Universidad de Santiago de Chile
Dpto Ingeniería Mecánica

Macroscopic and local thermal characterization from infrared images processing : Analytical modeling and field estimation

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PROJETO DE COOPERAÇÃO SUL-AMERICANA EM IDENTIFICAÇÃO DE PROPRIEDADES FÍSICAS EM TRANSFERÊNCIA DE CALOR E MASSA
Programa CNPq/PROSUL

Escola Sul-Americana em Identificação de Propriedades Físicas em Transferência de Calor e Massa – PROPFIS

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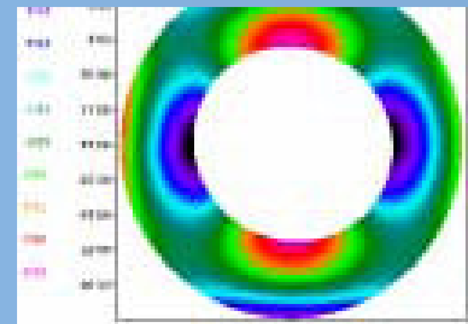
=
Chemical Engineering Laboratory for Finely Divided Solids, Energy & Environment

- South West of France
- Ministère de l'Economie, des Finances, et de l'Industrie
- Civil Engineers *(Major in Chemical Engineering)*

Background : New needs for full fields methods…

Spectacular Recent Progress

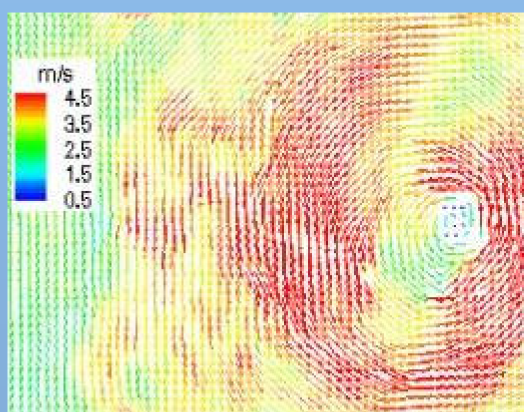
in field measurements…



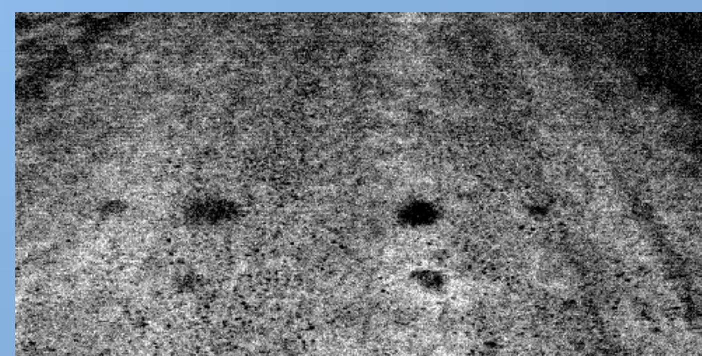
Vibrations Brake disks

Dr. Ettemeyer GmbH & Co, Germany

…Velocity, concentration, déformations, stress, temperature, density…



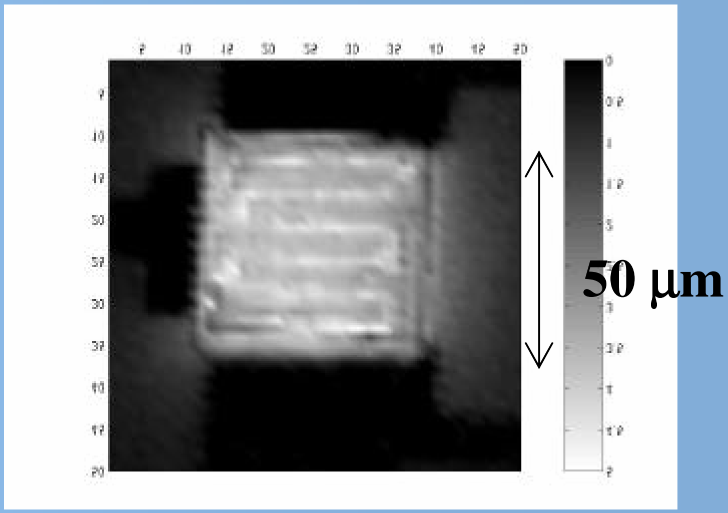
Particule Image Velocimetry(PIV)



Underground Landmines Detection from Thermal Imaging

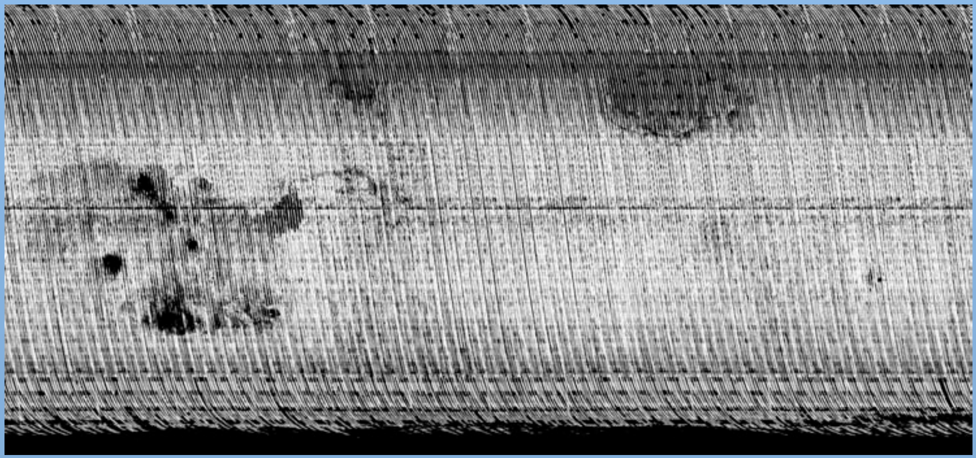
Background…

…From microscopic scale…



Temperature mapping of a micro heater
From photoreflectance imaging

…to astrophysic scale !



Mars Global Surveyor Thermal Emission Spectrometer
Thermal inertia mapping of Mars ground

...Background…

That is the « Full field methods » revolution...

Images are recorded from either :

1. One sensor scanning

Atomic force Microscopy, SThM, photoreflectance imaging, old fashion IR…

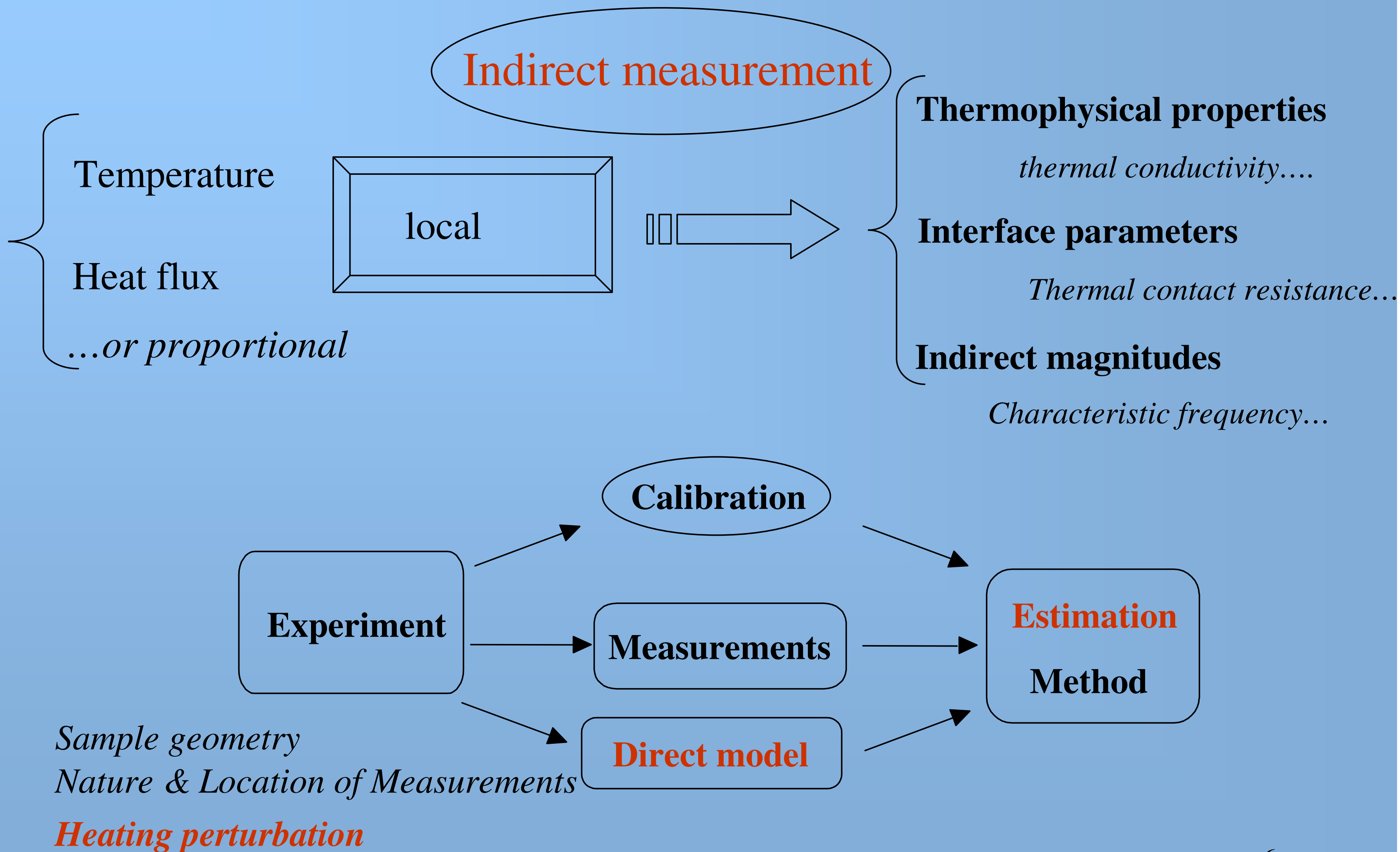
2. From a very high number of spatially distributed sensors

Focal Plane Array camera, PIV, Tomography X…

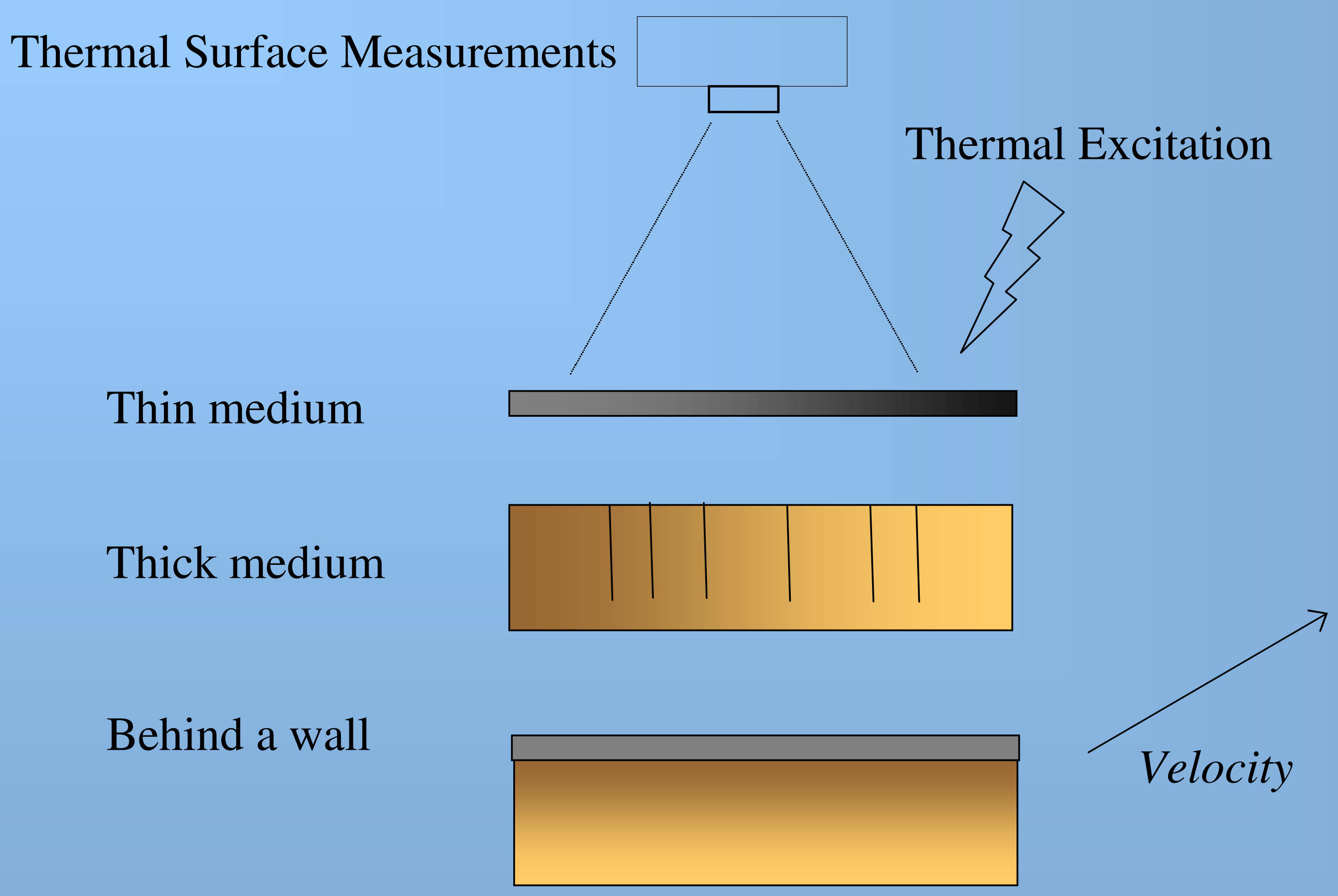
Challenge : how to process this intensive flux of data ?

Ex. : IR Camera 256 x 256 pixels 140 Hz 8 bits ⇨ ***8.75 MB/s***

Heat transfer parameters estimation



Our specific problem.....



Macroscopic and local thermal characterization

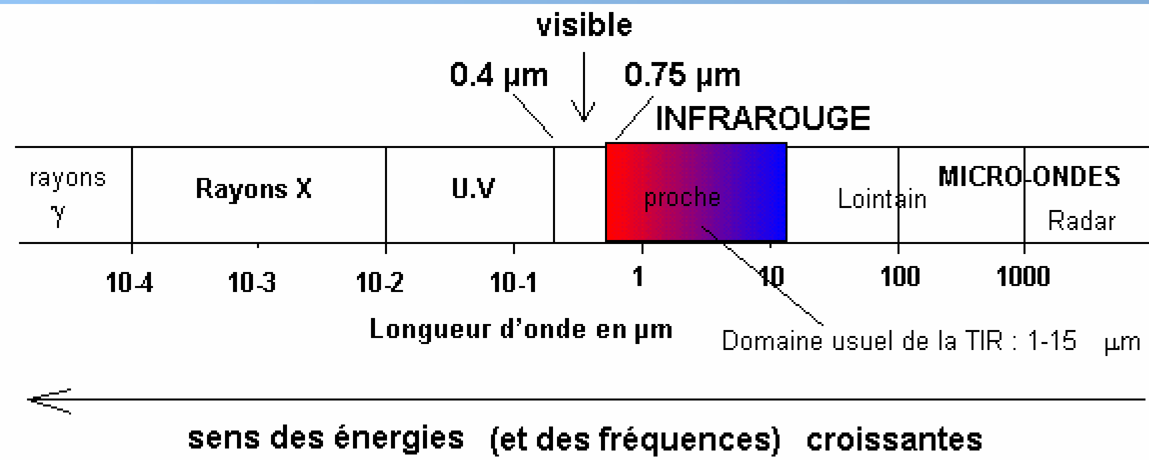
from infrared images processing :

Analytical modeling and field estimation

1. Introduction to infrared thermography
2. The Thermal Quadrupole Formalism
3. Field Estimation for Local Mapping
4. Macroscopic characterization from averaging
5. Conclusion

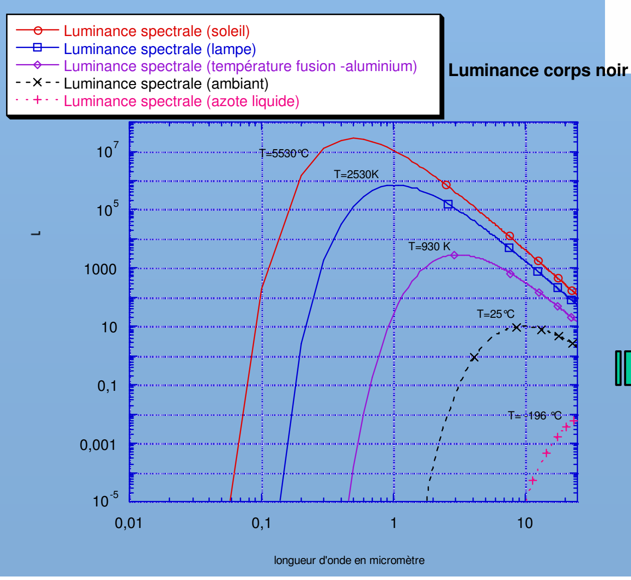
1. Introduction to infrared thermography

Thermal Radiation



1µm = 1 micromètre =0.000001m

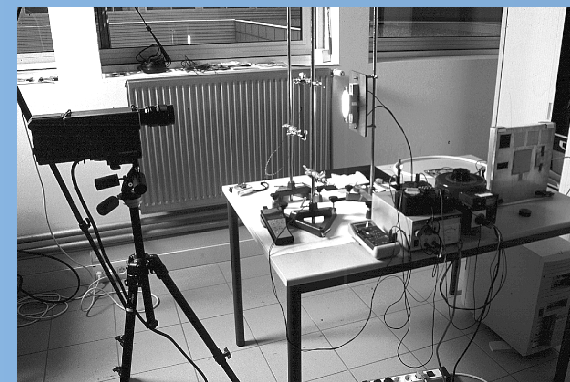
spectre visible : 0.4 µm = violet -bleu
0.47µm = bleu
0.55µm =vert- jaune
0.65µm = rouge..

