Universidad de Santiago de Chile
Dpto Ingeniería Mecánica

The Thermal Quadrupole Formalism.

Application to the estimation of thermophysical properties

by random heating

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PROJETO DE COOPERAÇÃO SUL-AMERICANA EM IDENTIFICAÇÃO DE PROPRIEDADES FÍSICAS EM TRANSFERÊNCIA DE CALOR E MASSA
Programa CNPq/PROSUL

Escola Sul-Americana em Identificação de Propriedades Físicas em Transferência de Calor e Massa – PROPFIS

ECOLE DES MINES D'ALBI CARMAUX

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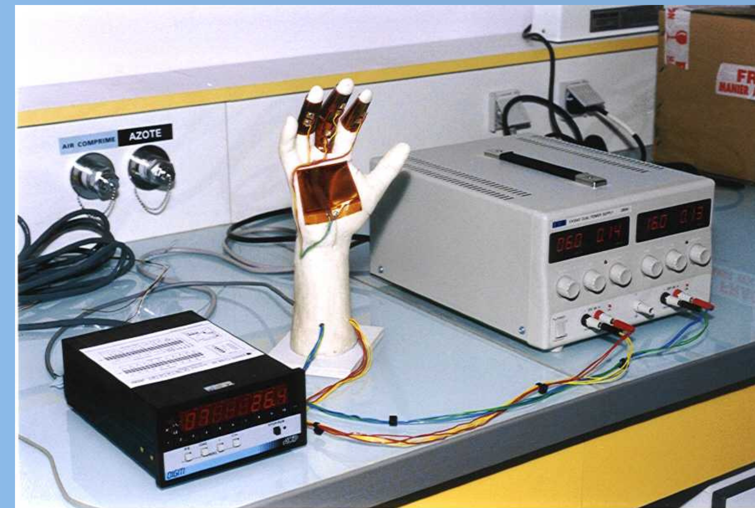
Chemical Engineering Laboratory for Finely Divided Solids

- South West of France
- Ministère de l'Economie, des Finances, et de l'Industrie
- Civil Engineers *(Major in Chemical Engineering)*

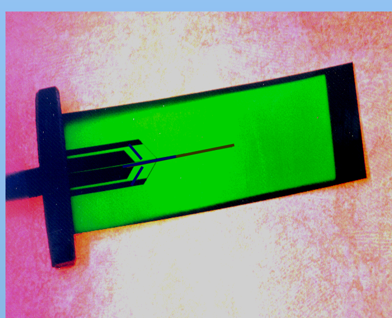
The Thermal Quadrupole Formalism.
Application to the estimation of thermophysical properties by random heating

1. The Thermal Quadrupole Formalism
2. Estimation of thermophysical properties by random heating
3. Thermal diffusivity mapping from spatial random heating
4. Conclusion

Motivation



Many Thermal Engineering problems
do not require the
knowledge of temperature and heat flux
in the whole domain



Heat transfer parameters measurement

Looking for
analytical relationships between
temperature and heat flux
at some given locations

Thermal Quadrupole Formalism

Carslaw & Jaeger	1959	*Laplace space, quadrupole network*
A. Degiovanni et al.	1988	*LEMTA, Nancy, France*
J.C. Batsale et al.	1994	*2D, 3D…Integral transforms*
D. Maillet et al.	2000	*Thermal Quadrupole Book*

A. Degiovanni
Conduction dans un «mur » multicouche avec sources : extension de la notion de quadripôle,
Int.J.Heat.Mass.Transfer. Vol 3, 553 - 557, 1988

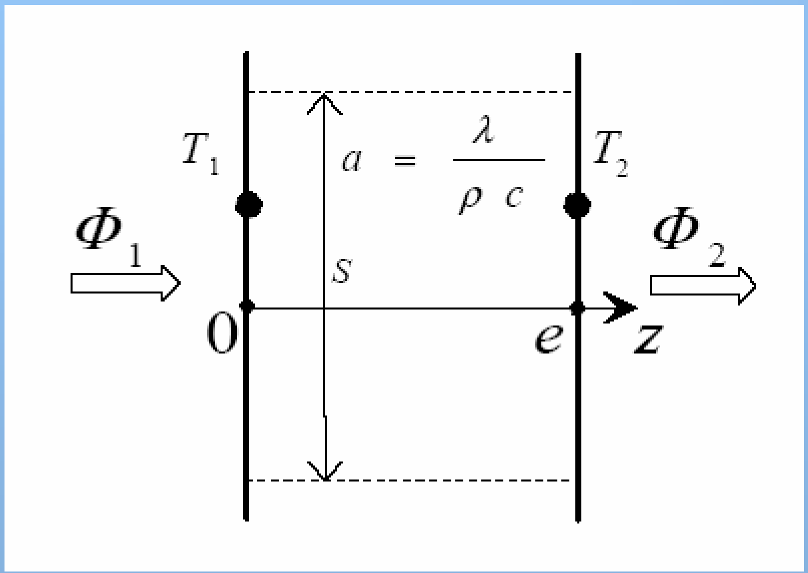
D. Maillet, S. André, J.C. Batsale, A. Degiovanni, C. Moyne
Thermal quadrupoles : Solving the heat equation through integral transforms
Wiley, London, 2000

Thermal Quadrupole Formalism

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} \quad \text{with} \quad T(z)=0 \text{ for } t=0$$

Laplace Transform

$$\frac{d^2\theta}{dz^2} - \frac{s}{a}\theta = 0$$



$$K = \sqrt{\frac{s}{a}}$$

$$\theta(z,s) = G_1 \cosh(Kz) + G_2 \sinh(Kz)$$

$$\phi(z,s) = -kS\frac{d\theta}{dz}$$

$$\phi(z,s) = -kSK\left(G_1 \sinh(Kz) + G_2 \cosh(Kz)\right)$$

Substitute the input/output boundary conditions : $T_1 ; \Phi_1$ *and* $T_2 ; \Phi_2$

…in order to eliminate the coefficients G_1 *and* G_2

Thermal Quadrupole Formalism

$$\begin{bmatrix}\theta_e(s)\\ \Phi_e(s)\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta_s(s)\\ \Phi_s(s)\end{bmatrix}$$

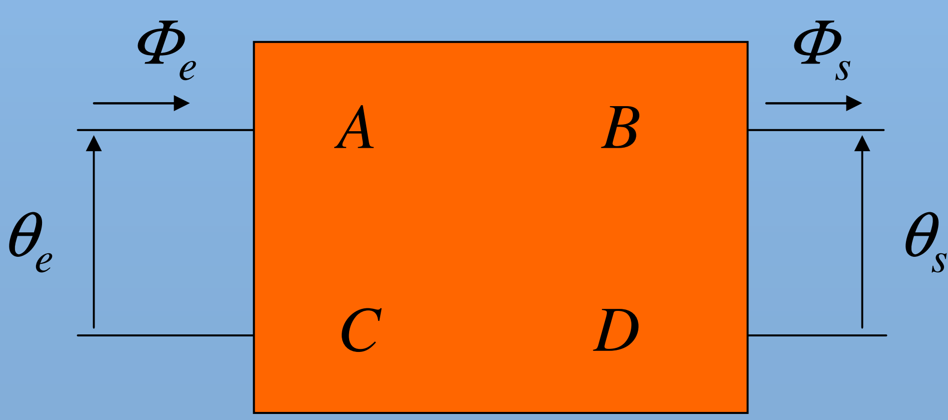
$$\begin{bmatrix}A = cosh(\,Ke\,) & B = \dfrac{sinh(\,Ke\,)}{kKS}\\ C = kKS\,sinh(\,Ke\,) & D = cosh(\,Ke\,)\end{bmatrix}$$

