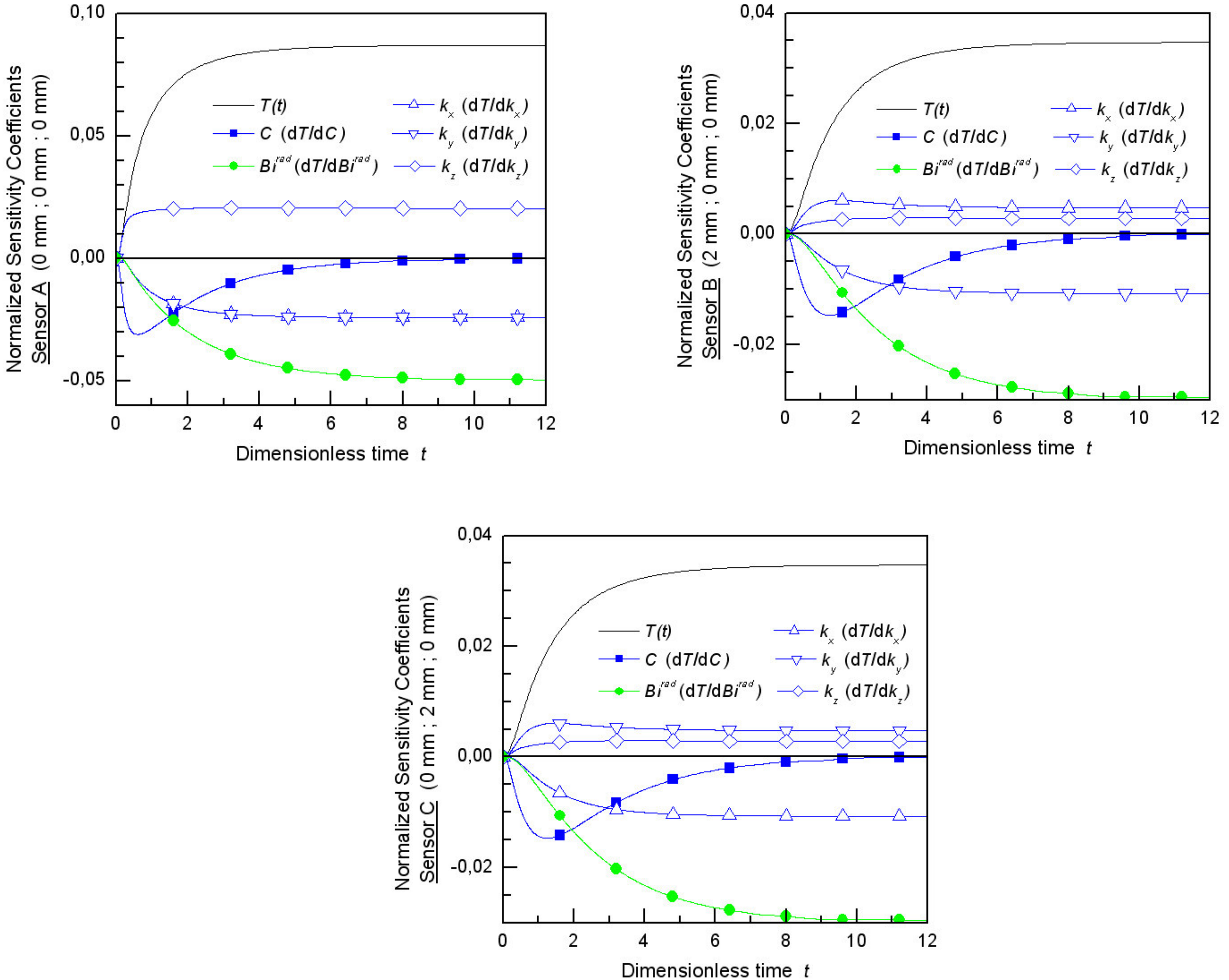
6. INVERSE PROBLEM

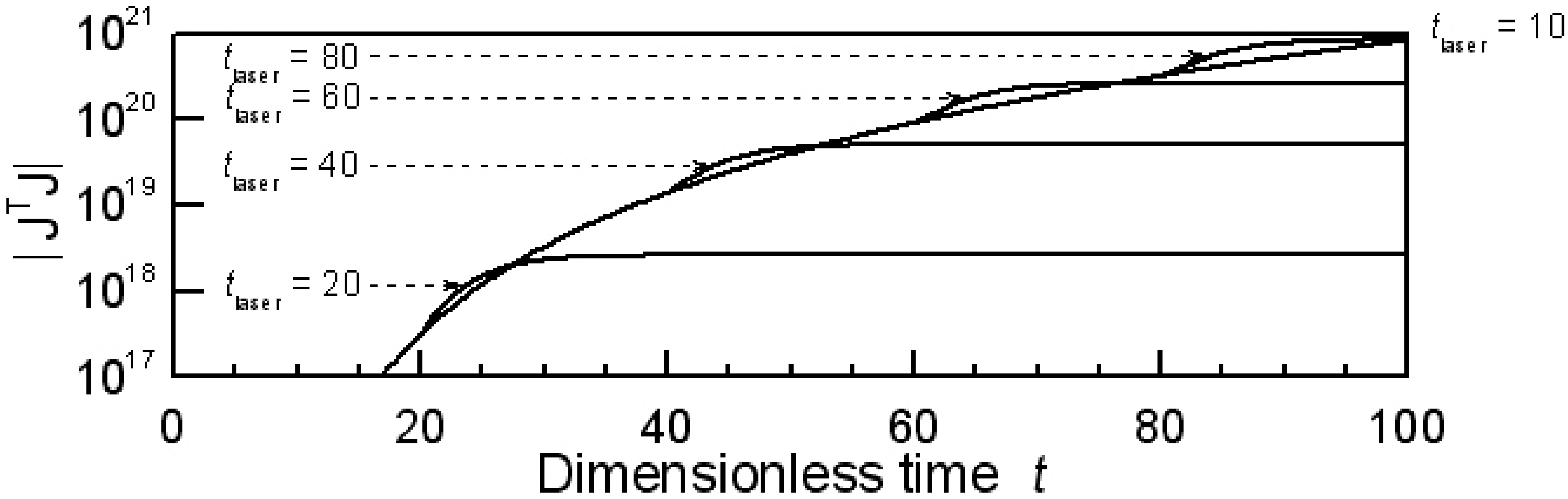
The sample was assumed to be a parallelepiped with dimensions $2a^* = 2b^* = 0.01$ m and $c^*=0.001$ m, heated by a laser with a power of 23 W and a Gaussian distribution. For the heat flux imposed by the laser, 99% of its power was assumed to be delivered within a circle with radius of 2 mm centered at the sample. The sample is assumed to be initially at the uniform temperature of 1800K, which is the same temperature of the surrounding environment.

6. INVERSE PROBLEM

$$q^{*}_{laser}(x^{*}, y^{*}, t^{*})$$




Normalized Sensitivity Coefficients
Sensor A (0 mm ; 0 mm ; 0 mm)
0,10
0,05
0,00
-0,05
0
2
4
6
8
10
12
Dimensionless time t
T(t)
C (dT/dC)
Bi^rad (dT/dBi^rad)
k_x (dT/dk_x)
k_y (dT/dk_y)
k_z (dT/dk_z)
Normalized Sensitivity Coefficients
Sensor B (2 mm ; 0 mm ; 0 mm)
0,04
0,02
0,00
-0,02
0
2
4
6
8
10
12
Dimensionless time t
T(t)
C (dT/dC)
Bi^rad (dT/dBi^rad)
k_x (dT/dk_x)
k_y (dT/dk_y)
k_z (dT/dk_z)
Normalized Sensitivity Coefficients
Sensor C (0 mm ; 2 mm ; 0 mm)
0,04
0,02
0,00
-0,02
0
2
4
6
8
10
12
Dimensionless time t
T(t)
C (dT/dC)
Bi^rad (dT/dBi^rad)
k_x (dT/dk_x)
k_y (dT/dk_y)
k_z (dT/dk_z)


$|J^TJ|$
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
t_{laser} = 80
t_{laser} = 60
t_{laser} = 40
t_{laser} = 20
t_{laser} = 10
0
20
40
60
80
100
Dimensionless time t