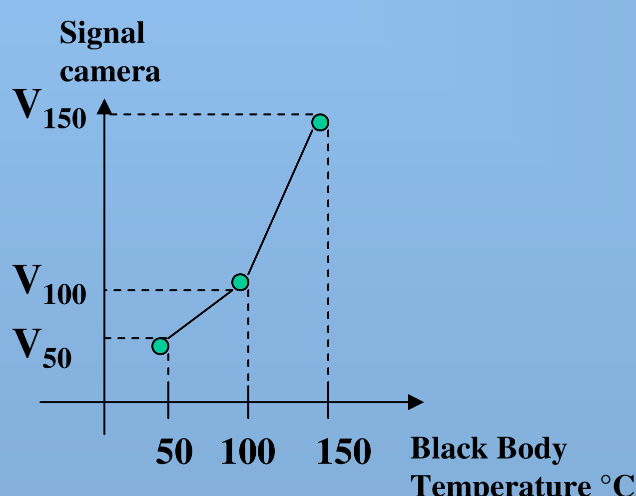


Infrared thermography : usually 1 – 15 µm

- Energy
- Sensitivity

Calibration : Black body



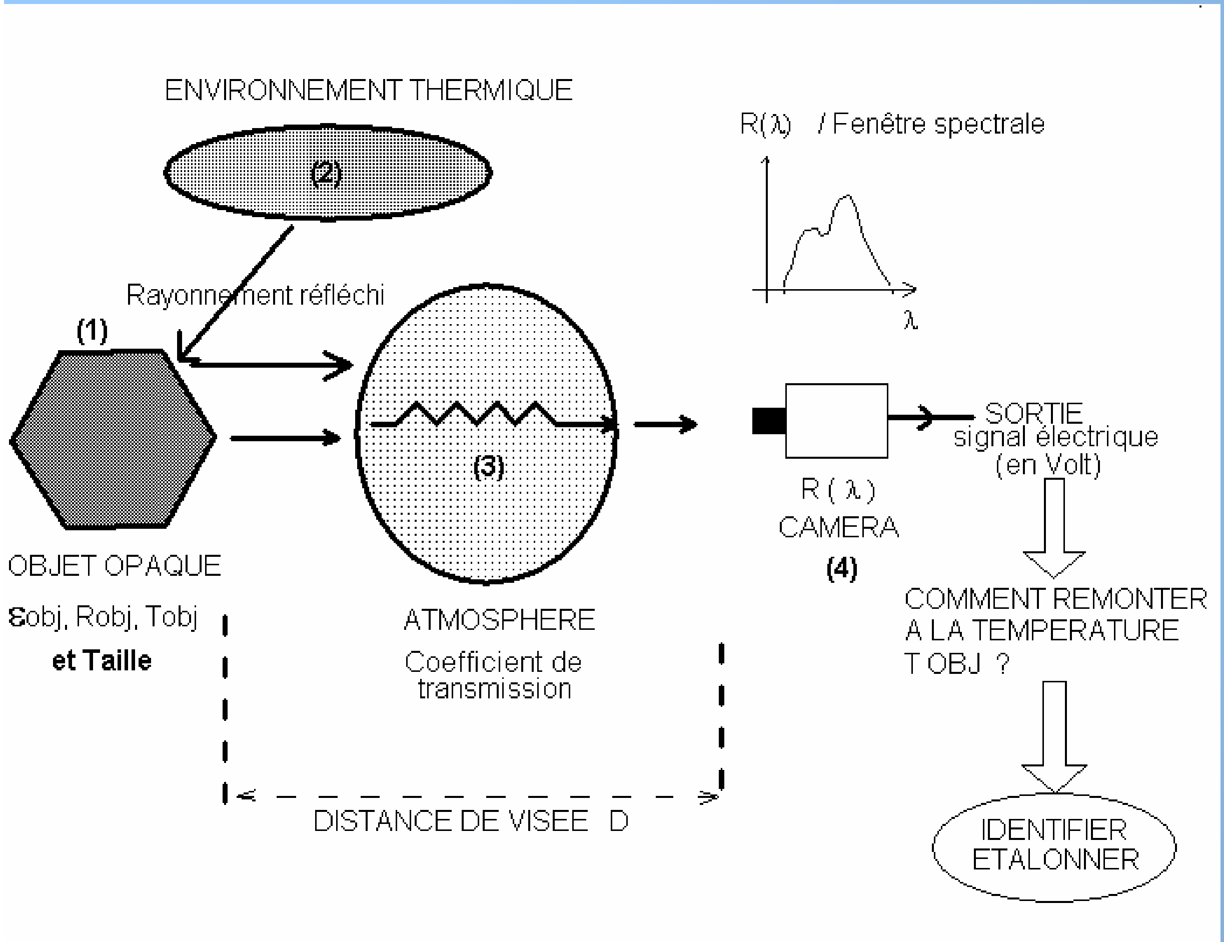


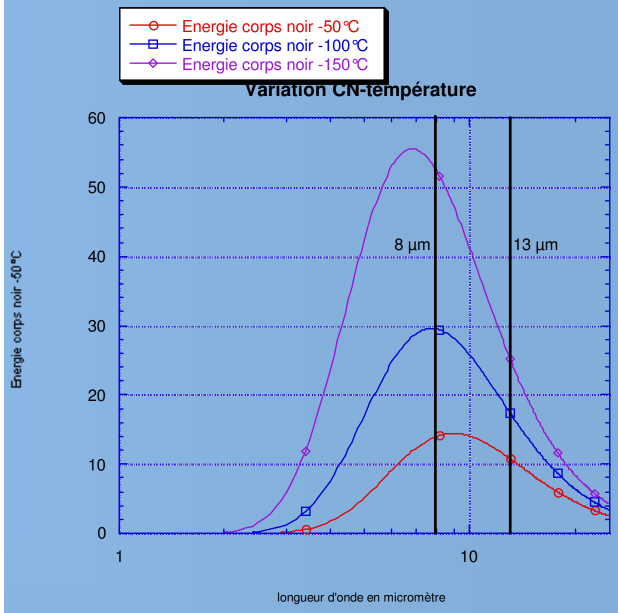
After calibration, the optical properties are necessary in order to measure the surface temperature of the sample :

Emissivity

$$\varepsilon = \frac{E_{objet}(T)}{E_{Corps\ noir}(T)}$$

Infrared measurement : typical situation

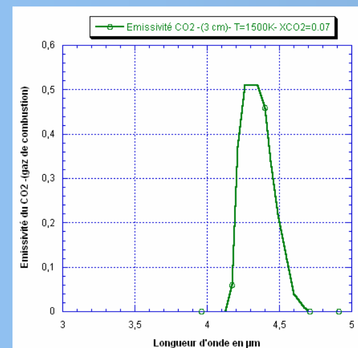




- Sensor : InSb, InGaAs, QWIP, microbolometric…
- Focal plane Array: 640x512 pixels or 320x256 pixels
- Typical : 150 – 400 Hz !!!
- thermal sensibility : 20 mK InSb, < 35 mK QWIP
- Spectral Sens. : 1 - 5 µm or 3 - 5 µm (InSb), 8,2 - 9,2 µm (QWIP)
- Integration time: about 10 µs
- Pitch : 30 µs

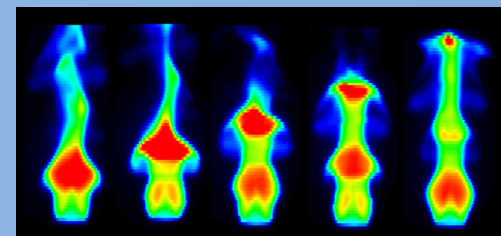
Combustion

Methane - Air



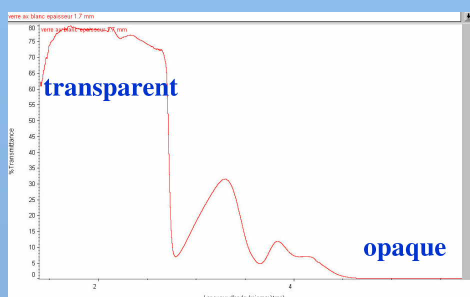
CO_2 spectral emissivity

3-5 µm



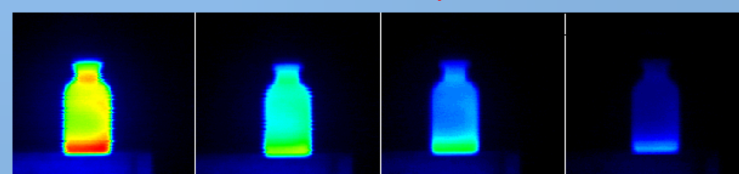
Flame instability

Glass bottle process

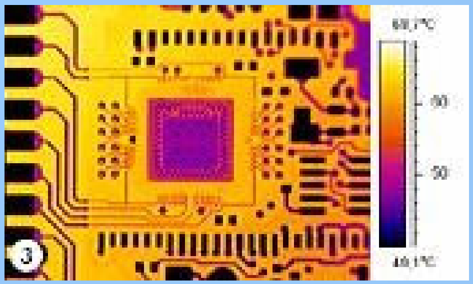


« Spectre » de transmission IR du verre

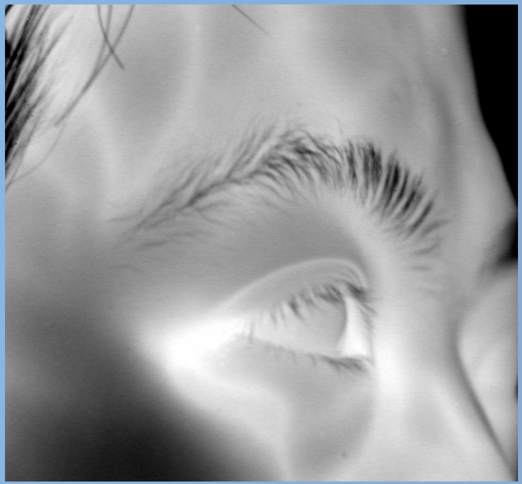
8 - 12 µm



Bottle Cooling / Surface Temperature

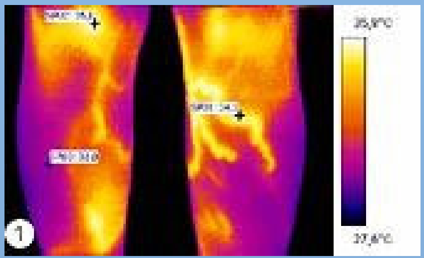


Microcomponent analysis

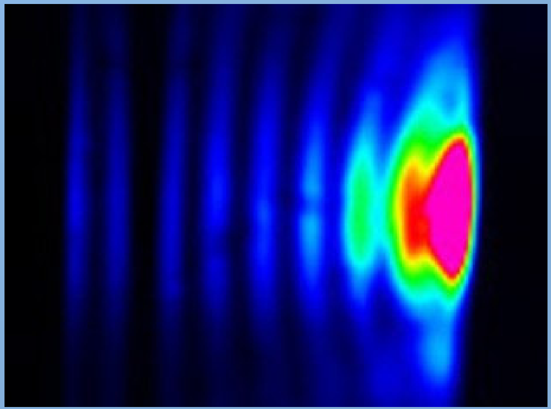


Skin & Fever

Art restoration



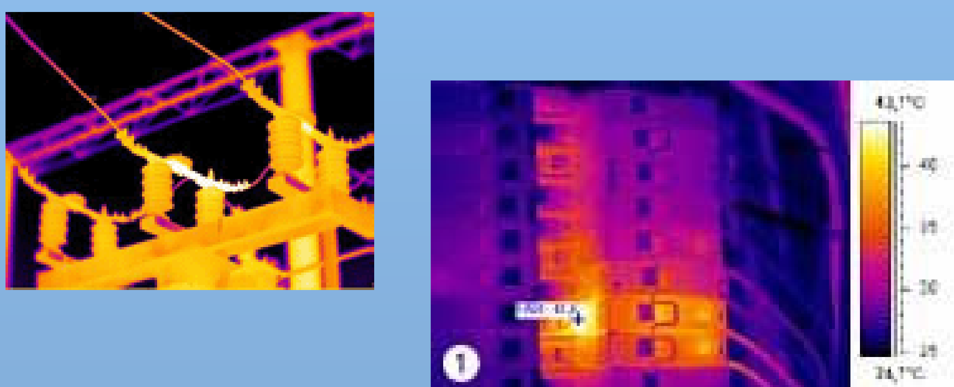
Vascular



Microwave heating

Multispectral imagery

Aircraft signature



Electrical control

Security control

Macroscopic and local thermal characterization

from infrared images processing :

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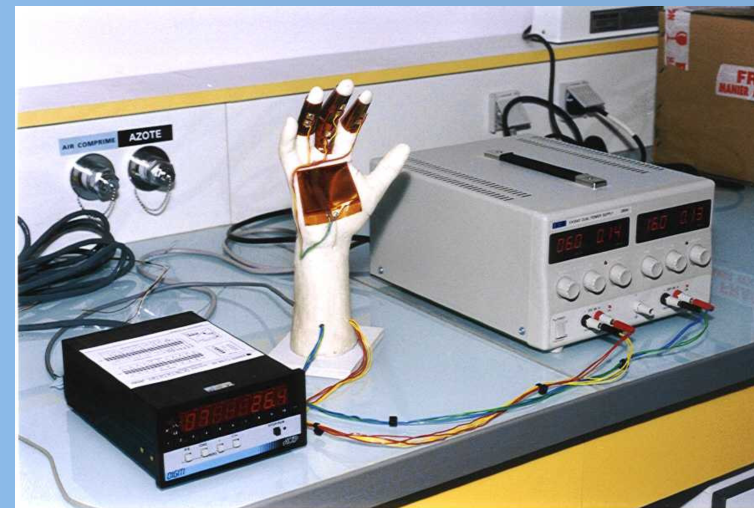
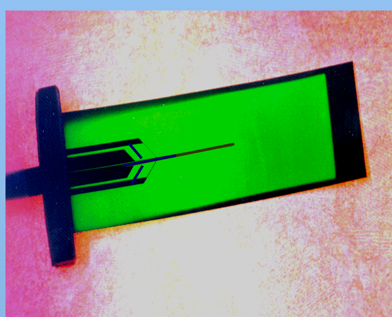
2. The Thermal Quadrupole Formalism



Motivation

Many Thermal Engineering problems
do not require the
knowledge of temperature and heat flux
in the whole domain



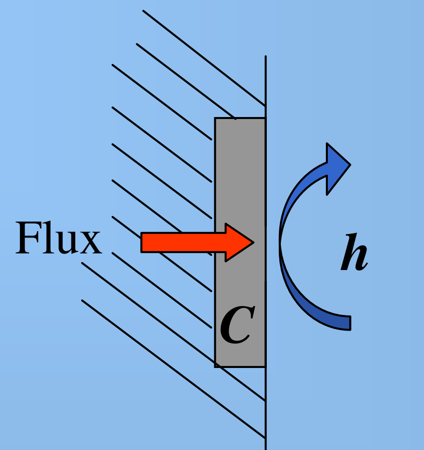
Heat transfer parameters measurement

Looking for
analytical relationships between
temperature and heat flux
at some given locations

Pseudo-random heating

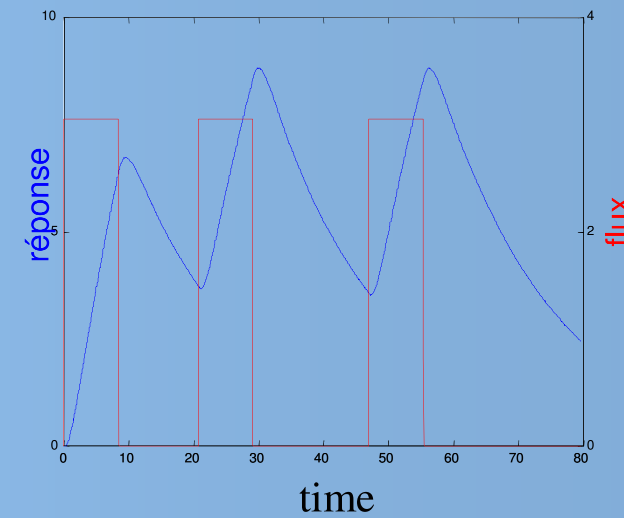
Convective coefficients mapping

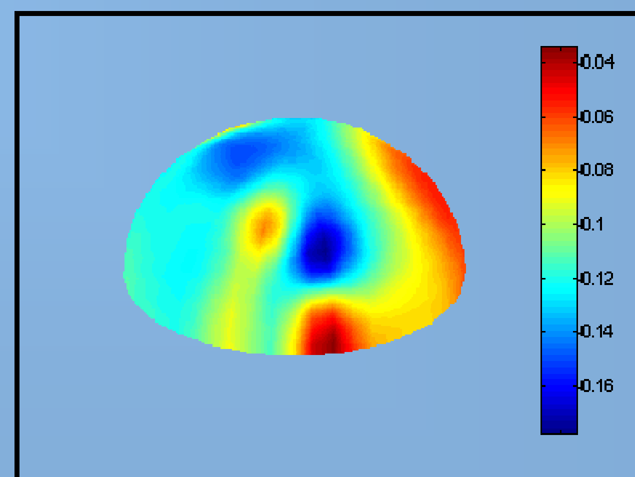
Thermal characterization
of cyclist casque



$$C\frac{dT}{dt} = \varphi(t) - hT$$

Characteristic frequencies are estimated : $\frac{h}{C}$





2. The Thermal Quadrupole Formalism

Carslaw & Jaeger	1959	*Laplace space, quadrupole network*
A. Degiovanni et al.	1988	*LEMTA, Nancy, France*
J.C. Batsale et al.	1994	*2D, 3D…Integral transforms*
D. Maillet et al.	2000	*Thermal Quadrupole Book*