OR

$$\begin{bmatrix}\theta_e(s)\\ \varphi_e(s)\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta_s(s)\\ \varphi_s(s)\end{bmatrix}$$

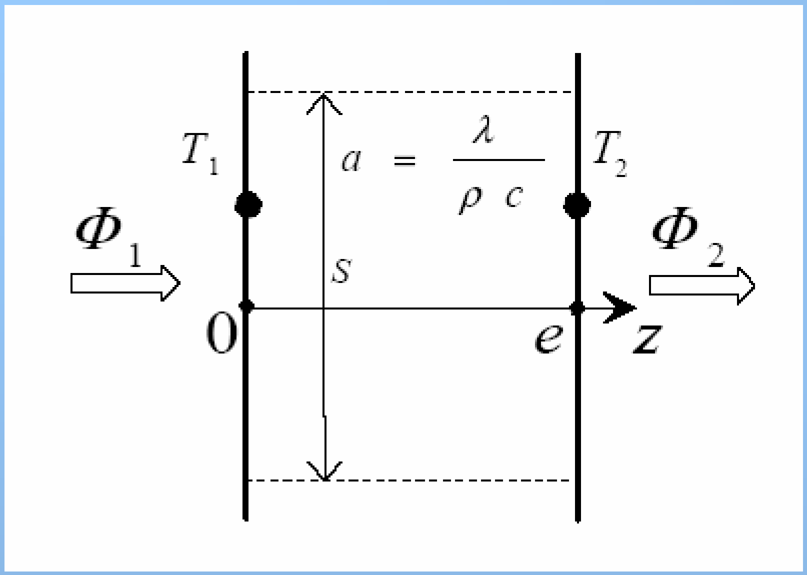
$$\begin{bmatrix}A = cosh(\,Ke\,) & B = \dfrac{sinh(\,Ke\,)}{kK}\\ C = kK\,sinh(\,Ke\,) & D = cosh(\,Ke\,)\end{bmatrix}$$

thermal conductivity (k), *thermal diffusivity* (K), *Thickness* (e)



Intrinsic linear relationship between input / output variables

Thermal Quadrupole Formalism



$$\begin{bmatrix} \theta_e(s) \\ \Phi_e(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_s(s) \\ \Phi_s(s) \end{bmatrix}$$

Well-posed problem

Two boundary conditions are known

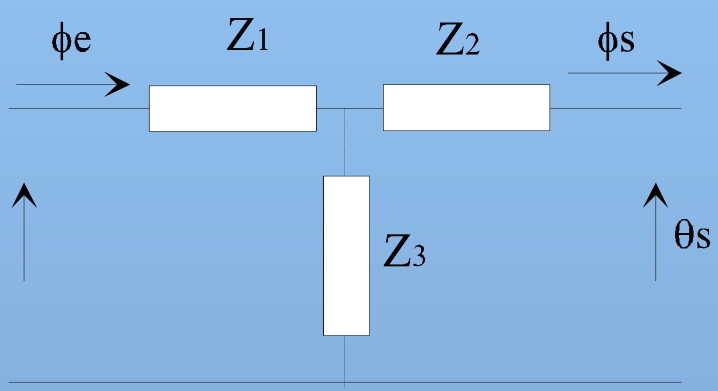
Two remaining equations given by the quadrupole

Thermal Quadrupole Formalism

$$\begin{bmatrix} \theta_e(s) \\ \Phi_e(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_s(s) \\ \Phi_s(s) \end{bmatrix}$$

$$AD - BC = 1 \qquad A = D$$

(Symmetrical system)



$$Z_1 = \frac{A-1}{C} \qquad Z_2 = \frac{D-1}{C} \qquad Z_3 = \frac{1}{C}$$

$$\frac{at}{e^2} >> 1 \quad \Rightarrow \quad s \to 0 \quad \Rightarrow \quad \begin{cases} Z_1 = Z_2 \to \dfrac{e}{2kS} \\ Z_3 \to \dfrac{1}{\rho c_p e S p} \end{cases}$$

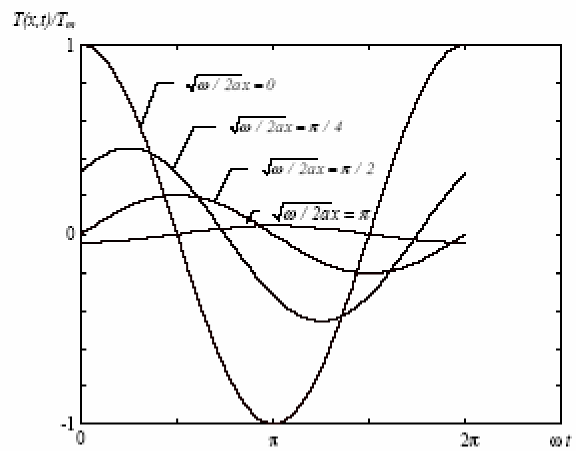
Thermal Quadrupole Formalism

$$T(z,t) = A(z,\omega)\cos\left(\omega t - d(z,\omega)\right)$$

$$\theta(z,\omega) = A(z,\omega)\exp\left(-j\,d(z,\omega)\right).$$

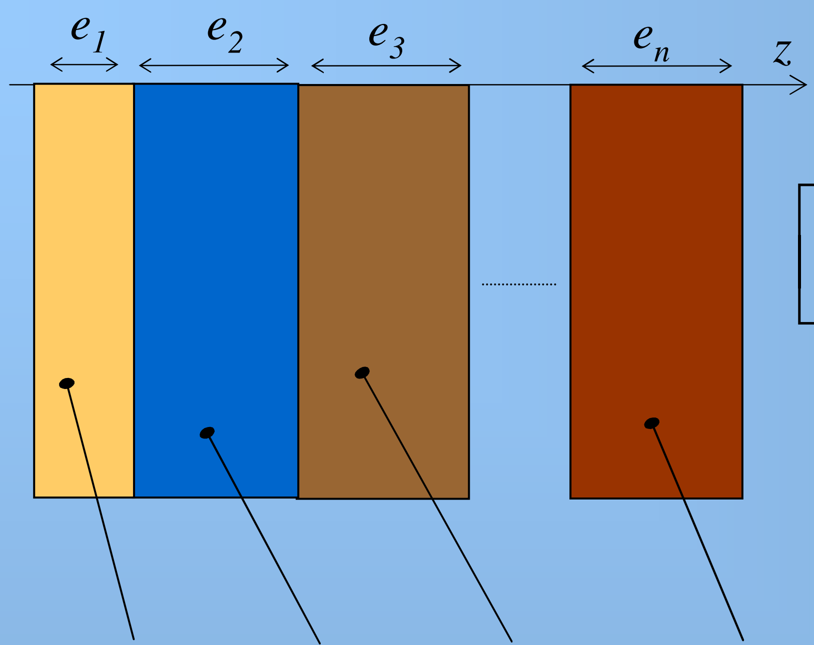
$$\frac{\partial^2\theta}{\partial z^2} = \frac{j\omega}{a}\,\theta$$

$$s \dashrightarrow j\,\omega$$



Time-dependent periodic case

Multilayer System



$$\begin{bmatrix}\theta_e\\ \phi_e\end{bmatrix} \qquad \begin{bmatrix}\theta_s\\ \phi_s\end{bmatrix}$$

$$\begin{bmatrix}\theta_e\\ \phi_e\end{bmatrix} = \begin{bmatrix}A\,B\\ C\,D\end{bmatrix}\begin{bmatrix}\theta_s\\ \phi_s\end{bmatrix}$$