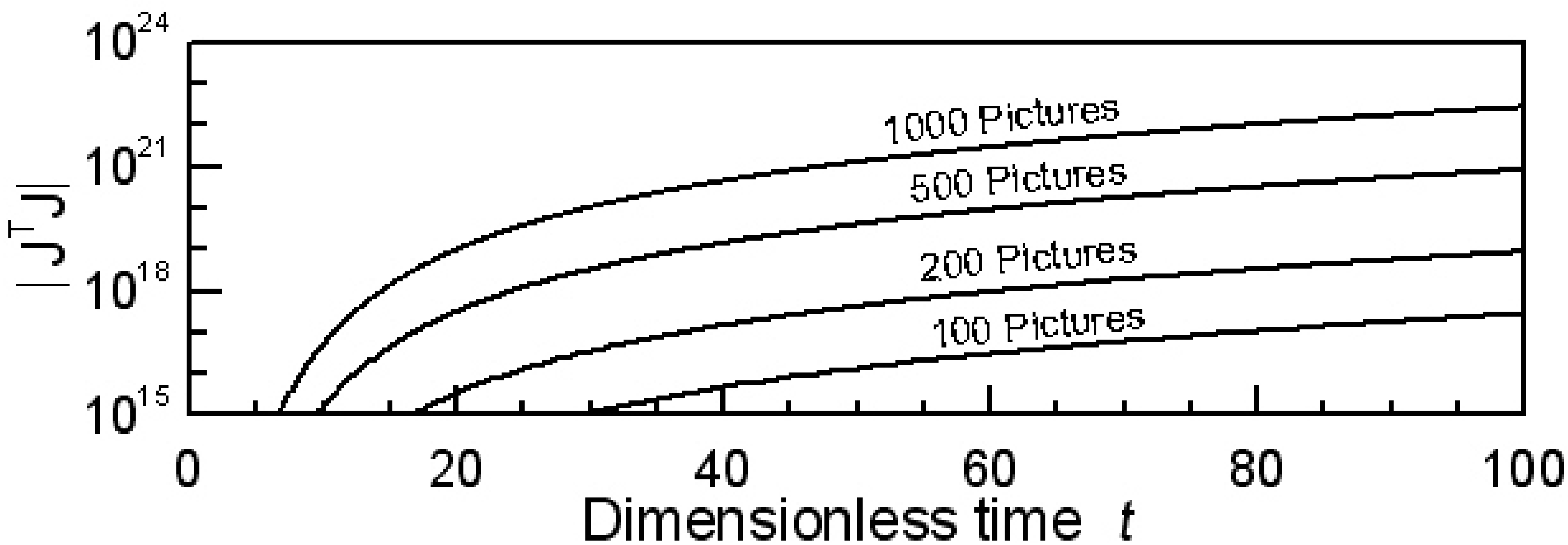

$|J^TJ|$
10^{24}
10^{21}
10^{18}
10^{15}
1000 Pictures
500 Pictures
200 Pictures
100 Pictures
0
20
40
60
80
100
Dimensionless time t

***Table 1*: Estimation techniques**

Technique	Objective Function	Method	Model for the Direct Problem	Model for the Gradient
1	Least-squares	Levenberg-Marquardt	Complete	Surrogate
2	Least-squares	Levenberg-Marquardt	Complete	Complete
3	Maximum a Posteriori	Gauss	Complete	Surrogate
4	Maximum a Posteriori	Gauss	Complete	Complete
5	Least-squares	1^{st} step: Hybrid 2^{nd} step: Levenberg-Marquardt	1^{st} step: Surrogate 2^{nd} step: Complete	1^{st} step: Surrogate 2^{nd} step: Surrogate
6	Least-squares	1^{st} step: Hybrid 2^{nd} step: Levenberg-Marquardt	1^{st} step: Surrogate 2^{nd} step: Complete	1^{st} step: Surrogate 2^{nd} step: Complete
7	1^{st} step: Least-squares 2^{nd} step:MAP	1^{st} step: Hybrid 2^{nd} step: Gauss	1^{st} step: Surrogate 2^{nd} step: Complete	1^{st} step: Surrogate 2^{nd} step: Surrogate
8	1^{st} step: Least-squares 2^{nd} step:MAP	1^{st} step: Hybrid 2^{nd} step: Gauss	1^{st} step: Surrogate 2^{nd} step: Complete	1^{st} step: Surrogate 2^{nd} step: Complete