A. Degiovanni
Conduction dans un «mur » multicouche avec sources : extension de la notion de quadripôle,
Int.J.Heat.Mass.Transfer. Vol 3, 553 - 557, 1988

D. Maillet, S. André, J.C. Batsale, A. Degiovanni, C. Moyne
Thermal quadrupoles : Solving the heat equation through integral transforms
Wiley, London, 2000

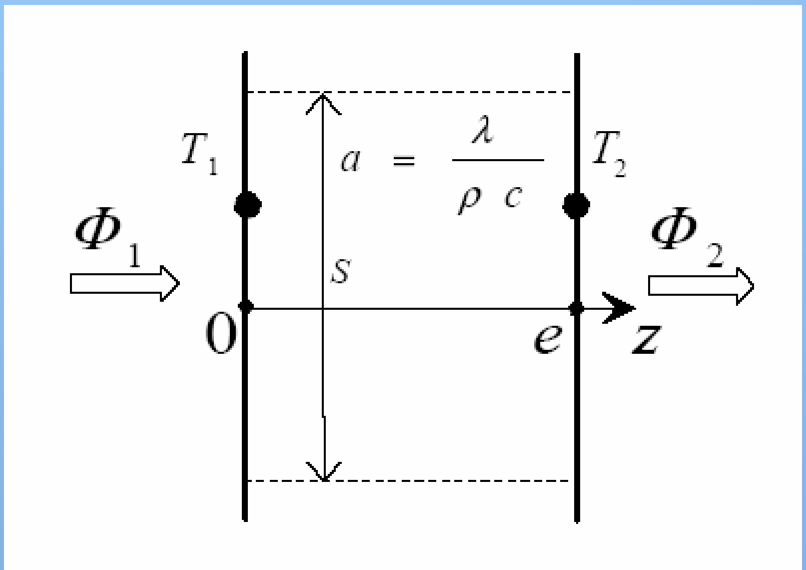
Thermal Quadrupole Formalism

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} \quad \text{with} \quad T(z)=0 \text{ for } t=0$$

Laplace Transform

$$\frac{d^2\theta}{dz^2} - \frac{s}{a}\theta = 0$$

$$K = \sqrt{\frac{s}{a}}$$



$$\theta(z,s) = G_1 \cosh(Kz) + G_2 \sinh(Kz)$$

$$\phi(z,s) = -kS\frac{d\theta}{dz}$$

$$\phi(z,s) = -kSK\left(G_1 \sinh(Kz) + G_2 \cosh(Kz)\right)$$

Substitute the input/output boundary conditions : $T_1 ; \Phi_1$ *and* $T_2 ; \Phi_2$

…in order to eliminate the coefficients G_1 *and* G_2

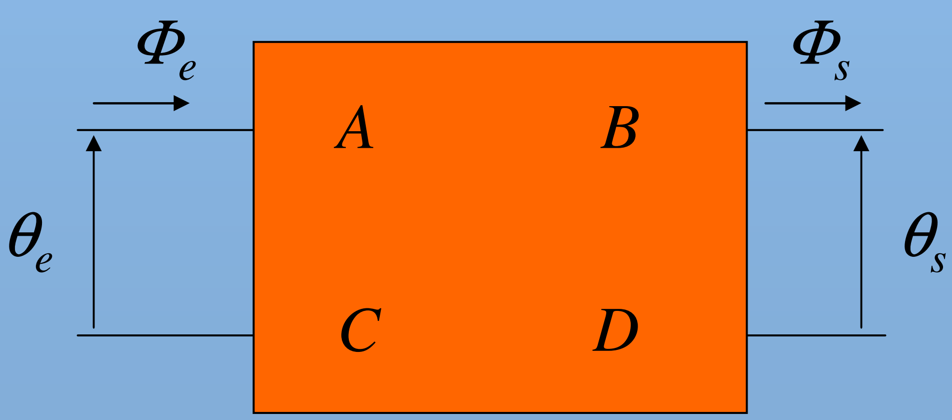
Thermal Quadrupole Formalism

$$\begin{bmatrix}\theta_e(s)\\ \Phi_e(s)\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta_s(s)\\ \Phi_s(s)\end{bmatrix} \qquad \begin{bmatrix}A = \cosh(Ke) & B = \dfrac{\sinh(Ke)}{kKS}\\ C = kKS\,\sinh(Ke) & D = \cosh(Ke)\end{bmatrix}$$

OR

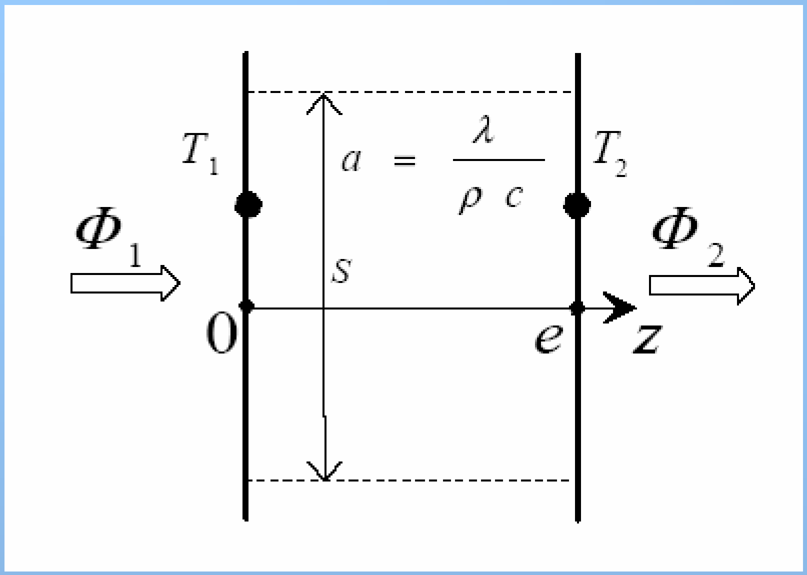
$$\begin{bmatrix}\theta_e(s)\\ \varphi_e(s)\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta_s(s)\\ \varphi_s(s)\end{bmatrix} \qquad \begin{bmatrix}A = \cosh(Ke) & B = \dfrac{\sinh(Ke)}{kK}\\ C = kK\,\sinh(Ke) & D = \cosh(Ke)\end{bmatrix}$$

thermal conductivity (k), *thermal diffusivity* (K), *Thickness* (e)



Intrinsic
linear relationship
between
input / output variables

Thermal Quadrupole Formalism



$$\begin{bmatrix}\theta_e(s)\\ \Phi_e(s)\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta_s(s)\\ \Phi_s(s)\end{bmatrix}$$

Well-posed problem

⇓

Two boundary conditions are known

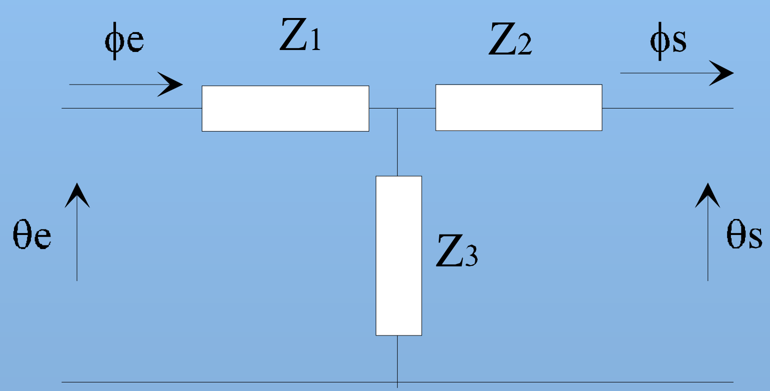
⇒ **Two remaining equations given by the quadrupole**

Thermal Quadrupole Formalism

$$\begin{bmatrix} \theta_e(s) \\ \Phi_e(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_s(s) \\ \Phi_s(s) \end{bmatrix}$$

$$AD - BC = 1 \qquad A = D$$

(Symmetrical system)

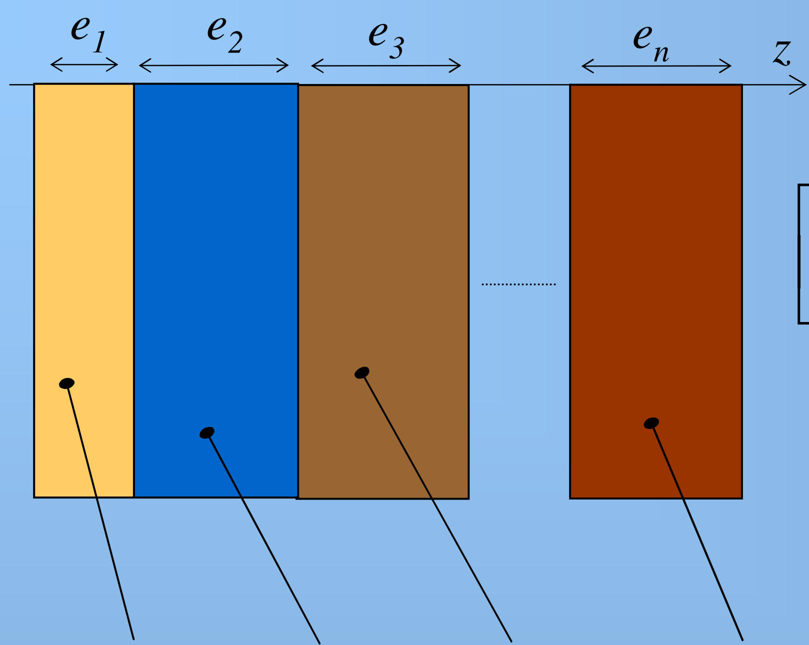


$$Z_1 = \frac{A-1}{C} \qquad Z_2 = \frac{D-1}{C} \qquad Z_3 = \frac{1}{C}$$

$$\frac{at}{e^2} >> 1 \quad \Rightarrow \quad s \rightarrow 0 \quad \Rightarrow \quad \begin{cases} Z_1 = Z_2 \rightarrow \dfrac{e}{2kS} \\ Z_3 \rightarrow \dfrac{1}{\rho c_p e S p} \end{cases}$$

Multilayer System

$$\begin{bmatrix}\theta_e \\ \phi_e\end{bmatrix}$$



$$\begin{bmatrix}\theta_s \\ \phi_s\end{bmatrix}$$

$$\begin{bmatrix}\theta_e \\ \phi_e\end{bmatrix} = \begin{bmatrix}A\,B \\ C\,D\end{bmatrix}\begin{bmatrix}\theta_s \\ \phi_s\end{bmatrix}$$

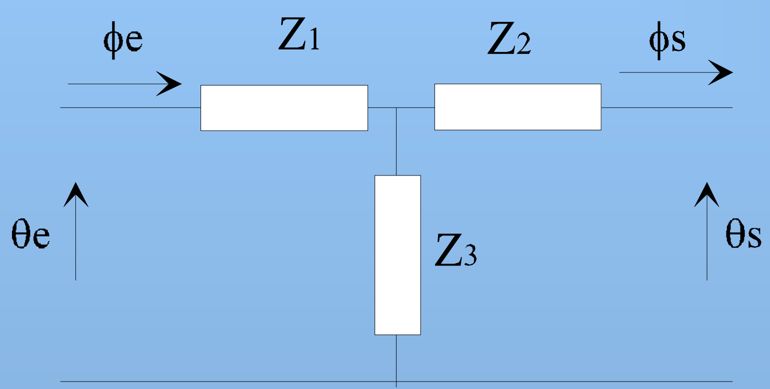
$$\begin{bmatrix}A\,B \\ C\,D\end{bmatrix} = \begin{bmatrix}A_1\,B_1 \\ C_1\,D_1\end{bmatrix}\begin{bmatrix}A_2\,B_2 \\ C_2\,D_2\end{bmatrix}\begin{bmatrix}A_3\,B_3 \\ C_3\,D_3\end{bmatrix}\ldots\begin{bmatrix}A_n\,B_n \\ C_n\,D_n\end{bmatrix}$$

…As well as the interface vectors :

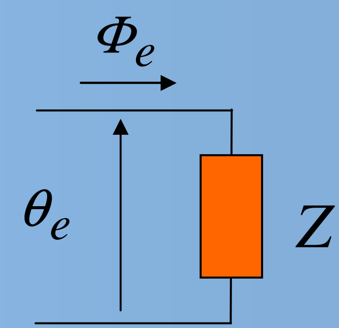
$$\begin{bmatrix}\theta_i \\ \phi_i\end{bmatrix} = \begin{bmatrix}A_i & B_i \\ C_i & D_i\end{bmatrix}\ldots\begin{bmatrix}A_n & B_n \\ C_n & D_n\end{bmatrix}\begin{bmatrix}\theta_s \\ \phi_s\end{bmatrix}$$

Semi-infinite medium

$$\begin{bmatrix}\theta_e(s)\\ \Phi_e(s)\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta_s(s)\\ \Phi_s(s)\end{bmatrix} \xrightarrow[e\to\infty]{} \theta_e = Z\Phi_e$$



$$\begin{cases} Z_1 \to Z/2 \\ Z_2 \to Z/2 \\ Z_3 \to 0 \end{cases}$$



$$Z = \frac{1}{S\sqrt{k\rho c}\sqrt{s}}$$

Interface conditions

Thermal contact resistance

$$T_1 - T_2 = R_c \phi$$

$$\begin{bmatrix} \theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & R_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \Phi_2 \end{bmatrix}$$

Newton B.C.

$$\phi = hS(T_1 - T_\infty)$$

$$\begin{bmatrix} \theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{hS} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_\infty \\ \Phi_\infty \end{bmatrix}$$

Heat Capacity condition

$$C \frac{dT}{dt} = \phi_1 - \phi_2$$

$$\begin{bmatrix} \theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Cs & 1 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \Phi_2 \end{bmatrix}$$

Internal heat sources and initial temperature imbalance

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} + \frac{g(z,t)}{k} \quad \text{with} \quad T(z)=T_0(z) \quad \text{for} \quad t=0$$

$$\frac{d^2\theta}{dz^2} + \frac{G(z,s)}{k} + \frac{T_0(z)}{a} - \frac{s}{a}\theta = 0$$

$$\theta(z,s) = G_1 \cosh(Kz) + G_2 \sinh(Kz) + \theta_{part}$$

$$\begin{bmatrix} \theta_e(s) \\ \Phi_e(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_s(s) \\ \Phi_s(s) \end{bmatrix} - \begin{bmatrix} X \\ Y \end{bmatrix}$$

Cylindrical coordinate system

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

⇓

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - \frac{s}{a}\theta = 0$$

$$\theta(z,s) = G_1 I_0(Kr) + G_2 K_0(Kr)$$

Bessel functions

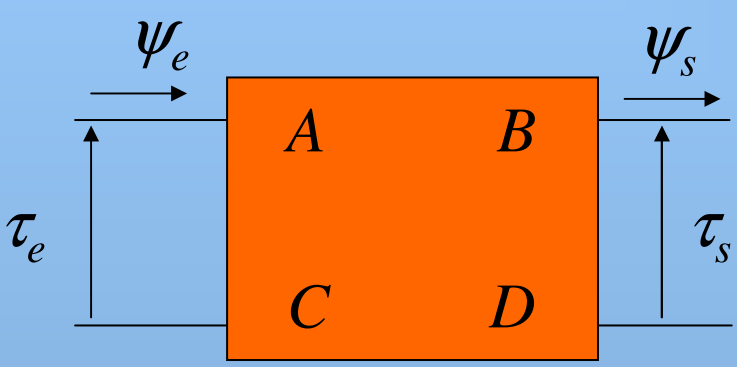
$$\phi = -kS\frac{d\theta}{dr} \qquad \text{with} \qquad S = 2\pi rL$$

Two or three dimensional cases

Laplace-Fourier Transform

$$\frac{1}{a}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2} \quad \Rightarrow \quad \tau(\beta_n,z,s)=\int_0^\infty\int_0^L T(x,z,t)\exp(-st)\cos(\beta_n x)\,dx\,dt$$

β_n = *Eigenvalues from boundary-value problem relative to x*



$$\frac{d^2\tau}{dx^2} - K\tau = 0$$

Generalized frequency

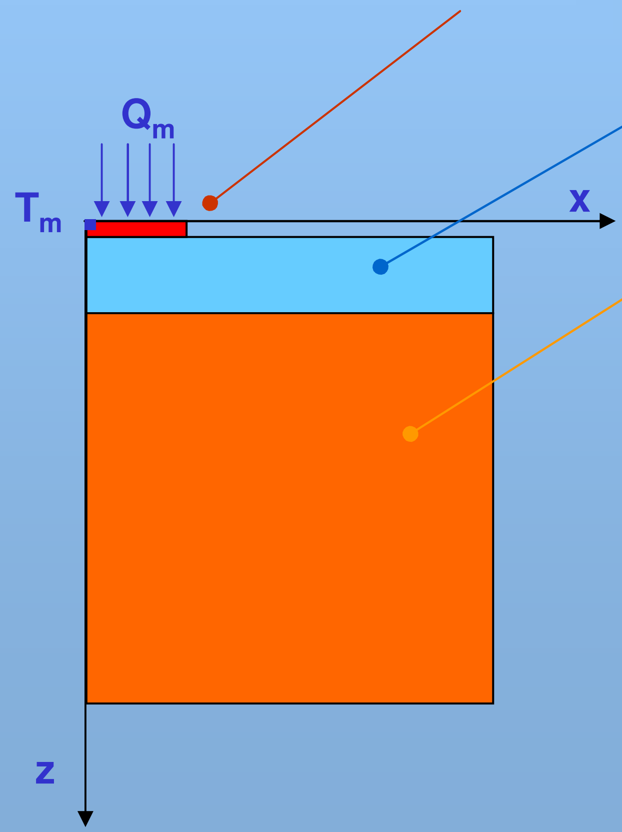
$$K=\sqrt{\beta_n^2+\frac{s}{a}}$$

$$\begin{bmatrix}\tau_e(s)\\ \psi_e(s)\end{bmatrix}=\begin{bmatrix}A(\beta_n,s) & B(\beta_n,s)\\ C(\beta_n,s) & D(\beta_n,s)\end{bmatrix}\begin{bmatrix}\tau_s(s)\\ \psi_s(s)\end{bmatrix}$$