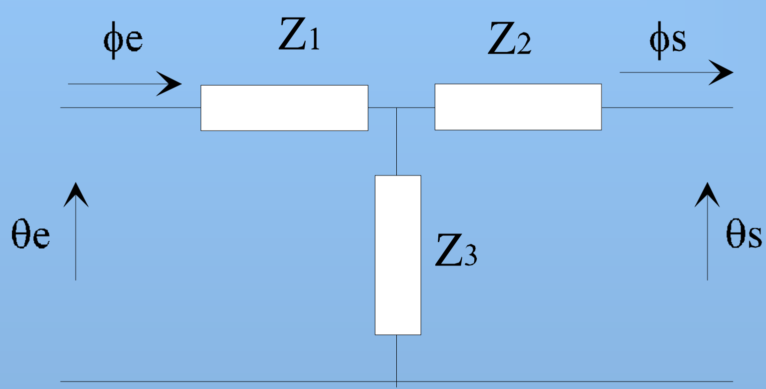
$$\begin{bmatrix}A\,B\\ C\,D\end{bmatrix} = \begin{bmatrix}A_1\,B_1\\ C_1\,D_1\end{bmatrix}\begin{bmatrix}A_2\,B_2\\ C_2\,D_2\end{bmatrix}\begin{bmatrix}A_3\,B_3\\ C_3\,D_3\end{bmatrix}\cdots\begin{bmatrix}A_n\,B_n\\ C_n\,D_n\end{bmatrix}$$

…As well as the interface vectors :

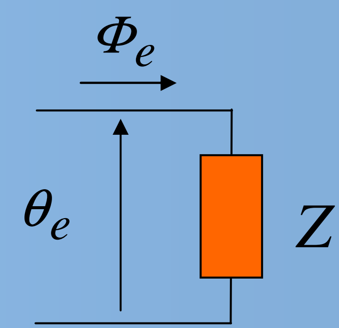
$$\begin{bmatrix}\theta_i\\ \phi_i\end{bmatrix} = \begin{bmatrix}A_i & B_i\\ C_i & D_i\end{bmatrix}\cdots\begin{bmatrix}A_n & B_n\\ C_n & D_n\end{bmatrix}\begin{bmatrix}\theta_s\\ \phi_s\end{bmatrix}$$

Semi-infinite medium

$$\begin{bmatrix}\theta_e(s)\\ \Phi_e(s)\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta_s(s)\\ \Phi_s(s)\end{bmatrix} \xrightarrow[e\to\infty]{} \theta_e = Z\Phi_e$$



$$\begin{cases} Z_1 \to Z/2 \\ Z_2 \to Z/2 \\ Z_3 \to 0 \end{cases}$$



$$Z = \frac{1}{S\sqrt{k\rho c}\sqrt{s}}$$

Interface conditions

Thermal contact resistance

$$T_1 - T_2 = R_c \phi$$

$$\begin{bmatrix} \theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & R_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \Phi_2 \end{bmatrix}$$

Newton B.C.

$$\phi = hS(T_1 - T_\infty)$$

$$\begin{bmatrix} \theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{hS} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_\infty \\ \Phi_\infty \end{bmatrix}$$

Heat Capacity condition

$$C \frac{dT}{dt} = \phi_1 - \phi_2$$

$$\begin{bmatrix} \theta_1 \\ \Phi_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Cs & 1 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \Phi_2 \end{bmatrix}$$

Internal heat sources and initial temperature imbalance

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} + \frac{g(z,t)}{k} \quad \text{with} \quad T(z)=T_0(z) \quad \text{for} \quad t=0$$

⇓

$$\frac{d^2\theta}{dz^2} + \frac{G(z,s)}{k} + \frac{T_0(z)}{a} - \frac{s}{a}\theta = 0$$

$$\theta(z,s) = G_1 \cosh(Kz) + G_2 \sinh(Kz) + \theta_{part}$$

$$\begin{bmatrix} \theta_e(s) \\ \Phi_e(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_s(s) \\ \Phi_s(s) \end{bmatrix} - \begin{bmatrix} X \\ Y \end{bmatrix}$$

Cylindrical coordinate system

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - \frac{s}{a}\theta = 0$$

$$\theta(z,s) = G_1 I_0(Kr) + G_2 K_0(Kr)$$

Bessel functions

$$\phi = -kS\frac{d\theta}{dr} \qquad \text{with} \qquad S = 2\pi rL$$

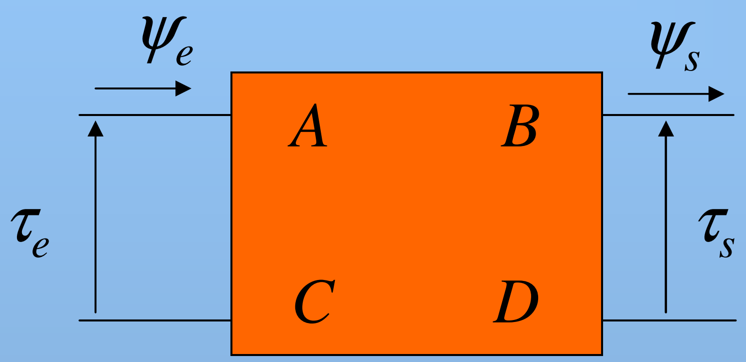
Two or three dimensional cases

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}$$

Laplace-Fourier Transform

$$\tau(\beta_n, z, s) = \int_0^{\infty}\int_0^{L} T(x,z,t)\exp(-st)\cos(\beta_n x)\,dx\,dt$$

β_n = *Eigenvalues from boundary-value problem relative to x*



$$\frac{d^2\tau}{dx^2} - K\tau = 0$$

Generalized frequency

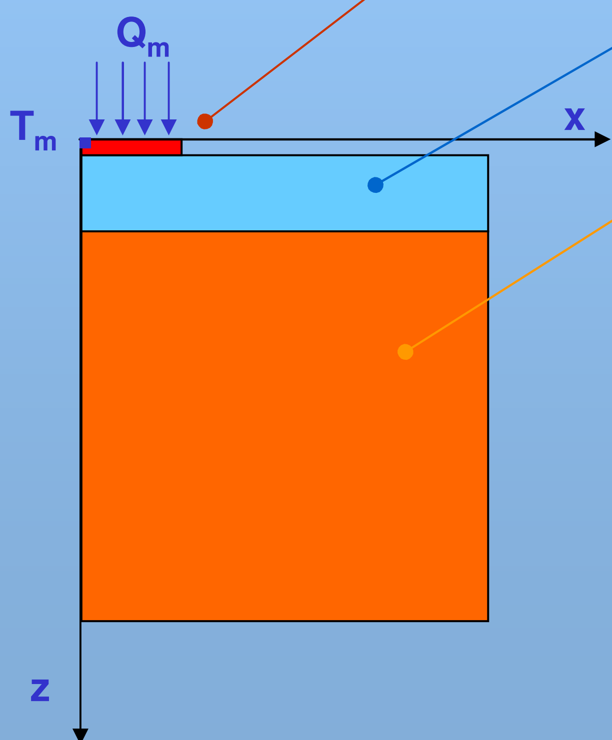
$$K = \sqrt{\beta_n^2 + \frac{s}{a}}$$

$$\begin{bmatrix} \tau_e(s) \\ \psi_e(s) \end{bmatrix} = \begin{bmatrix} A(\beta_n, s) & B(\beta_n, s) \\ C(\beta_n, s) & D(\beta_n, s) \end{bmatrix} \begin{bmatrix} \tau_s(s) \\ \psi_s(s) \end{bmatrix}$$