***Table 2**: Results obtained with an initial guess close to the exact parameters*

$\left(C^{*0} = 2.8\times10^{6}\ \text{J/m}^{3}.\text{K},\ k_x^{*0} = k_y^{*0} = k_z^{*0} = 8\ \text{W/m.K},\ h^{rad*0} = 800\ \text{W/m}^{2}.\text{K}\right)$

Technique	Number of Iterations	CPU Time	Estimates				
			$C^{*}\times10^{6}$ $\text{Jm}^{-3}\text{K}^{-1}$	k_x^{*} $\text{Wm}^{-1}\text{K}^{-1}$	k_y^{*} $\text{Wm}^{-1}\text{K}^{-1}$	k_z^{*} $\text{Wm}^{-1}\text{K}^{-1}$	h^{rad*} $\text{Wm}^{-2}\text{K}^{-1}$
1	16	5h35m18s	2.51 ± 0.03	4.99 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5
2	16	6h14m04s	2.51 ± 0.03	5.00 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5
3	13	4h33m16s	2.51 ± 0.03	5.00 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5
4	6	2h36m5s	2.51 ± 0.03	5.00 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5
5	50	1h17m26s	2.19	5.74	5.80	3.6	1246
	15	5h50m55s	2.51 ± 0.03	4.99 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5
6	50	1h17m26s	2.19	5.74	5.80	3.6	1246
	16	6h33m02s	2.51 ± 0.03	5.00 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5
7	50	1h17m26s	2.19	5.74	5.80	3.6	1246
	11	4h09m59s	2.51 ± 0.03	5.00 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5
8	50	1h17m26s	2.19	5.74	5.80	3.6	1246
	4	1h59m11s	2.51 ± 0.03	5.00 ± 0.07	5.01 ± 0.07	5.0 ± 0.2	1373 ± 5

***Table 3**: Results obtained with an initial guess far from the exact parameters*

$\left(C^{*0} = 0.1\times10^{6}\ \mathrm{J/m^3.K},\ k_x^{*0} = k_y^{*0} = k_z^{*0} = 50\ \mathrm{W/m.K},\ h^{rad*0} = 5\ \mathrm{W/m^2.K}\right)$

Technique	Number of Iterations	CPU Time	Estimates				
			$C^* \times 10^6$ $\mathrm{Jm^{-3}K^{-1}}$	k_x^* $\mathrm{Wm^{-1}K^{-1}}$	k_y^* $\mathrm{Wm^{-1}K^{-1}}$	k_z^* $\mathrm{Wm^{-1}K^{-1}}$	h^{rad*} $\mathrm{Wm^{-2}K^{-1}}$
1	NC	-	-	-	-	-	-
2	NC	-	-	-	-	-	-
3	NC	-	-	-	-	-	-
4	NC	-	-	-	-	-	-
5	50 21	1h19m46s 7h44m03s	2.04 2.51 ± 0.03	5.22 4.99 ± 0.07	5.26 5.01 ± 0.07	2.9 5.0 ± 0.2	1224 1373 ± 5
6	50 16	1h19m46s 6h32m44s	2.04 2.51 ± 0.03	5.22 5.00 ± 0.07	5.26 5.01 ± 0.07	2.9 5.0 ± 0.2	1224 1373 ± 5
7	50 10	1h19m46s 3h54m49s	2.04 2.51 ± 0.03	5.22 5.00 ± 0.07	5.26 5.01 ± 0.07	2.9 5.0 ± 0.2	1224 1373 ± 5
8	50 5	1h19m46s 2h22m53s	2.04 2.51 ± 0.03	5.22 5.00 ± 0.07	5.26 5.01 ± 0.07	2.9 5.0 ± 0.2	1224 1373 ± 5

7. CONCLUSIONS

• The use of a surrogate model for the gradient did not affect the accuracy of the estimated parameters and may cause an increase on the number of iterations and CPU time, due to the loss of computational accuracy.

• The two-step approach was necessary to reach convergence if initial guesses far from the exact parameters were used in the inverse analysis.