J.C Batsale, D. Maillet, A.Degiovanni
Extension de la notion de quadripôle thermique à l'aide de transformations intégrales : calcul du transfert thermique au travers d'un défaut plan bidimensionnel, Int.J.Heat.Mass.Transfer. Vol 37, 111 - 127, 1994

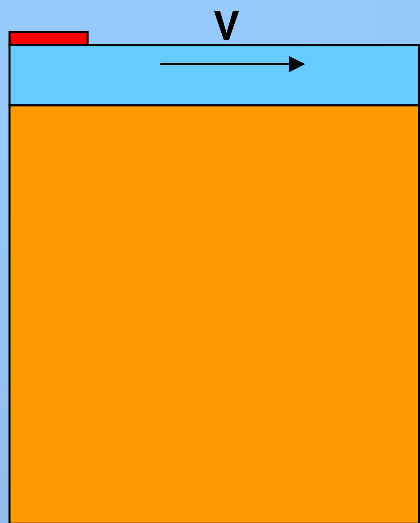
Multilayer example : super insulating materials characterization

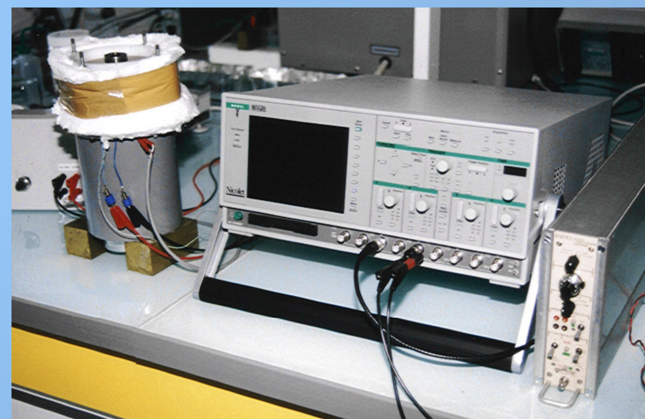
$$\begin{bmatrix} \tau_m(\beta_n,s) \\ \varphi_m(\beta_n,s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (\rho c_p)_s e_s s & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \tau(\beta_n, e_s + e, s) \\ \lambda_2 \sqrt{\frac{p}{a_2} + \alpha_n^2}\, \tau(\beta_n, e_s + e, s) \end{bmatrix}$$



Monolithic silica aerogel

Extension for thermal charaterization of liquids in Couette flow



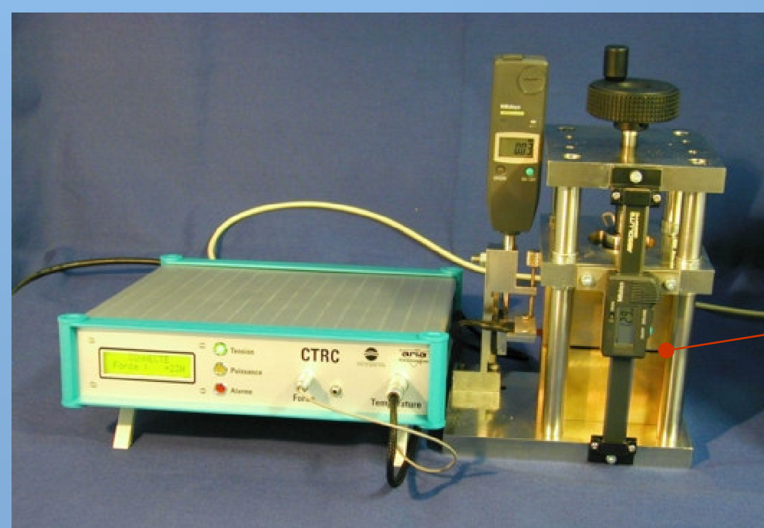


Thermal conductivity

Viscosity

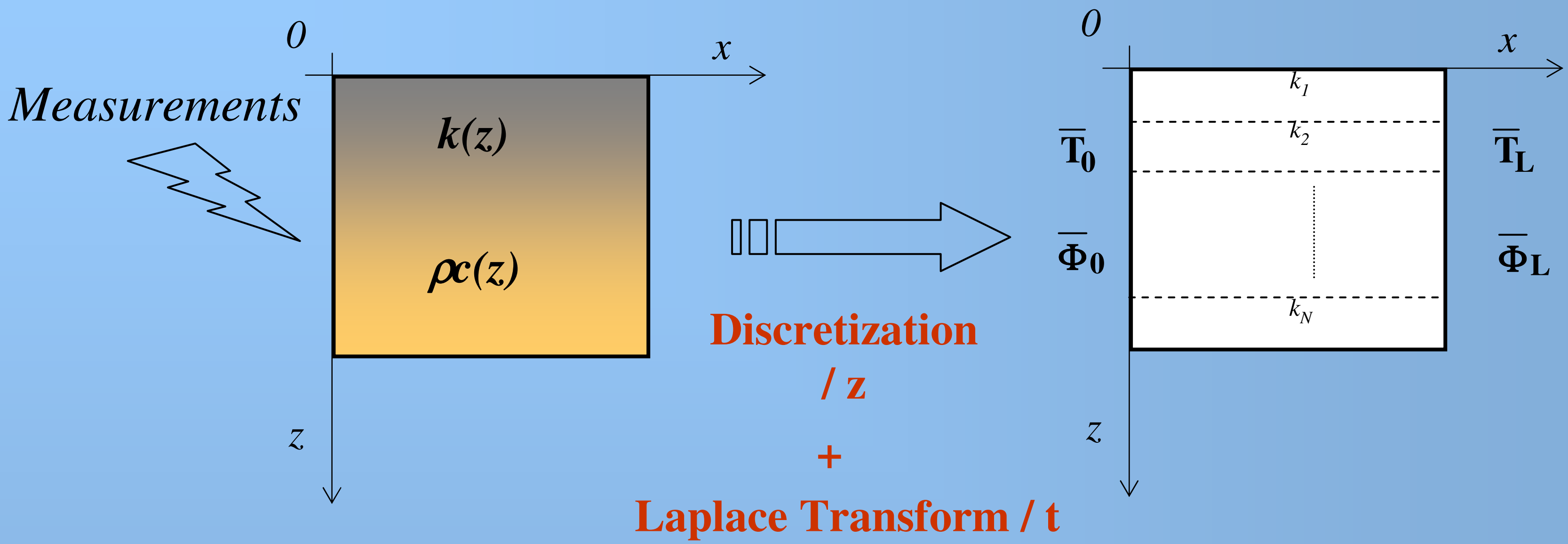
Transfer of technology :

« Capthermic » start-up



Compressible material

Semi-analycal extension for heterogeneous media



$$k(z)\frac{\partial^2 T}{\partial x^2}+\frac{\partial}{\partial z}\left(k(z)\frac{\partial T}{\partial z}\right)=(\rho c)(z)\frac{\partial T}{\partial t} \quad \Longrightarrow \quad \mathbf{M}_s\left(\mathbf{M}_{//}+\mathbf{G}s\right)\overline{\mathbf{T}}-\frac{d^2\overline{\mathbf{T}}}{dx^2}=\mathbf{0}$$

Matrix relative to transfer in the x direction

transverse conduction versus z

Semi-analycal extension for heterogeneous media

1. Diagonalization

$$\mathbf{M}_s(\mathbf{M}_{//} + \mathbf{G}s) = \mathbf{P}\mathbf{\Omega}\mathbf{P}^{-1}$$

$$\mathbf{V} = \mathbf{P}^{-1}\overline{\mathbf{T}}$$

2. Resolution in the eigenvalues space

$$\mathbf{\Omega}\mathbf{V} - \frac{d^2\mathbf{V}}{dx^2} = \mathbf{0}$$

$$\mathbf{J}_{\mathbf{V}} = -dz\frac{d\mathbf{V}}{dx}$$

$$\mathbf{A}_{\mathbf{V}} = \mathbf{D}_{\mathbf{V}} = cosh(\sqrt{\mathbf{\Omega}}L)$$

$$\mathbf{B}_{\mathbf{V}} = sinh(\sqrt{\mathbf{\Omega}}L)\left(\sqrt{\mathbf{\Omega}}dz\right)^{-1}$$

$$\mathbf{C}_{\mathbf{V}} = \left(dz\sqrt{\mathbf{\Omega}}\right)sinh(\sqrt{\mathbf{\Omega}}L)$$

3. Return to temperature / flux basis

$$\mathbf{A} = \mathbf{P}\mathbf{A}_{\mathbf{V}}\mathbf{P}^{-1}$$

$$\mathbf{B} = \mathbf{P}\mathbf{B}_{\mathbf{V}}(\mathbf{K}\mathbf{P})^{-1}$$

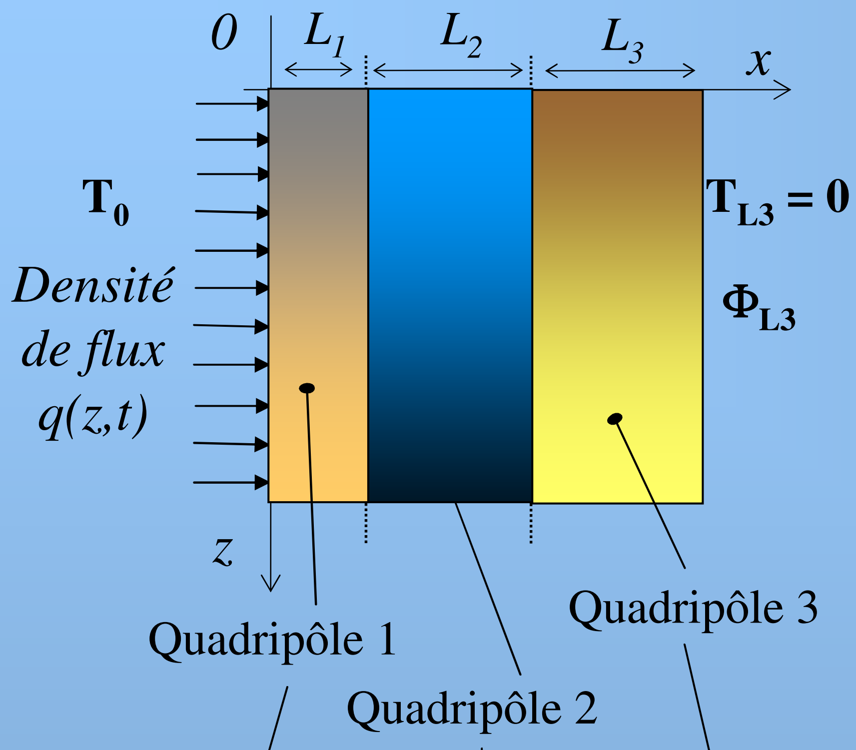
$$\mathbf{C} = \mathbf{K}\mathbf{P}\mathbf{C}_{\mathbf{V}}\mathbf{P}^{-1}$$

$$\mathbf{D} = \mathbf{K}\mathbf{P}\mathbf{D}_{\mathbf{V}}(\mathbf{K}\mathbf{P})^{-1}$$

$$\overline{\mathbf{\Phi}} = -dz\mathbf{K}\frac{d\overline{\mathbf{T}}}{dx} = \mathbf{K}\mathbf{P}\mathbf{J}_{\mathbf{V}}$$

$$\begin{bmatrix}\overline{\mathbf{T}}\\ \overline{\mathbf{\Phi}}\end{bmatrix}_{x1} = \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{T}}\\ \overline{\mathbf{\Phi}}\end{bmatrix}_{x2}$$

Implementation of the method



Direct computation with N points

(Numerical methods => N^2)

$$\begin{bmatrix} T_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} 0 \\ \Phi_{L3} \end{bmatrix}$$

Wall temperature field as a function of the input heat flux

Macroscopic and local thermal characterization

from infrared images processing :

Analytical modeling and field estimation

1. Introduction to infrared thermography
2. The Thermal Quadrupole Formalism
3. Field Estimation for Local Mapping
4. Macroscopic characterization from averaging
5. Conclusion

Linear least squares
Maximum likelyhood Estimator

$$T = X\beta$$

Hypothesis :

- zero mean and additive errors

- β constant and unknown before the estimation and X_{ij} known without error

- constant variance (σ known) and uncorrelated errors

$$S = (\mathbf{Y} - X\hat{\boldsymbol{\beta}})^t.(\mathbf{Y} - X\hat{\boldsymbol{\beta}})$$

Estimator

$$\hat{\beta} = (X^t X)^{-1} X^t \hat{Y}$$

Estimation error

$$\mathrm{cov}(e_\beta) = (X^t X)^{-1} \sigma^2$$

Example : Non-Stationary Signal -Estimation of one parameter

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ \vdots \\ f_N \end{bmatrix} \beta$$

$$\hat{\beta} = \frac{\sum_{i=1}^{N} f_i \hat{T}_i}{\sum_{i=1}^{N} f_i^{\,2}}$$

$$\sigma_\beta^{\,2} = \frac{\sigma^2}{\sum_{i=1}^{N} f_i^{\,2}}$$

$$\left\| f(t) \right\| = \sqrt{\int_0^{t_{\max}} f^2(t)\,dt}$$

$$\sigma_\beta^{\,2} = \frac{t_{\max}\,\sigma^2}{N \left\| f \right\|^2}$$

-If *N* small, T_i must be chosen as f is maximum

- T_i regularly spaced, *N* must be chosen <u>as great as possible!</u>

Estimation of several parameters

(f and g assumed to be orthogonal)

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \underbrace{\begin{bmatrix} f_1 & g_1 \\ f_2 & g_2 \\ \vdots & \vdots \\ f_N & g_N \end{bmatrix}}_{\boldsymbol{X}} \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\hat{\beta}_1 = \frac{\left(\boldsymbol{f}^t \cdot \hat{\boldsymbol{T}}\right)}{\left(\boldsymbol{f}^t \cdot \boldsymbol{f}\right)} \qquad \hat{\beta}_2 = \frac{\left(\boldsymbol{g}^t \cdot \hat{\boldsymbol{T}}\right)}{\left(\boldsymbol{g}^t \cdot \boldsymbol{g}\right)}$$

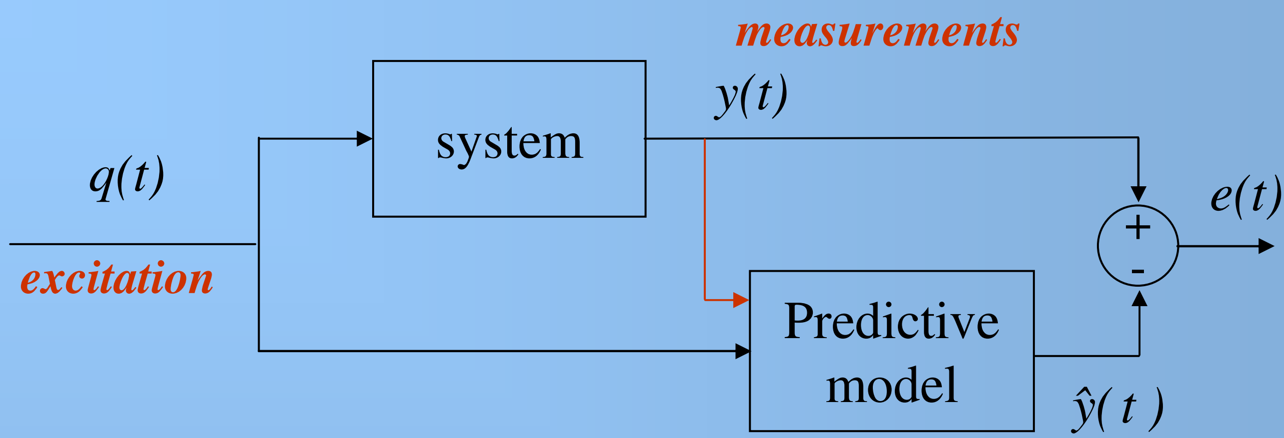
$$\operatorname{cov}\left(\hat{\boldsymbol{B}}\right) = \sigma^2 \begin{pmatrix} \left(\boldsymbol{f}^t \boldsymbol{f}\right)^{-1} & 0 \\ 0 & \left(\boldsymbol{g}^t \boldsymbol{g}\right)^{-1} \end{pmatrix}$$

$$\operatorname{cov}\left(\hat{\boldsymbol{B}}\right) \approx \sigma^2 \left(\frac{N}{t_{\max}}\right)^{-1} \begin{pmatrix} \|f\|^{-2} & 0 \\ 0 & \|g\|^{-2} \end{pmatrix} \qquad \operatorname{cond}\left(\operatorname{cov}\left(\hat{\boldsymbol{B}}\right)\right) \approx \frac{\|f\|^2}{\|g\|^2}$$

- T_i regularly spaced, N must be chosen as great as possible!

-The conditioning number is non-dependant on N !