J.C Batsale, D. Maillet, A.Degiovanni
Extension de la notion de quadripôle thermique à l'aide de transformations intégrales :
calcul du transfert thermique au travers d'un défaut plan bidimensionnel,
Int.J.Heat.Mass.Transfer. Vol 37, 111 - 127, 1994

Multilayer example : super insulating materials characterization

$$\begin{bmatrix} \tau_m(\beta_n, s) \\ \varphi_m(\beta_n, s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (\rho c_p)_s e_s s & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \tau(\beta_n, e_s + e, s) \\ \lambda_2 \sqrt{\frac{p}{a_2} + \alpha_n^2}\, \tau(\beta_n, e_s + e, s) \end{bmatrix}$$



Monolithic silica aerogel

Multilayer example : super insulating materials characterization

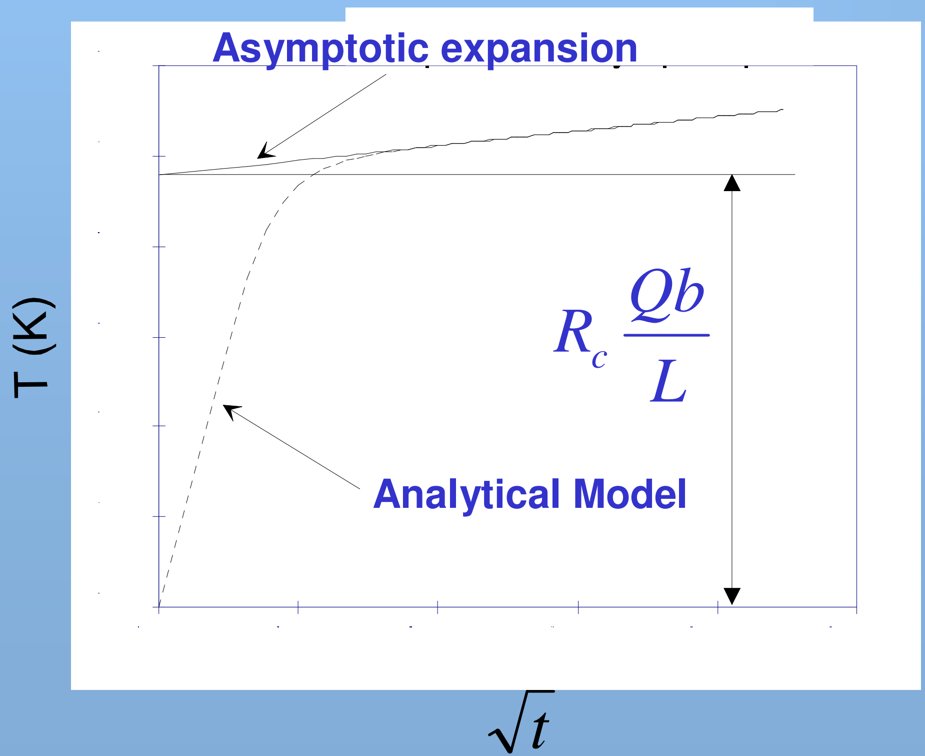
For long times :

$$-(\rho c_p)_s e_s = 0$$

$$-\rho c e = 0$$

$$T(0,0,t) = \frac{Qb}{L\sqrt{\pi}\sqrt{\lambda_2(\rho c_p)_2}}\sqrt{t} + Rc\frac{Qb}{L}$$

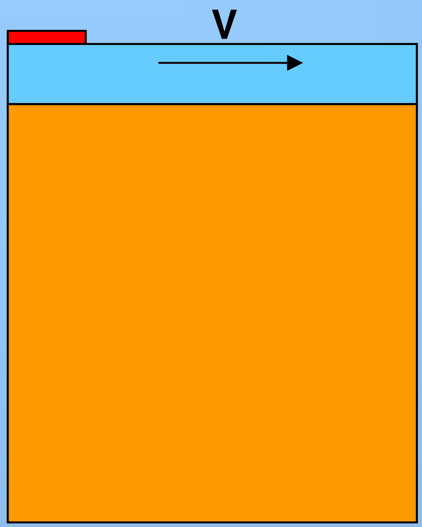
$$R_c = \frac{e}{\lambda} + \frac{2}{\lambda b}\sum_{n=1}^{\infty}\frac{th(\alpha_n e)}{\alpha_n}\frac{\sin(\alpha_n b)}{\alpha_n}$$

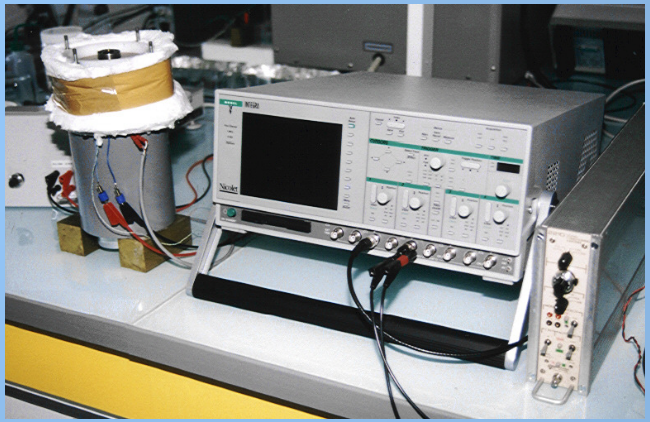


Initial guess for *k*

as a quite good approximation

Extension for thermal charaterization of liquids in Couette flow



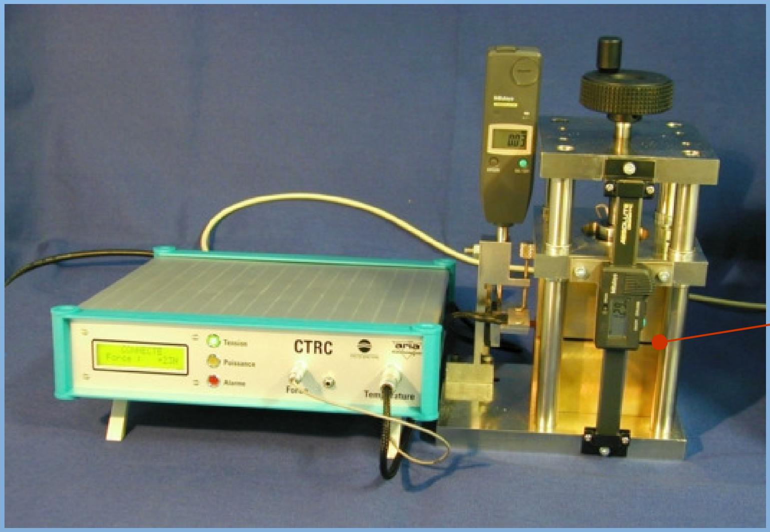


Thermal conductivity

Viscosity

Transfer of technology :