Linear estimation : minimization of the prediction error *e(t)*



The regression matrix is filled with measurements

Sampled system $$y(t_k) = \mathbf{H}(t_k)\boldsymbol{\beta} + e(t_k)$$

where $\hat{y}(t_k) = \mathbf{H}(t_k)\boldsymbol{\beta}$

n successive measurements $$\mathbf{Y}_n = \mathbf{H}_n\boldsymbol{\beta} + \mathbf{E}_n$$

OLS Estimation $$\hat{\boldsymbol{\beta}} = \left(\mathbf{H}_n^t\mathbf{H}_n\right)^{-1}\mathbf{H}_n^t\mathbf{Y}_n$$

Linear estimation : minimization of the prediction error *e(t)*

The cost = biased estimador

$$E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta} + \left(E(\mathbf{H}_n^t \mathbf{H}_n)\right)^{-1} E(\mathbf{H}_n^t \mathbf{E}_n)$$

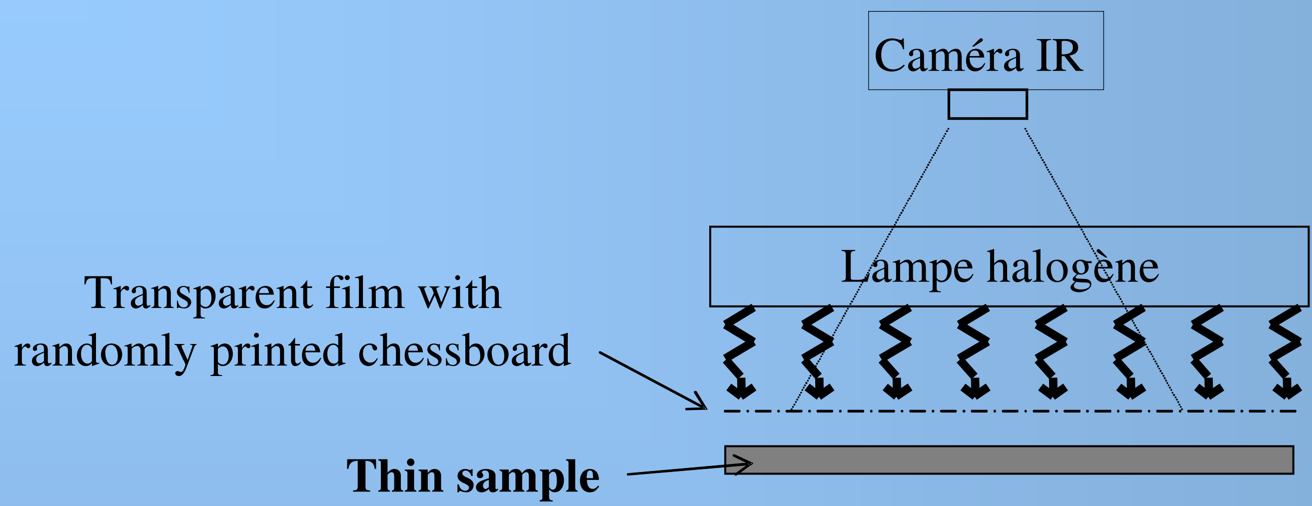
Bias is zero if :

$E(\mathbf{H}_n^t \mathbf{H}_n)$ non singular & *e(t)* is a white noise

or

The input *q(t)* is independant of *e(t)* and *H(t)* does not depend of *y(t)*

Thermal diffusivity mapping from spatial random heating



$$\rho c(x,y)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x,y)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(x,y)\frac{\partial T}{\partial y}\right) - \frac{2h}{e}(T - T_\infty)$$

Discretization

$$\mathbf{T}^{\mathbf{t}+\Delta t} - \mathbf{T}^{\mathbf{t}} = \mathbf{A}.*\,\Delta\mathbf{T}^{\mathbf{t}} + \mathbf{C}^{-1}.*\,\delta_{\mathbf{x}}\mathbf{K}.*\,\delta_{\mathbf{x}}\mathbf{T}^{\mathbf{t}} + \mathbf{C}^{-1}.*\,\delta_{y}\mathbf{K}.*\,\delta_{y}\mathbf{T}^{\mathbf{t}} - \mathbf{H}.*(\mathbf{T}^{\mathbf{t}} - T_\infty)$$

Thermal diffusivity mapping from spatial random heating

$$\hat{\mathbf{T}}' - \hat{\mathbf{T}} = \begin{bmatrix} \hat{\mathbf{T}}^{\mathbf{t}_0+\Delta\mathbf{t}} - \hat{\mathbf{T}}^{\mathbf{t}_0} \\ \cdot \\ \hat{\mathbf{T}}^{\mathbf{t}+\Delta\mathbf{t}} - \hat{\mathbf{T}}^{\mathbf{t}} \\ \cdot \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} \Delta\hat{\mathbf{T}}^{\mathbf{t}_0} & \delta_x \hat{\mathbf{T}}^{\mathbf{t}_0} & \delta_y \hat{\mathbf{T}}^{\mathbf{t}_0} & \hat{\mathbf{T}}^{\mathbf{t}_0} - T_\infty \\ \cdot & \cdot & \cdot & \cdot \\ \Delta\hat{\mathbf{T}}^{\mathbf{t}} & \delta_x \hat{\mathbf{T}}^{\mathbf{t}} & \delta_y \hat{\mathbf{T}}^{\mathbf{t}} & \hat{\mathbf{T}}^{\mathbf{t}} - T_\infty \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

$$\hat{\beta} = \left(\mathbf{X}^t \mathbf{X}\right)^{-1} \mathbf{X}^t \left(\hat{\mathbf{T}}' - \hat{\mathbf{T}}\right)$$

Point by point estimation

$\mathbf{X}^t \mathbf{X}$ = 4x4 matrix

$$\beta_{ij} = \begin{bmatrix} a_{ij} \\ \dfrac{\delta_x k_{ij}}{(\rho c)_{ij}} \\ \dfrac{\delta_y k_{ij}}{(\rho c)_{ij}} \\ H_{ij} \end{bmatrix}$$

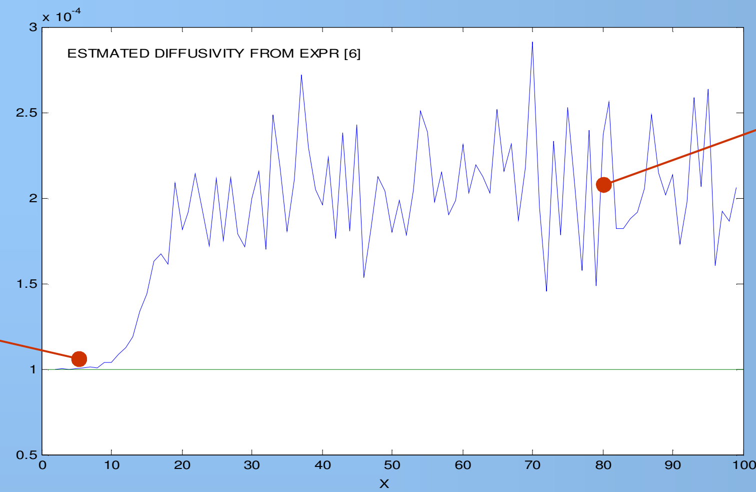
Sequential implementation of the sums
(Recursive estrimation)

Simplified model

$$\hat{\beta} \equiv \mathbf{A} \quad \Rightarrow \quad \hat{\mathbf{A}} = \frac{\sum_{i=0}^{n} \Delta\hat{\mathbf{T}}^{t_i} .* \left(\hat{\mathbf{T}}^{t_i+\Delta t} - \hat{\mathbf{T}}^{t_i}\right)}{\sum_{i=0}^{n} \left(\Delta\hat{\mathbf{T}}^{t_i}\right)^2}$$

Thermal diffusivity mapping from spatial random heating

Correct estimation in the perturbated region



Important bias in the unperturbated region

Homogeneous plate with local heating

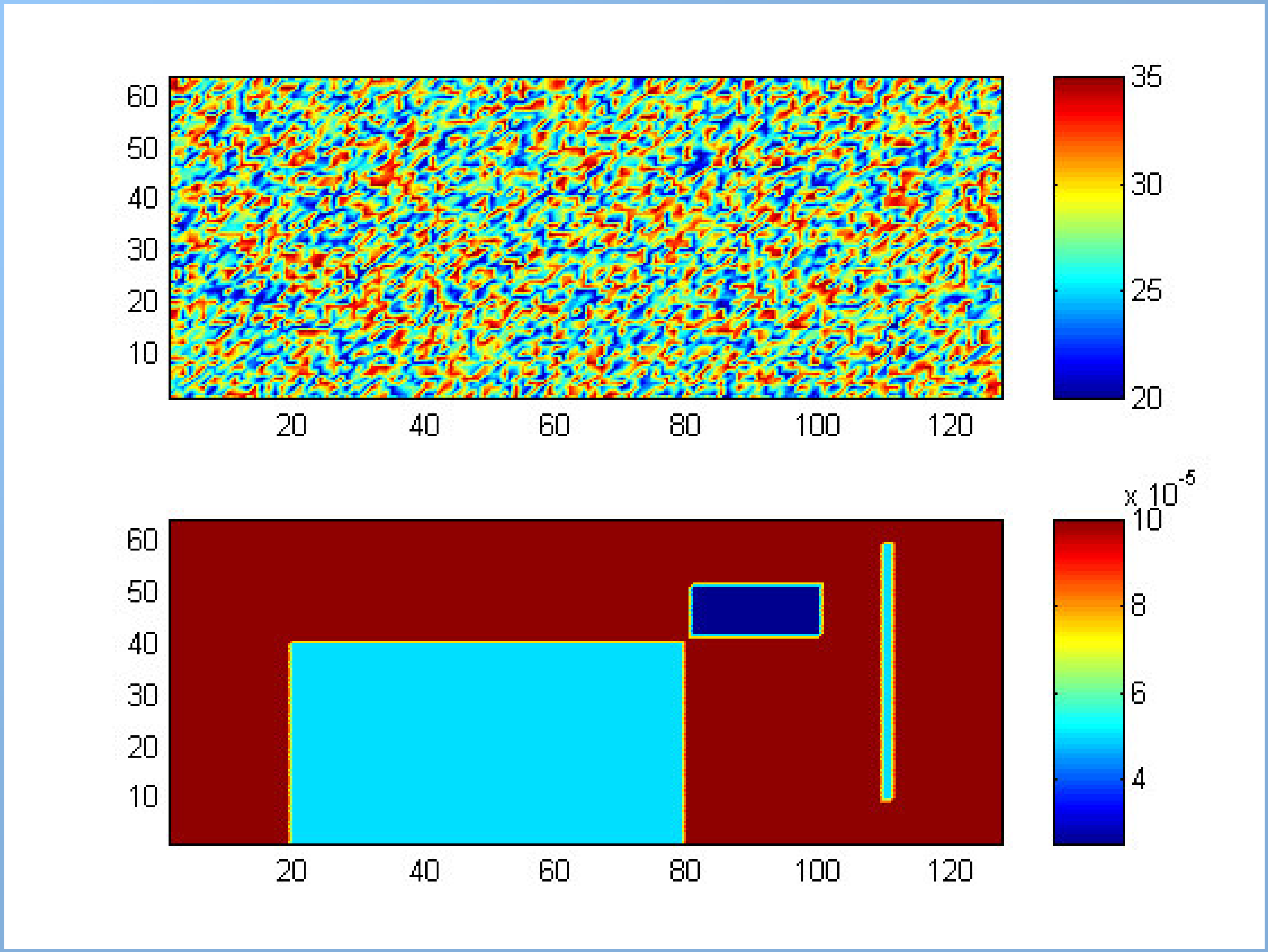
Periodic heating ⇒ Periodic bias in the unperturbated points

$$\hat{\mathbf{A}} = \frac{\sum_{i=0}^{n} \Delta\hat{\mathbf{T}}^{t_i} .* \left(\hat{\mathbf{T}}^{t_i+\Delta t} - \hat{\mathbf{T}}^{t_i}\right)}{\sum_{i=0}^{n} \left(\Delta\hat{\mathbf{T}}^{t_i}\right)^2}$$

Thermal diffusivity mapping from spatial random heating

Initial randomly distributed temperature field

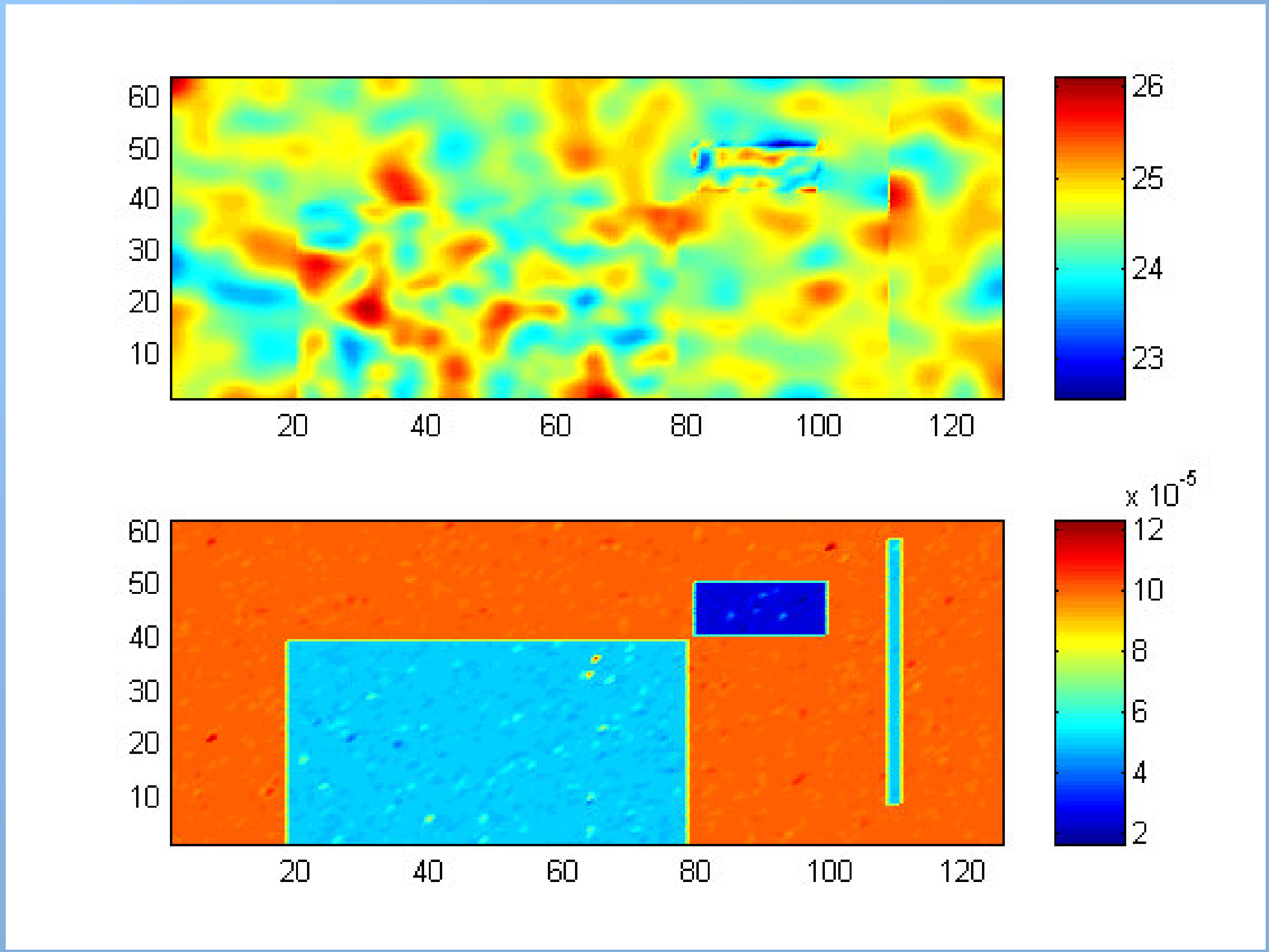
Sample



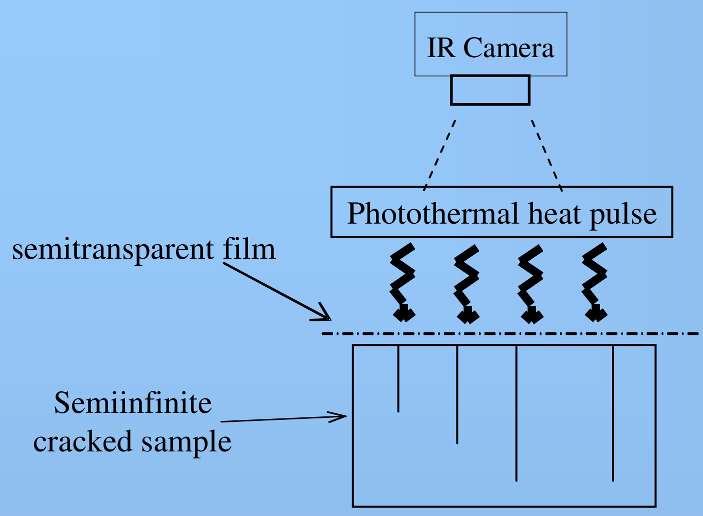
Thermal diffusivity mapping from spatial random heating