« Capthermic » start-up



Compressible material

Main characteristics of the Quadrupole formalism

Analytical relationships in the transformed space

Asymptotic expansions
Simplified models

Direct local relationships between measurement points

No grid = it is not necessary to compute the solution in the whole domain

No time discretization

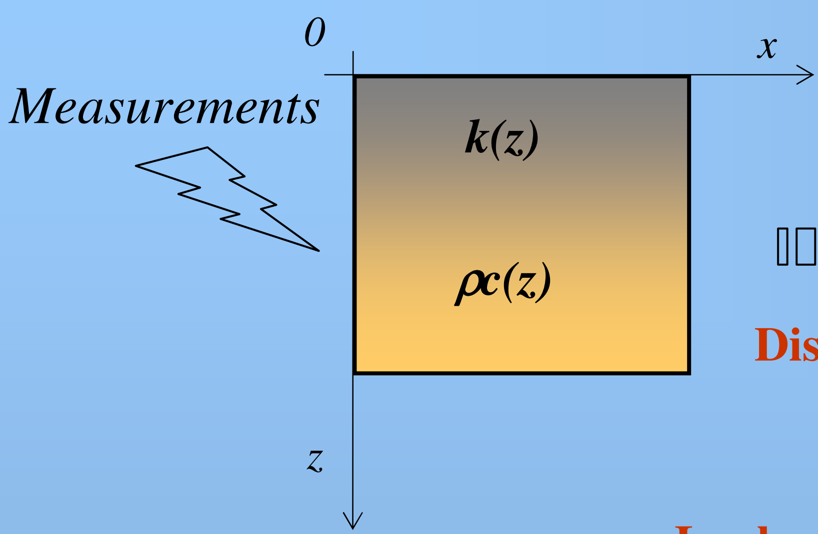
No accumulation of errors / t

Multilayer systems

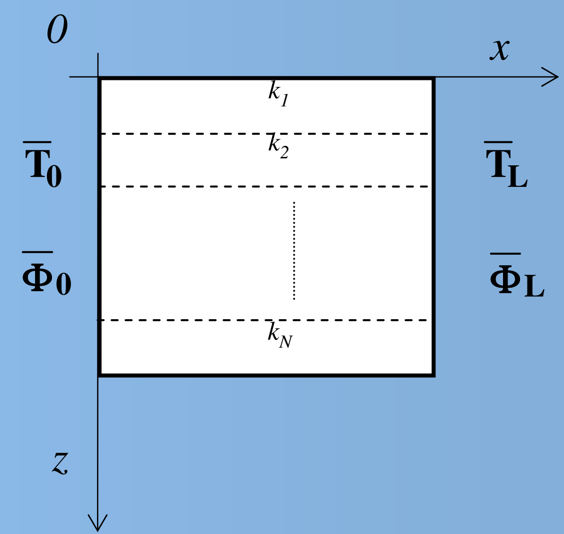
Matix multiplication

Exclusively limited to linear systems

Semi-analycal extension for heterogeneous media



Discretization
/ z
+
Laplace Transform / t



$$k(z)\frac{\partial^2 T}{\partial x^2} + \frac{\partial}{\partial z}\left(k(z)\frac{\partial T}{\partial z}\right) = (\rho c)(z)\frac{\partial T}{\partial t} \quad \Rightarrow \quad \mathbf{M}_s\left(\mathbf{M}_{//} + \mathbf{G}s\right)\overline{\mathbf{T}} - \frac{d^2\overline{\mathbf{T}}}{dx^2} = \mathbf{0}$$

Matrix relative to transfer in the x direction

transverse conduction versus z

Semi-analycal extension for heterogeneous media

1. Diagonalization

$$\mathbf{M}_s(\mathbf{M}_{//}+\mathbf{G}s)=\mathbf{P\Omega P}^{-1}$$

$$\mathbf{V}=\mathbf{P}^{-1}\overline{\mathbf{T}}$$

2. Resolution in the eigenvalues space

$$\mathbf{\Omega V}-\frac{d^2\mathbf{V}}{dx^2}=\mathbf{0}$$

$$\mathbf{J_V}=-dz\frac{d\mathbf{V}}{dx}$$

$$\mathbf{A_V}=\mathbf{D_V}=cosh(\sqrt{\mathbf{\Omega}}L)$$

$$\mathbf{B_V}=sinh(\sqrt{\mathbf{\Omega}}L)\left(\sqrt{\mathbf{\Omega}}dz\right)^{-1}$$

$$\mathbf{C_V}=\left(dz\sqrt{\mathbf{\Omega}}\right)sinh(\sqrt{\mathbf{\Omega}}L)$$

3. Return to temperature / flux basis

$$\mathbf{A}=\mathbf{PA_V P}^{-1}$$

$$\mathbf{B}=\mathbf{PB_V}(\mathbf{KP})^{-1}$$

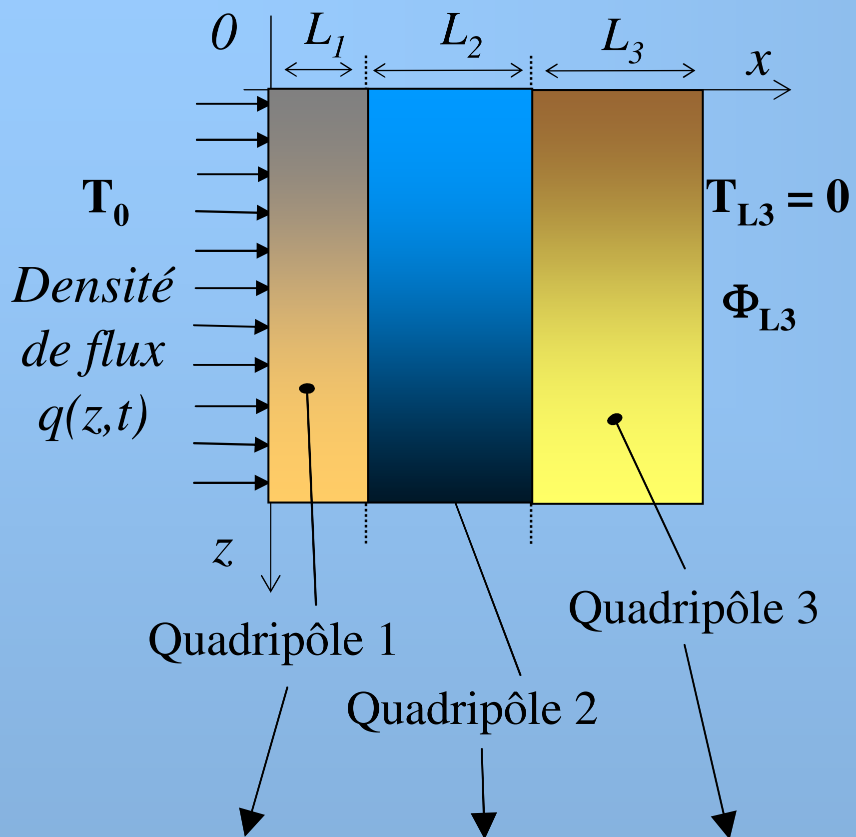
$$\mathbf{C}=\mathbf{KPC_V P}^{-1}$$

$$\mathbf{D}=\mathbf{KPD_V}(\mathbf{KP})^{-1}$$

$$\overline{\mathbf{\Phi}}=-dz\mathbf{K}\frac{d\overline{\mathbf{T}}}{dx}=\mathbf{KPJ_V}$$

$$\begin{bmatrix}\overline{\mathbf{T}}\\ \overline{\mathbf{\Phi}}\end{bmatrix}_{x1}=\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{T}}\\ \overline{\mathbf{\Phi}}\end{bmatrix}_{x2}$$