Final temperature field

Estimated thermal diffusivity field



Thermal diffusivity mapping from spatial random heating



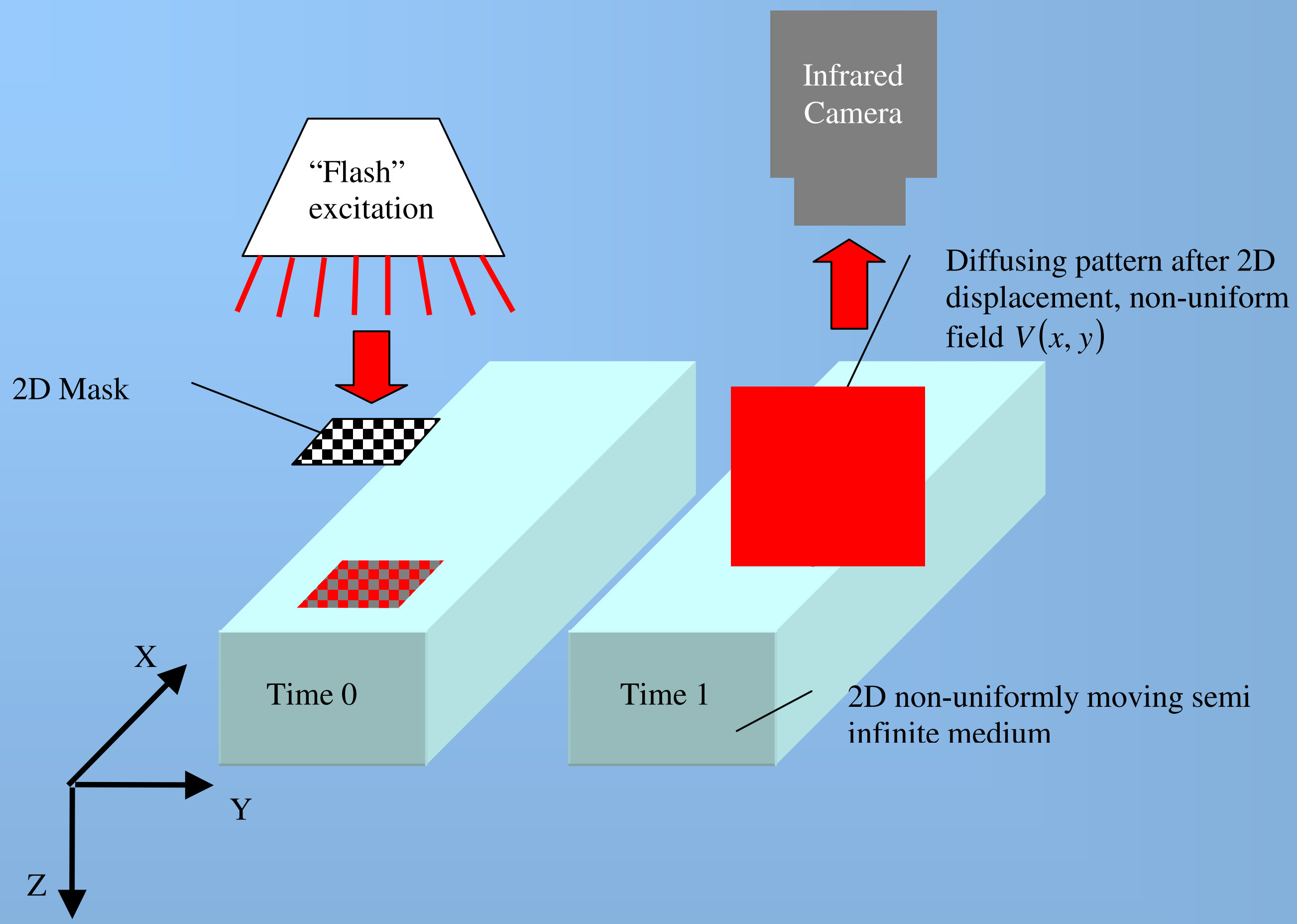
Thick Sample (semi-infinite)

$$T(x,y,z,t) = T_{x,y}(x,y,t).T_z(z,t)$$ Separability

$$T_{x,y}(x,y,t) = T(x,y,z=0,t).\sqrt{t}$$ New Observable variable

$$\hat{\boldsymbol{\beta}}_{ML} = \left(\hat{\mathbf{X}}'\mathbf{t}^{-1}\hat{\mathbf{X}}\right)^{-1}\hat{\mathbf{X}}'\mathbf{t}^{-1}\left(\hat{\mathbf{Y}}^{\mathbf{t}+\Delta\mathbf{t}} - \hat{\mathbf{Y}}^{\mathbf{t}}\right)$$

Velocity and diffusion mapping for a moving solid



$$\frac{\partial T(x,y,z,t)}{\partial t}+V_x\frac{\partial T(x,y,z,t)}{\partial x}+V_y\frac{\partial T(x,y,z,t)}{\partial y}=a\cdot\left(\frac{\partial^2 T(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T(x,y,z,t)}{\partial z^2}\right)$$

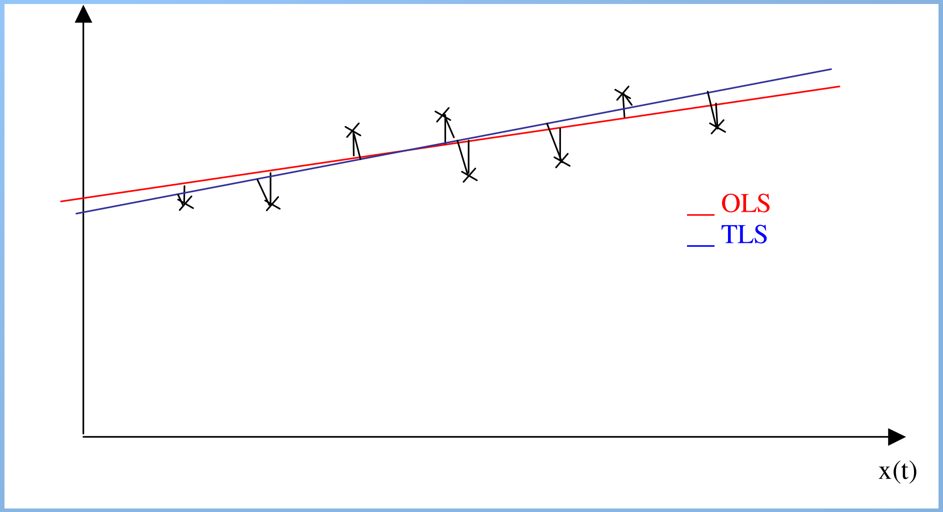
Total Least Square Estimation

$$\frac{\partial T_{x,y}(x,y,t)}{\partial t}+V_x\frac{\partial T_{x,y}(x,y,t)}{\partial x}+V_y\frac{\partial T_{x,y}(x,y,t)}{\partial y}=a\cdot\left(\frac{\partial^2 T_{x,y}(x,y,t)}{\partial x^2}+\frac{\partial^2 T_{x,y}(x,y,t)}{\partial y^2}\right)$$

$$X\boldsymbol{\beta}=\mathbf{Y} \quad\Longrightarrow\quad \begin{bmatrix}\mathbf{X} & \mathbf{-1}\end{bmatrix}\begin{bmatrix}\boldsymbol{\beta}\\ \mathbf{Y}\end{bmatrix}=\mathbf{0}$$

$$D(x,y)\quad (x,y)=0$$

$$\begin{cases} (x,y)=\begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \frac{\partial T}{\partial x}(x,y,t_i) & \frac{\partial T}{\partial y}(x,y,t_i) & -\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)(x,y,t) & \frac{\partial T}{\partial t}(x,y,t_i) \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \\ (x,y)=\begin{bmatrix} V_x(x,y) & V_y(x,y) & a(x,y) & 1\end{bmatrix}^T \end{cases}$$


OLS
TLS
x(t)

Total Least Square Estimation

$$\left\{ \quad (x,y), \ \| \quad (x,y) \quad (x,y)\|^{2} = min \right\}$$

With the constraint $\| \quad (x,y)\|^{2} = 1$

$$\left\{ \quad (x,y), \ \| \quad (x,y) \quad (x,y)\|^{2} + \lambda(x,y)\left(1 - \| \quad (x,y)\|^{2}\right) = min \right\}$$

Lagrange multipiers

$$(x,y) = \quad (x,y)^{T} \quad (x,y)$$

Total Least Square Estimation

Minimum for $(x,y)\quad (x,y)=\lambda(x,y)\quad (x,y)$

$V_{min}(x,y)$ $\qquad\qquad$ $\lambda_{min}(x,y)$

Eigenvector associated with the minimum eigenvalue

BUT $\quad \lambda_N \geq \lambda_{N-1} \geq .. > \lambda_p \approx .. \approx \lambda_0 \approx 0$

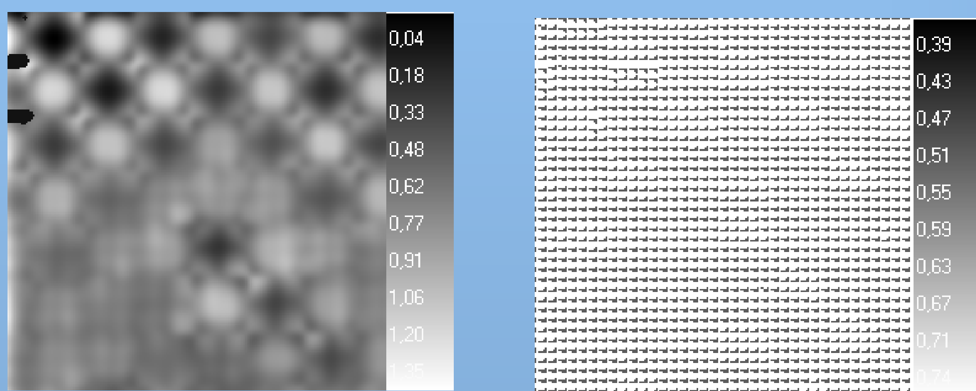
Threshold ?

Noise subspace dimension ?

- spanned by the eigenvectors of the “close to zero” eigenvalues -

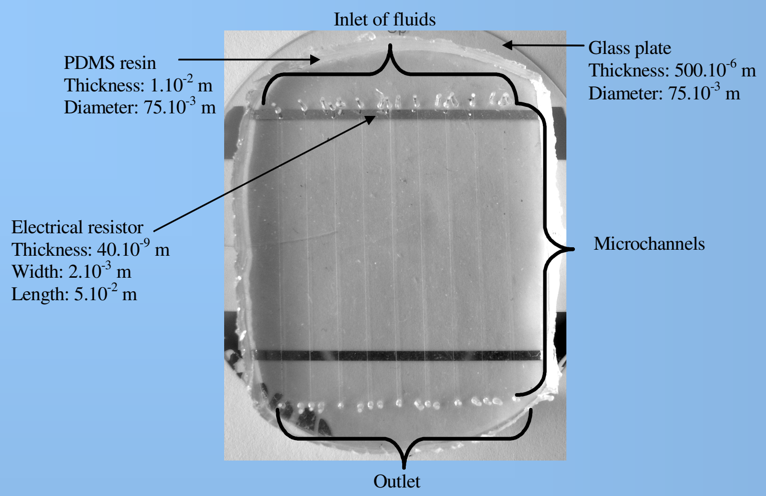
Velocity and diffusion mapping for a moving solid

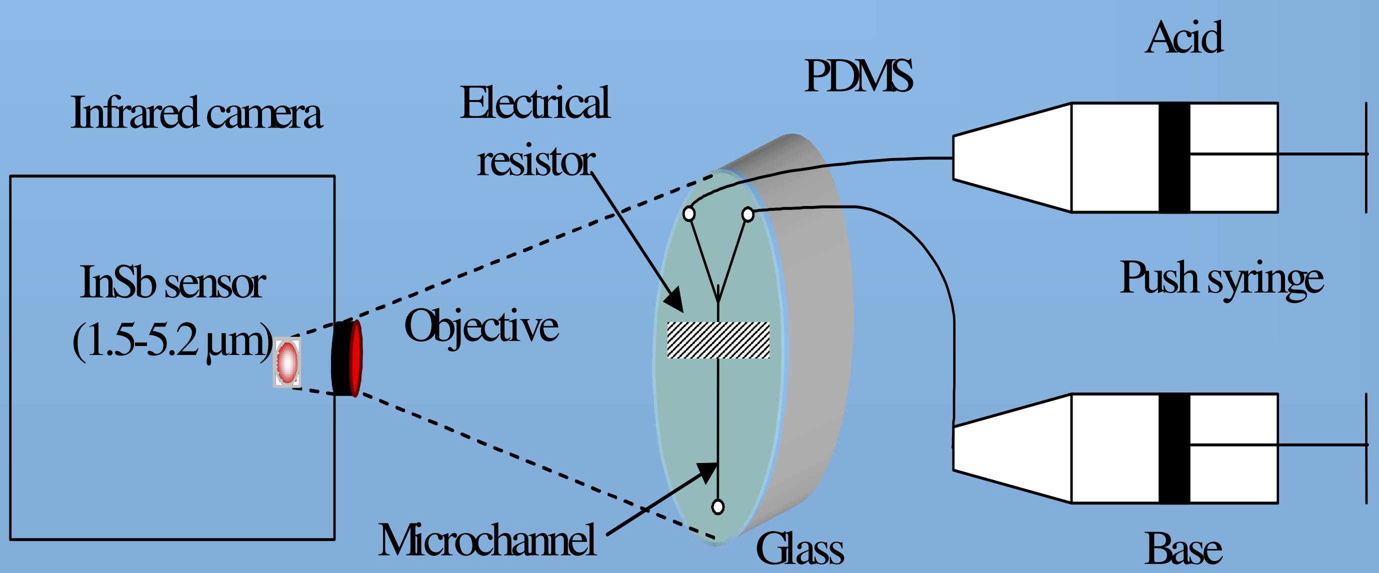
Infrared sequence showing a moving and diffusing pattern



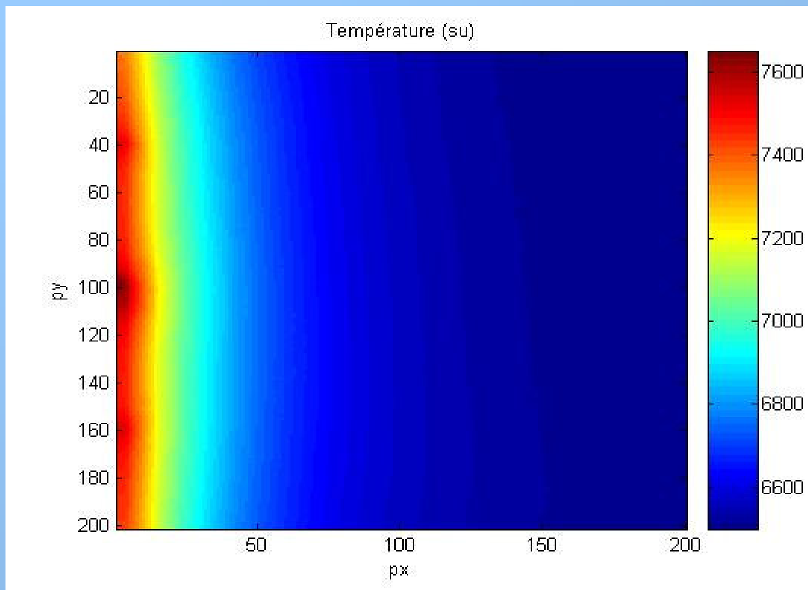
Diffusivity and velocity mapping from previous image sequence sampled at 25 Hz

Processing of temperature fields in microfluidic chips



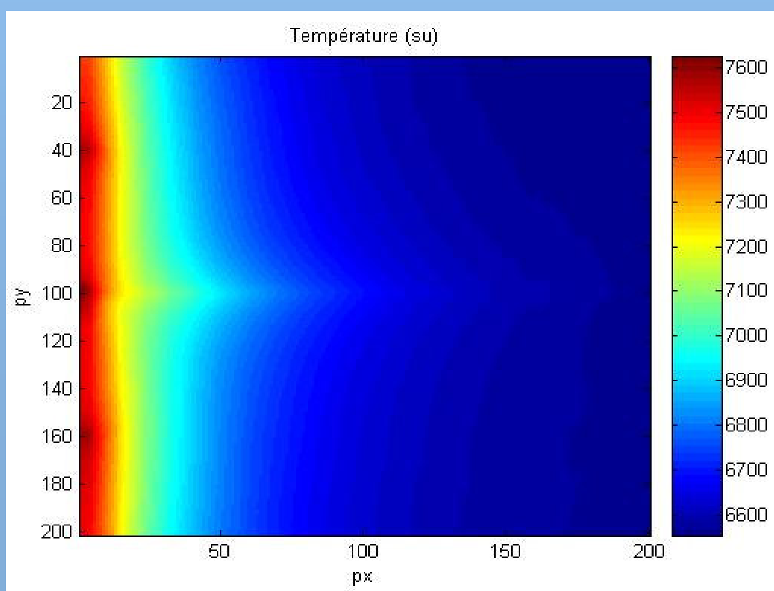


Temperature fields



Without flow

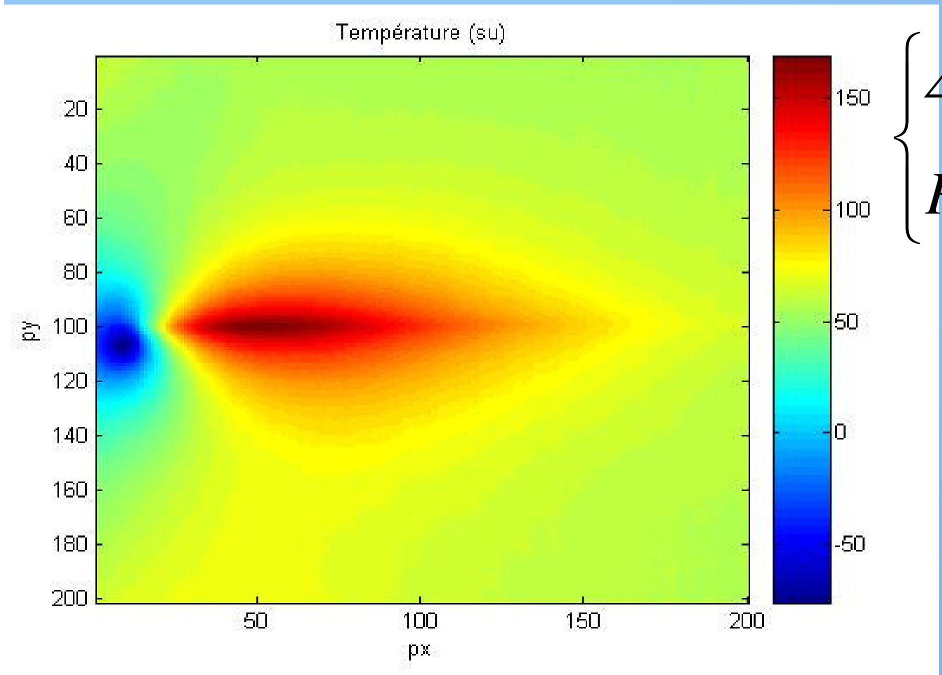
$$\frac{\partial^2 T^0}{\partial x^2} + \frac{\partial^2 T^0}{\partial y^2} - H(x,y)T^0 + \phi(x,y) = 0$$