Implementation of the method



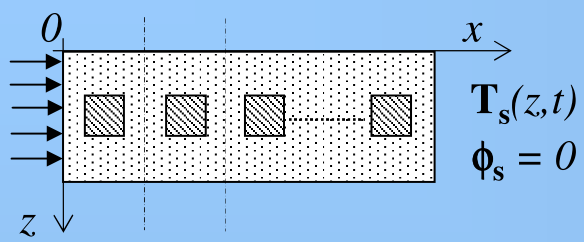
Direct computation with N points

(Numerical methods => N^2)

$$\begin{bmatrix} T_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} 0 \\ \Phi_{L3} \end{bmatrix}$$

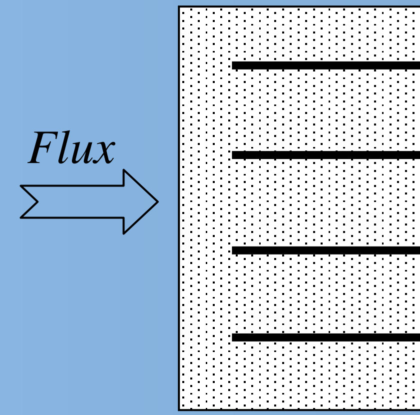
Wall temperature field as a function of the input heat flux

Some examples of applications



Periodic structures

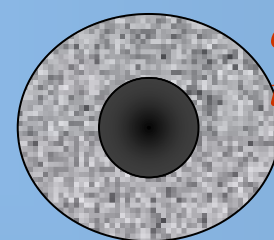
Optimization of the wall température field



Diffusive inserts

Homogénéisation en fonction du nombre de couches

Construction of the matrix $\mathbf{M}_{//}$ in the r direction

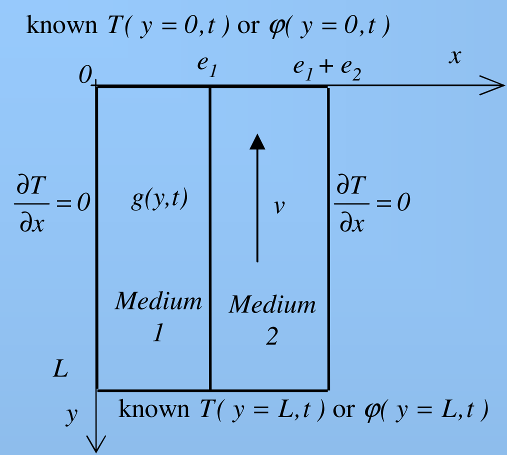


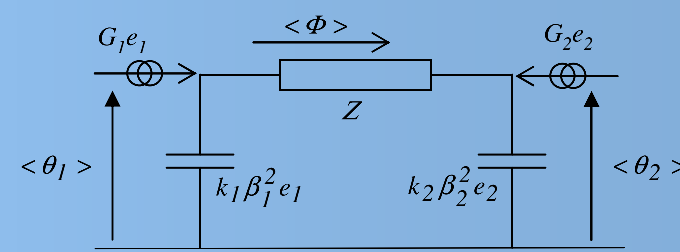
Radial discretization
(Reinforced composite fibre)

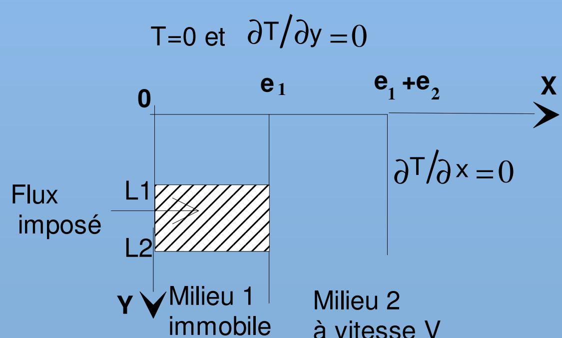
Coupled Equations : Analytical solutions in a quadrupole form

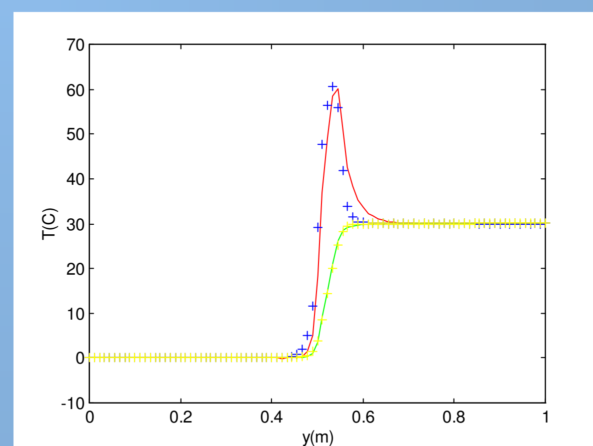
$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} \bar{\theta} \\ \bar{T} \end{bmatrix} - \frac{1}{x^p} \begin{bmatrix} D_\theta & D_T \\ 0 & a^* \end{bmatrix} \frac{d}{dx}\left(x^p \frac{d}{dx}\left(\begin{bmatrix} \bar{\theta} \\ \bar{T} \end{bmatrix} \right)\right) = \mathbf{0}$$

Mean Temperature analytical quadrupole









The Thermal Quadrupole Formalism.
Application to the estimation of thermophysical properties by random heating

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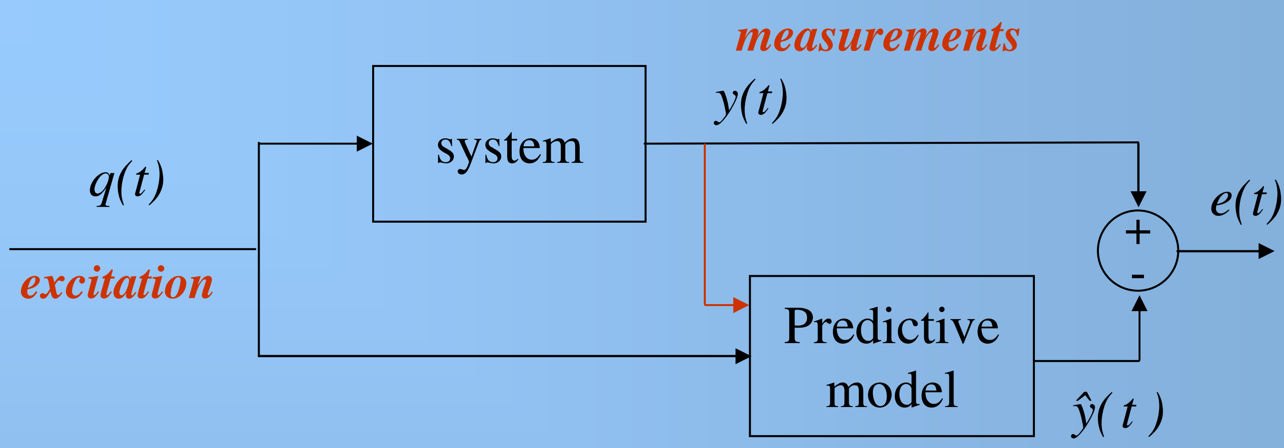
4. Conclusion