With flow

$$\frac{V(x,y)}{a}\frac{\partial T}{\partial x} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \phi(x,y)$$

Temperature difference between initial and perturbed field



$$\begin{cases} \Delta\left(T^0_{i,j}\right) + \phi_{i,j} = 0 \\ Pe_{i,j}\,\delta\left(T_{i,j}\right) = \Delta\left(T_{i,j}\right) + \phi_{i,j} = \Delta\left(T_{i,j} - T^0_{i,j}\right) \end{cases}$$

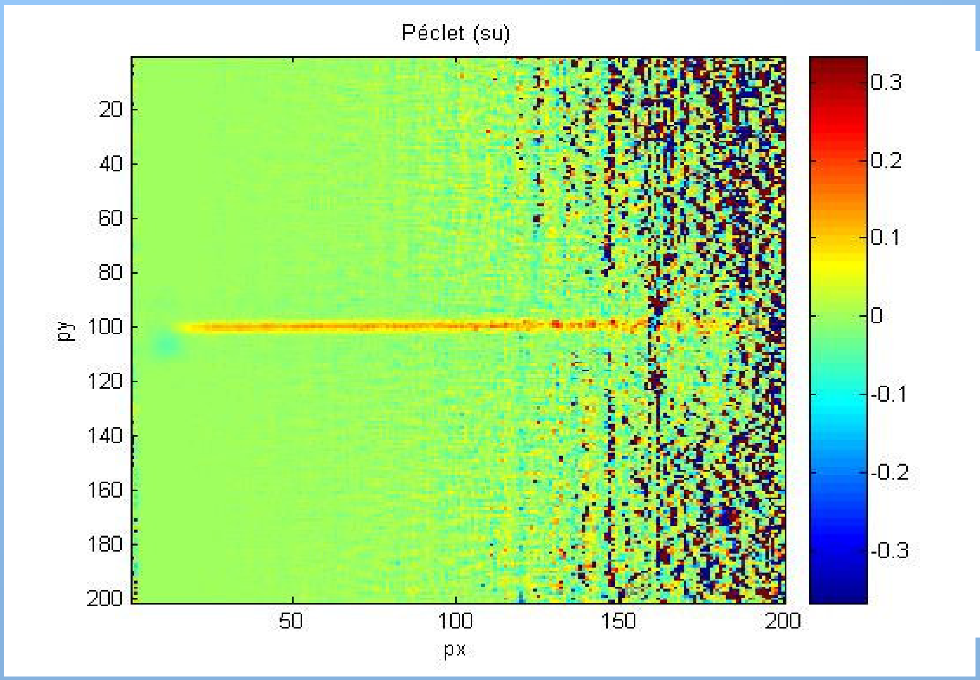
$$Pe_{i,j} = \frac{\Delta\left(T_{i,j} - T^0_{i,j}\right)}{\delta\left(T_{i,j}\right)}$$

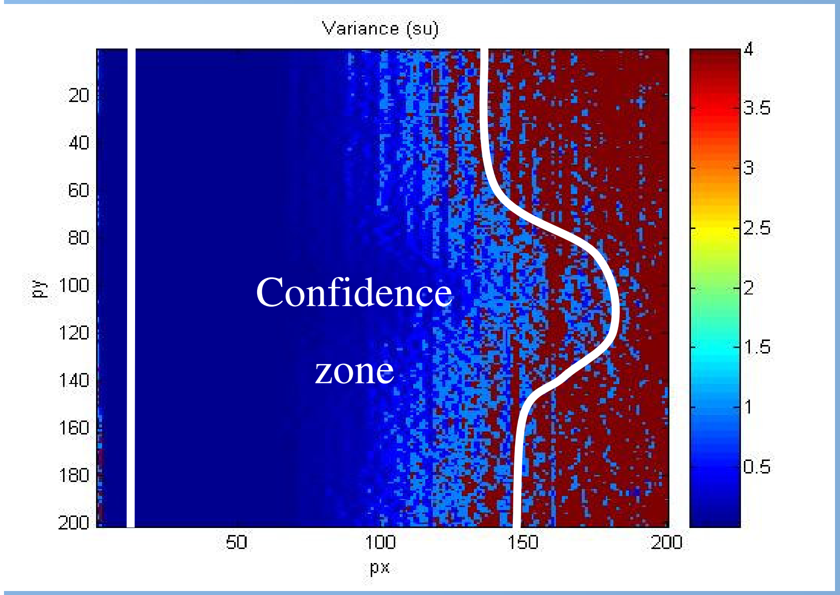
Ficticious temperature field :

$$\delta\left(T_{i,j} - T^0_{i,j}\right)Pe_{i,j} = \Delta\left(T_{i,j} - T^0_{i,j}\right) - \delta\left(T^0_{i,j}\right)Pe_{i,j}$$

Artificial source term

Peclet field $Pe_{i,j}$ estimation

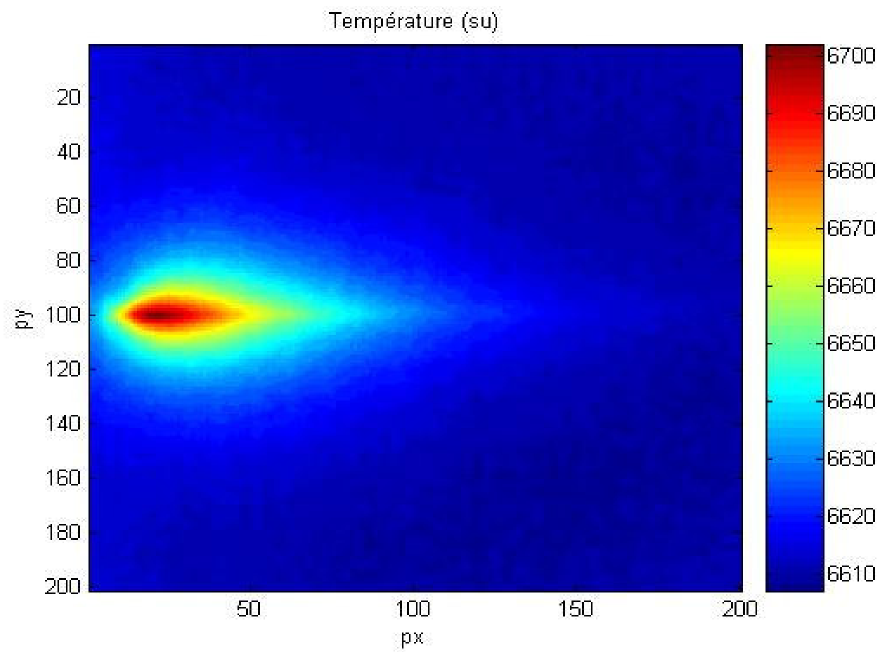


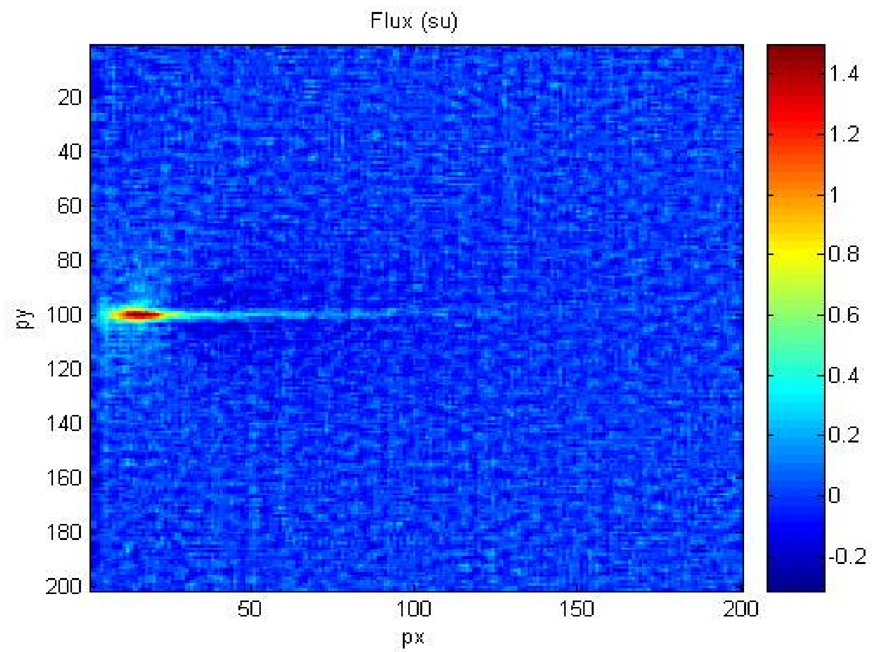


$$Pe_{i,j} = \frac{\Delta\left(T_{i,j} - T^{0}_{i,j}\right)}{\delta\left(T_{i,j}\right)}$$

$$var\left(Pe_{i,j}\right) = 40\,\sigma^{2}\,\delta T_{i,j}^{-2}$$

Application to a chemical reaction characterization

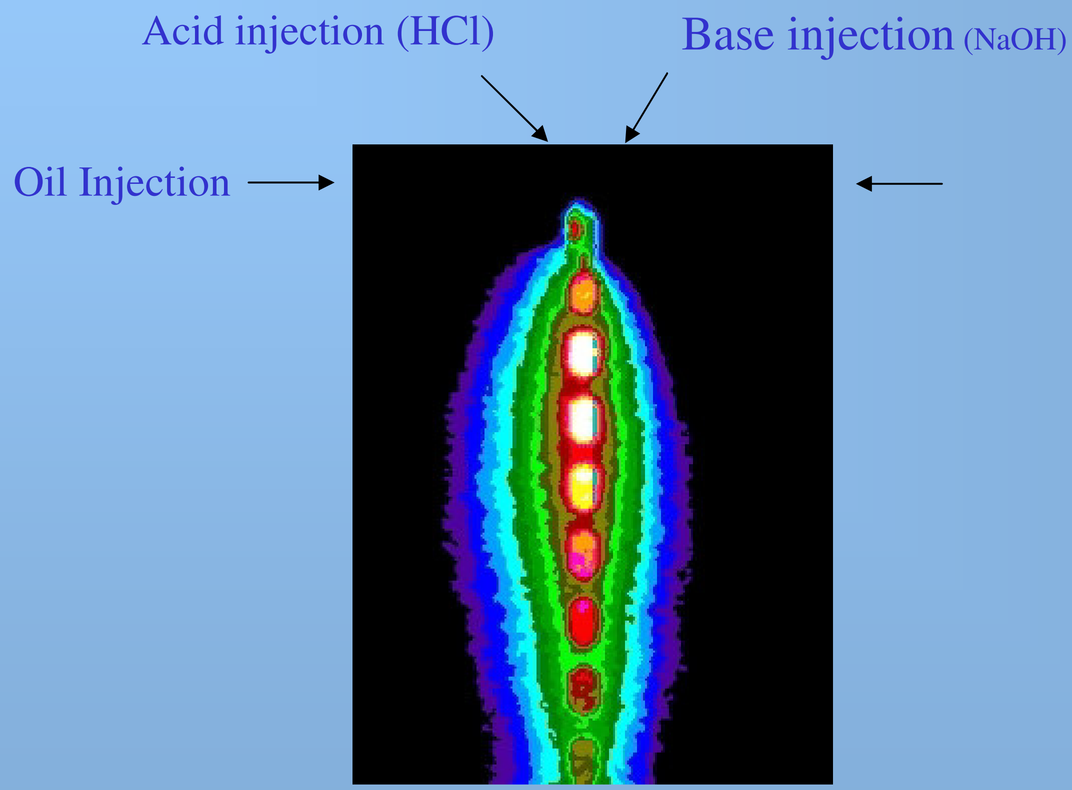




Temperature field T^c, at Q = 1000 µlh

Chemical source term at Q = 1000 µlh

-Reactive Droplets

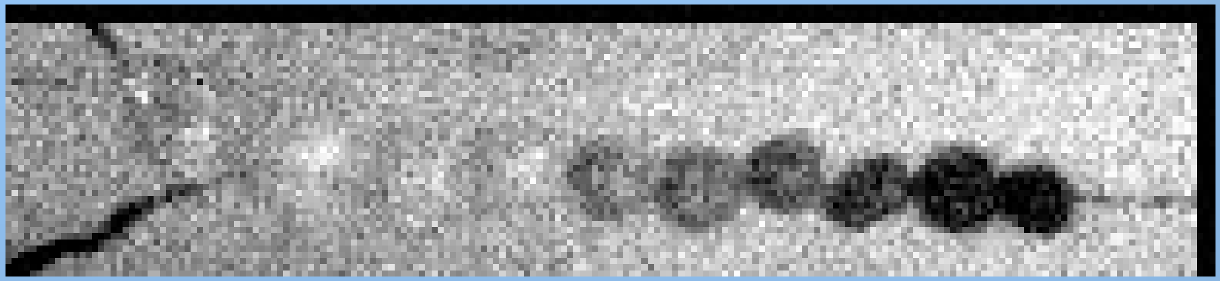


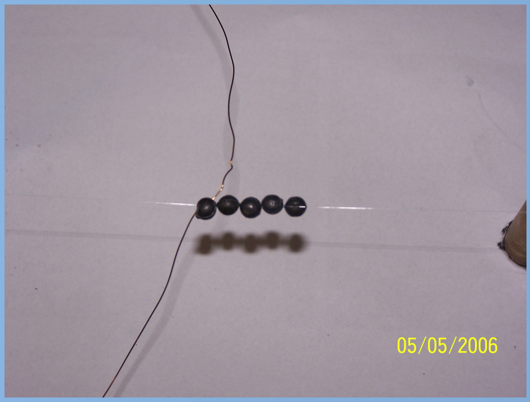
Quasi instantaneous mixing
1 droplet = 1 microreactor
Intensification of the experiments!

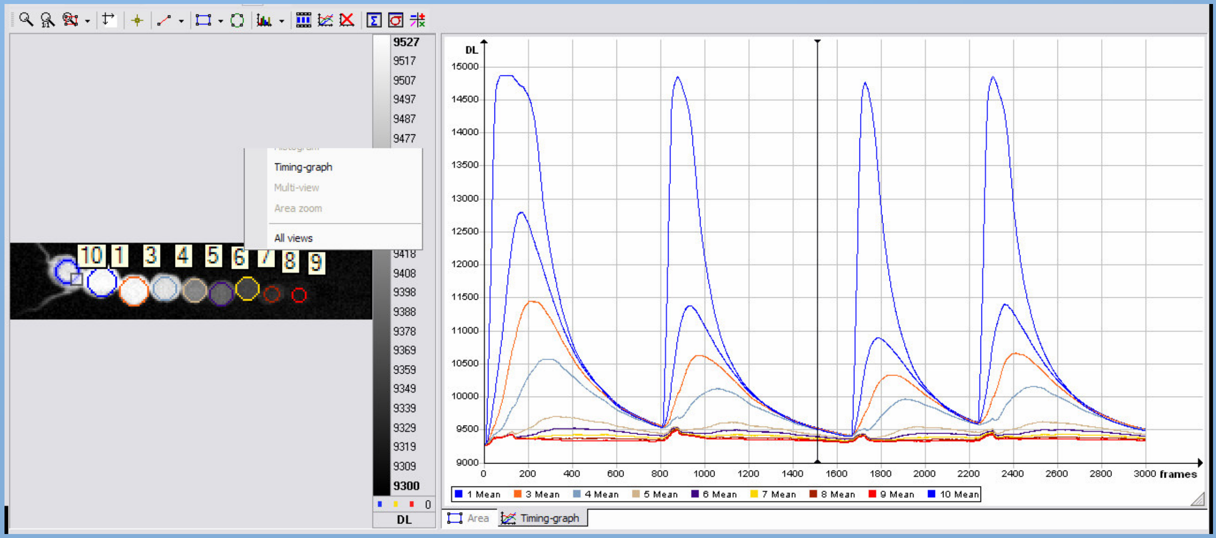
-Granular or dispersed media

Lead grains of 1mm of diameter (reduction of the ratio volume/contact surface)

Applications to thermomechanics







Macroscopic and local thermal characterization

from infrared images processing :