Linear estimation : minimization of the prediction error *e(t)*



The regression matrix is filled with measurements

Sampled system $$y(t_k) = \overbrace{\mathbf{H}(t_k)\boldsymbol{\beta}}^{\hat{y}(t_k)} + e(t_k)$$

n successive measurements $$\mathbf{Y}_n = \mathbf{H}_n\boldsymbol{\beta} + \mathbf{E}_n$$

OLS Estimation $$\hat{\boldsymbol{\beta}} = \left(\mathbf{H}_n^t\mathbf{H}_n\right)^{-1}\mathbf{H}_n^t\mathbf{Y}_n$$

Linear estimation : minimization of the prediction error *e(t)*

The cost = biased estimador

$$E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta} + \left(E(\mathbf{H}_n^t \mathbf{H}_n)\right)^{-1} E(\mathbf{H}_n^t \mathbf{E}_n)$$

Bias is zero if :

$E(\mathbf{H}_n^t \mathbf{H}_n)$ non singular

&

e(t) is a white noise

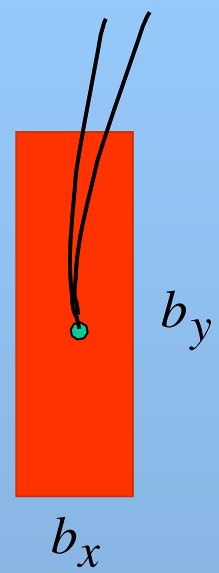
or

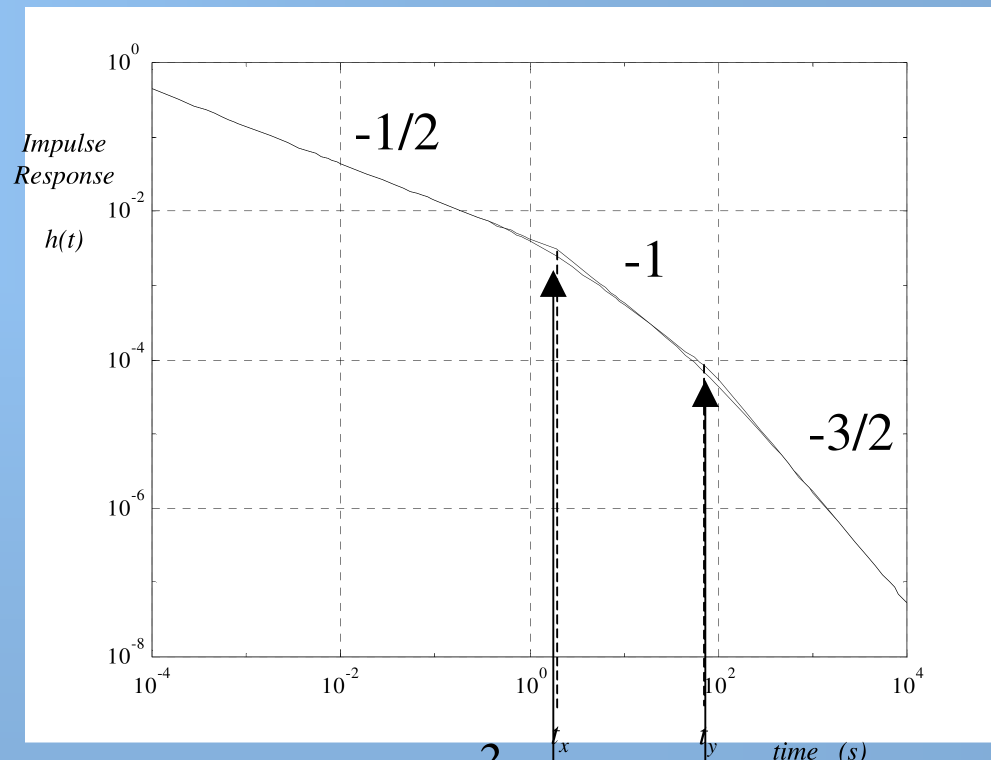
The input *q(t)* is independant of *e(t)*
and *H(t)* does not depend of *y(t)*

Pseudo-random heating

Thermophysical properties measurement in semi-infinite medium

$$h_{green}(t) = \frac{1}{b\sqrt{\pi t}}.erf\left(\frac{b_x}{\sqrt{4at}}\right).erf\left(\frac{b_y}{\sqrt{4at}}\right)$$





You can choose
the characteristic times
with
the probe's lengths

$$t_x = \frac{b_x^2}{\pi a}$$

$$t_y = \frac{b_y^2}{\pi a}$$

Pseudo-random heating

Simplified model corresponding to the asymtptotical behaviour

$$H_0(s) = \frac{1}{b\sqrt{s} + \frac{k}{K}}$$

…with the thermal probe's effect

$$\begin{bmatrix} H_m \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ C_s s & 1 \end{bmatrix} \begin{bmatrix} 1 & R_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} H_0 \phi_0 \\ \phi_0 \end{bmatrix}$$