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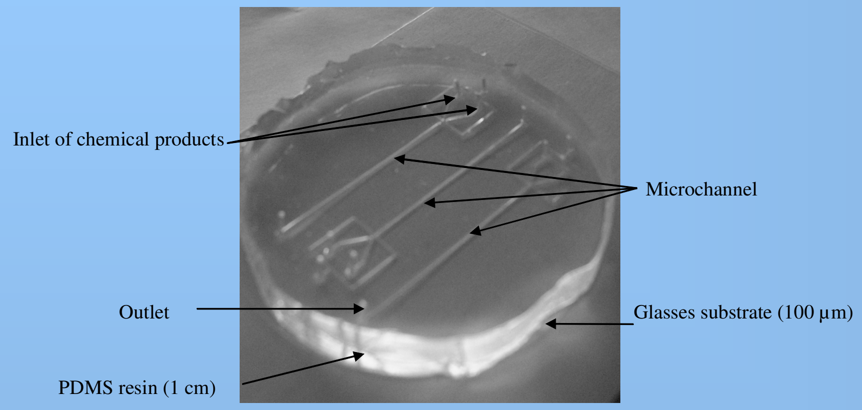
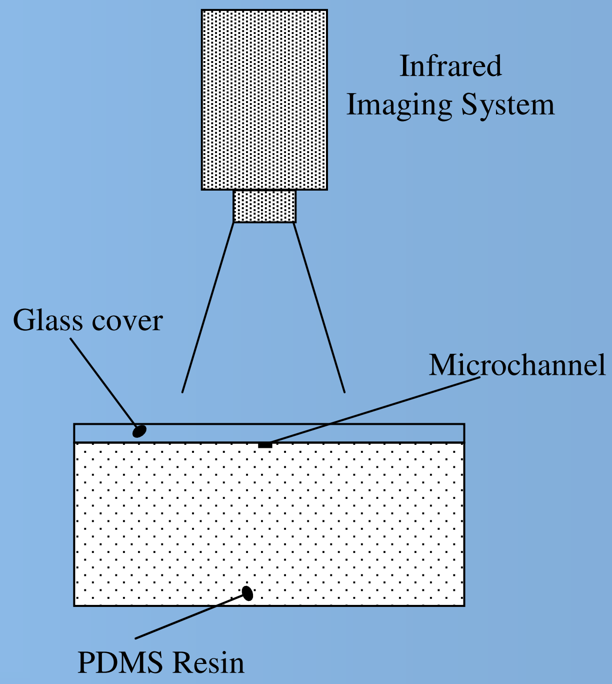
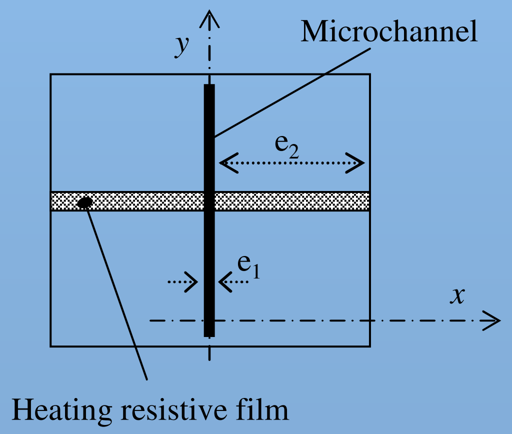
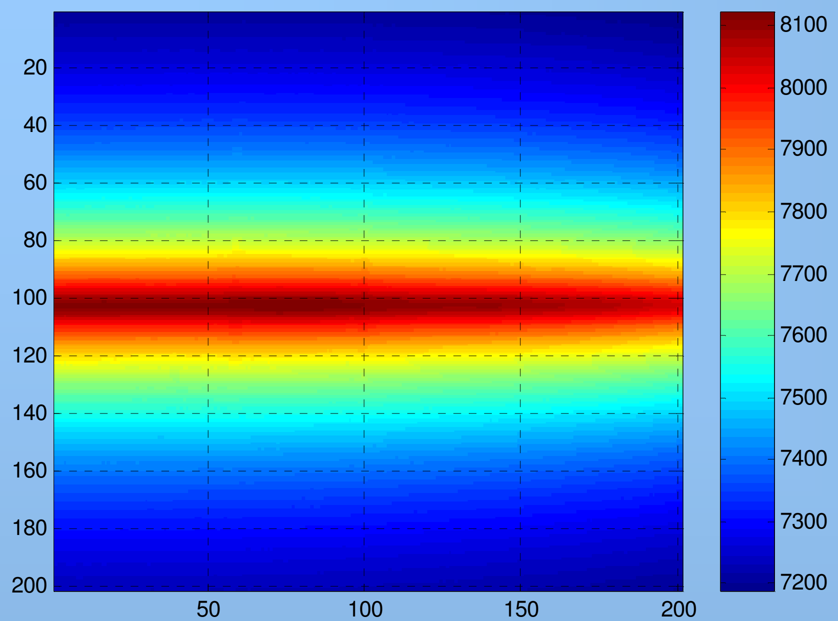
Macroscopic characterization from averaging

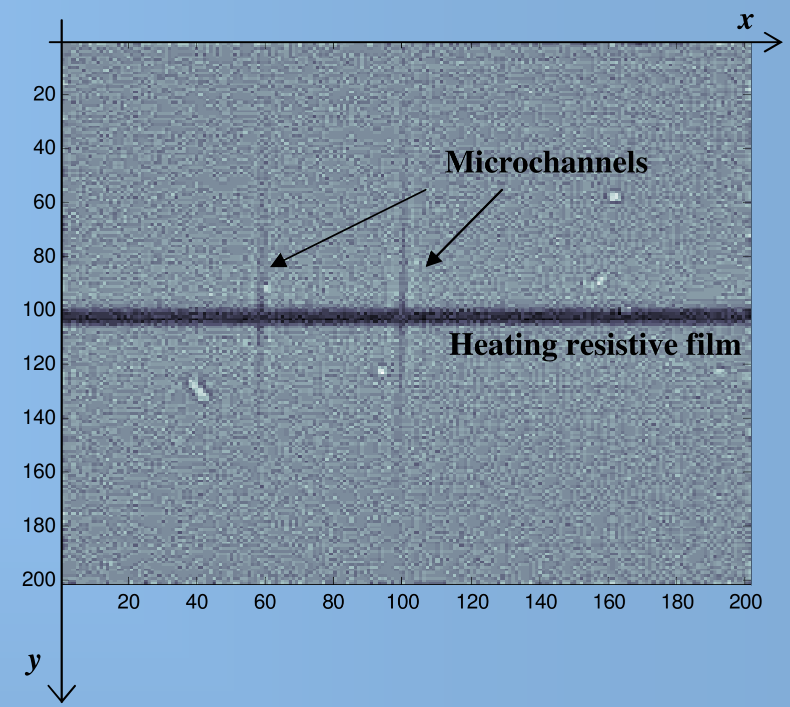






Macroscopic characterization from averaging





$$\frac{\partial}{\partial x}\left(k\frac{\partial Y}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial Y}{\partial y}\right)+\frac{g}{k}-HY=0$$

$$\Delta Y=-\frac{\partial k}{\partial x}\frac{\partial Y}{\partial x}-\frac{\partial k}{\partial y}\frac{\partial Y}{\partial y}-\frac{g}{k}+HY$$

Macroscopic characterization from averaging

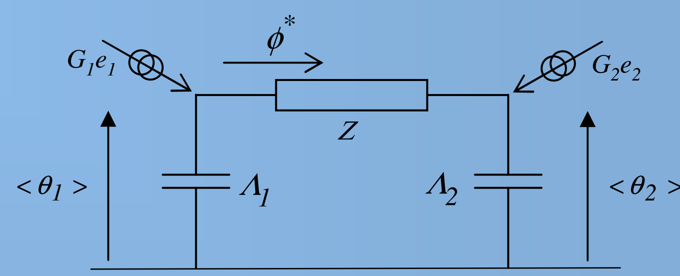
$$\frac{\partial^2 T_i}{\partial x^2}+\frac{\partial^2 T_i}{\partial y^2}+\frac{g_i(y,t)}{k_i}=\frac{1}{a_i}\frac{\partial T_i}{\partial t}+\frac{v_i}{a_i}\frac{\partial T_i}{\partial y}$$

Two Temperature Model :

$$k_1\beta_1^2 e_1 < \theta_1 > = G_1 e_1 - \phi^*$$

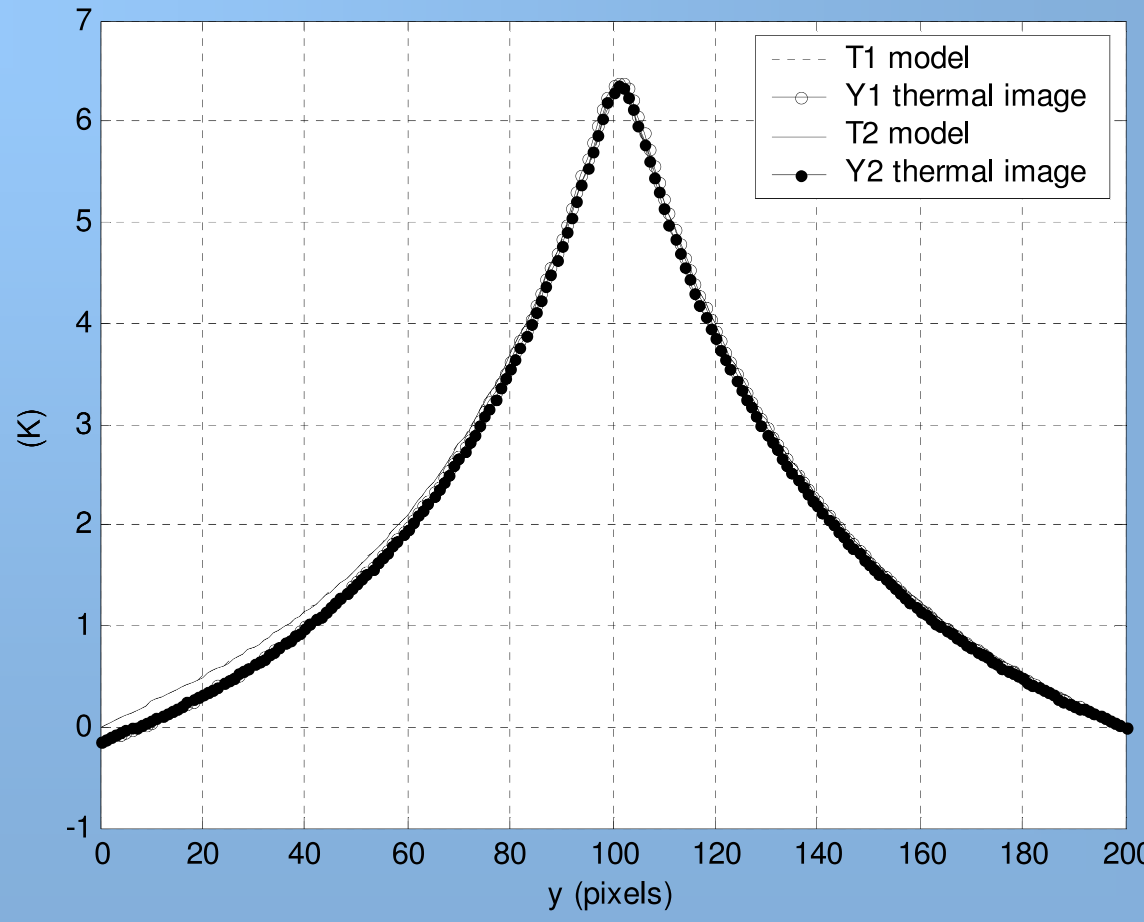
$$k_2\beta_2^2 e_2 < \theta_2 > = G_2 e_2 + \phi^*$$

$$< \theta_1 > - < \theta_2 > = Z\phi^*$$

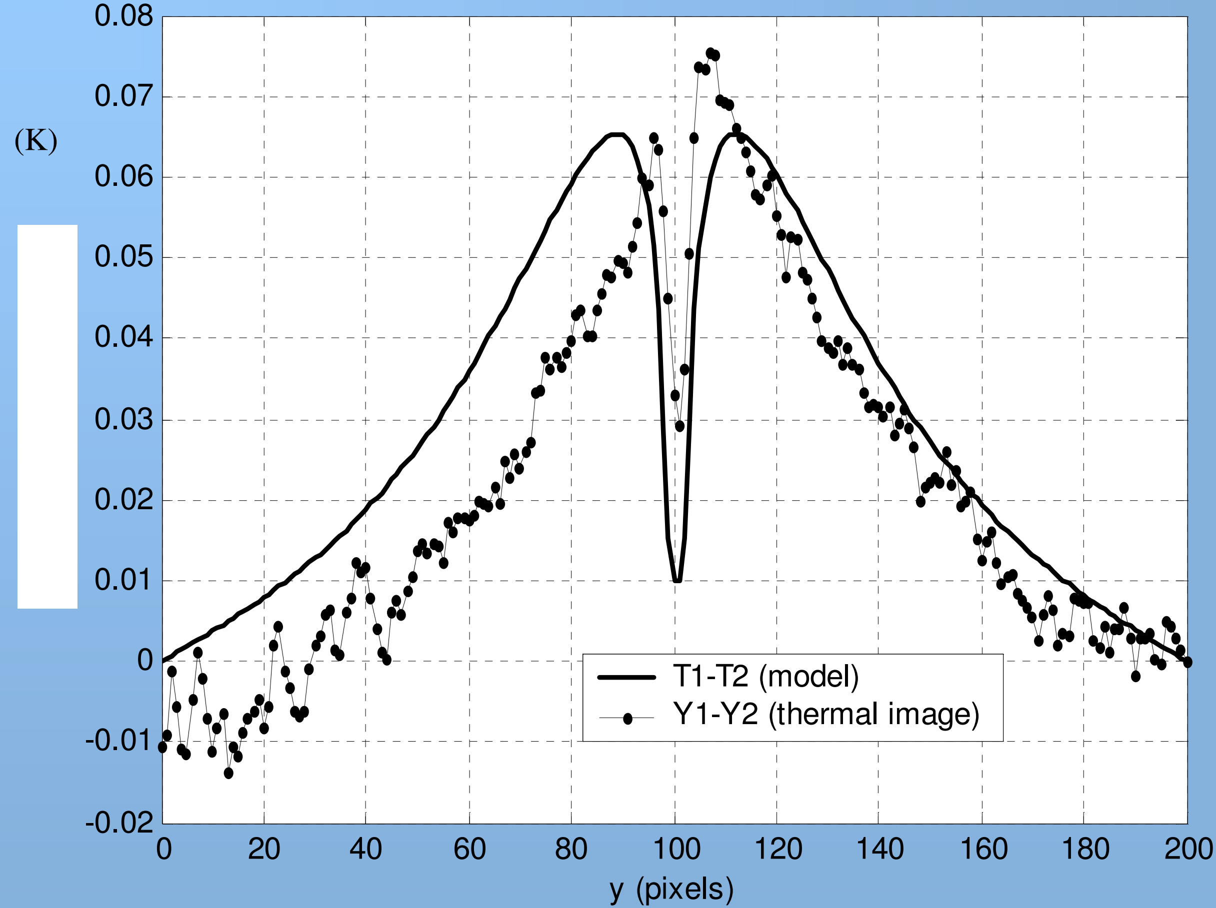


Macroscopic characterization from averaging

Fitted with Heat losses H

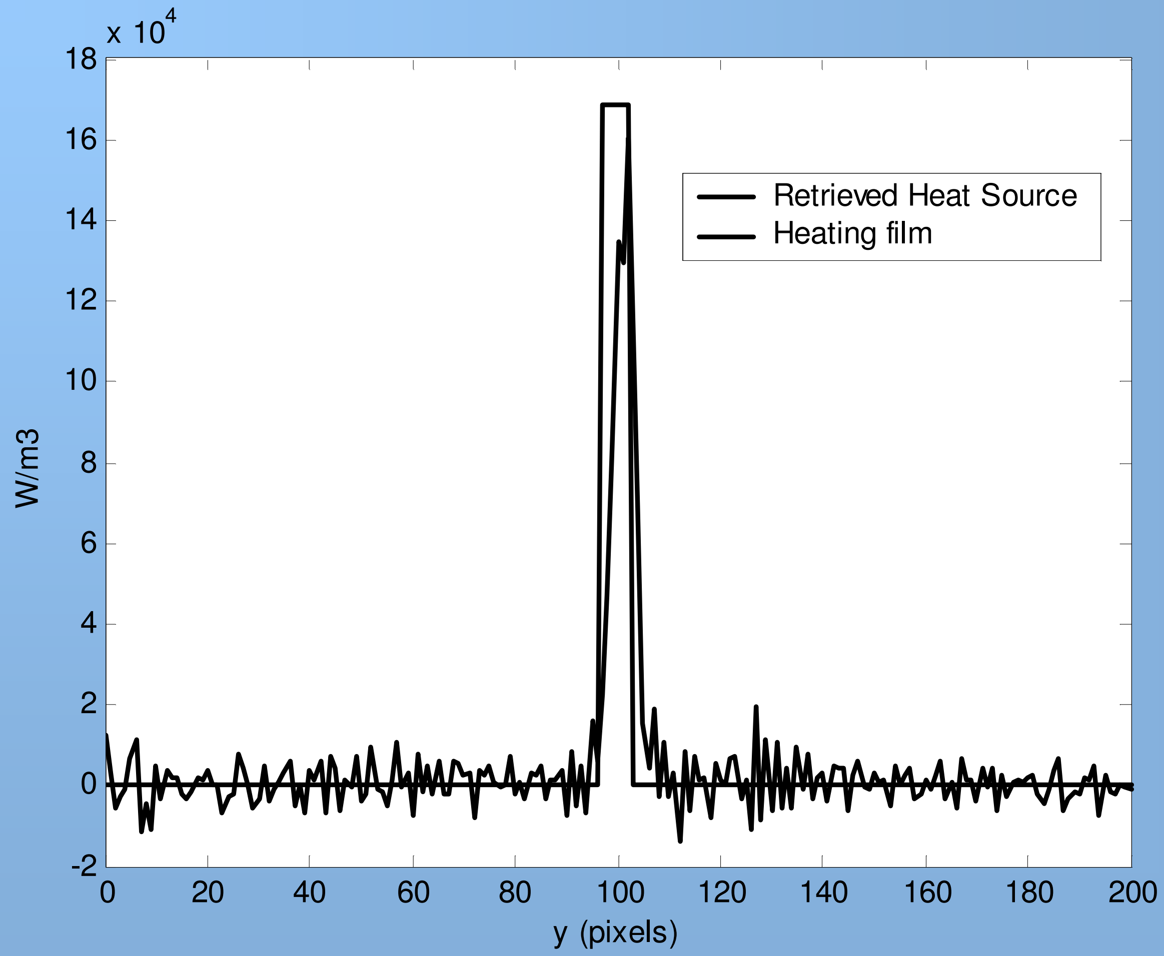


Macroscopic characterization from averaging



Average Temperature Difference

Macroscopic characterization from averaging



Retrieved Heat Source

Conclusion and perspectives

Biodiagnostic : Stress Thermal Signature Analysis

Microwire thermal conductivity

Time and space correlation analysis

SVD analysis

GITT

A new ill-posed problem…

Chile = Pisco sour

Brasil = Caipirinhia

Francia = Kir royal !

Argentina = ???