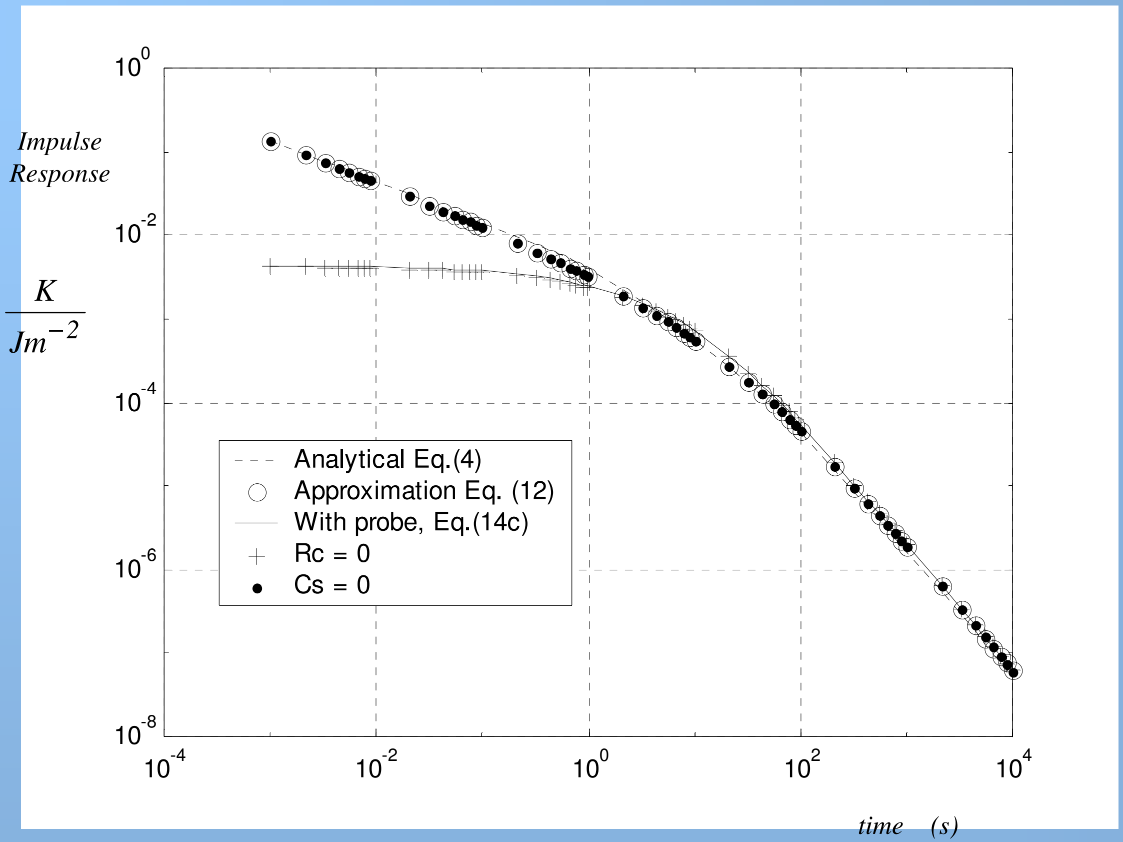
…probe's temperature as a function of the input heat flux

$$\frac{k}{K} + b s^{1/2} \overline{T}_m + \alpha_2 s \overline{T}_m + b R_c C_s s^{3/2} \overline{T}_m = \beta_0 + b R_c s^{1/2} \overline{q}$$

…Fractional derivative equation

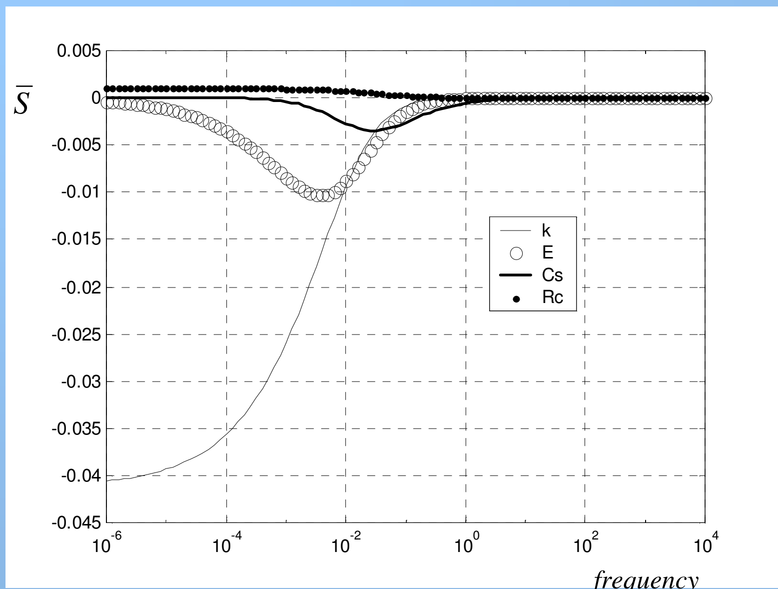
$$\frac{k}{K} + b \mathrm{D}^{1/2} T_m(t) + \alpha_2 \mathrm{D}^1 T_m(t) + b R_c C_s \mathrm{D}^{3/2} T_m(t) = \beta_0 + b R_c \mathrm{D}^{1/2} q(t)$$

Pseudo-random heating

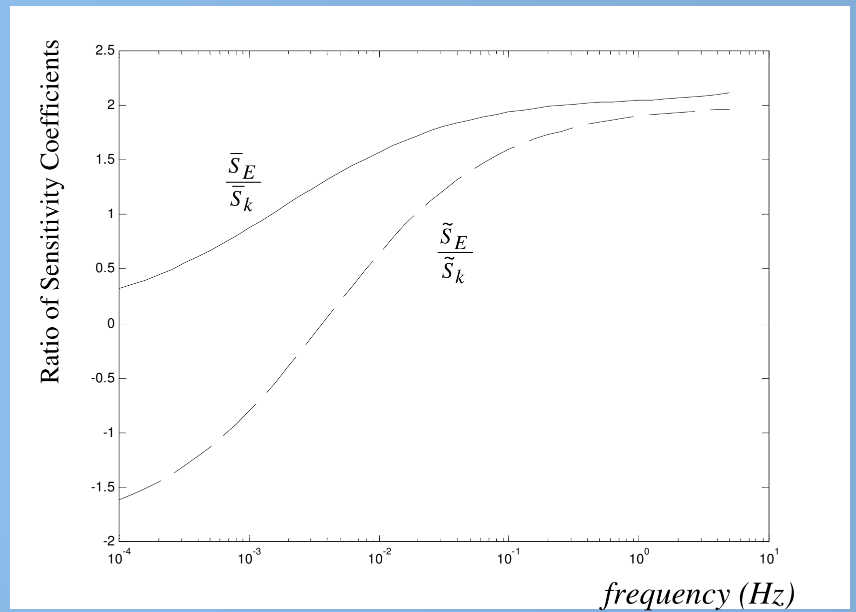


Comparison of impulse responses

Pseudo-random heating

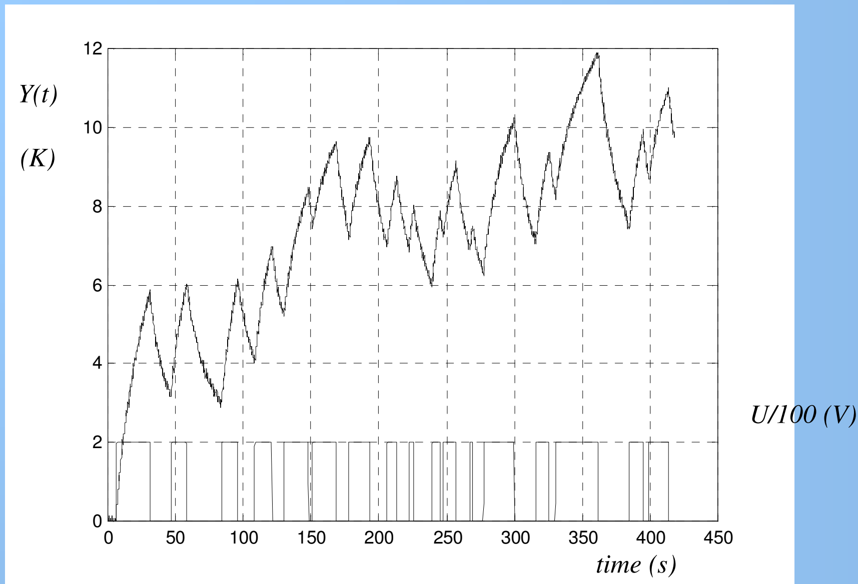


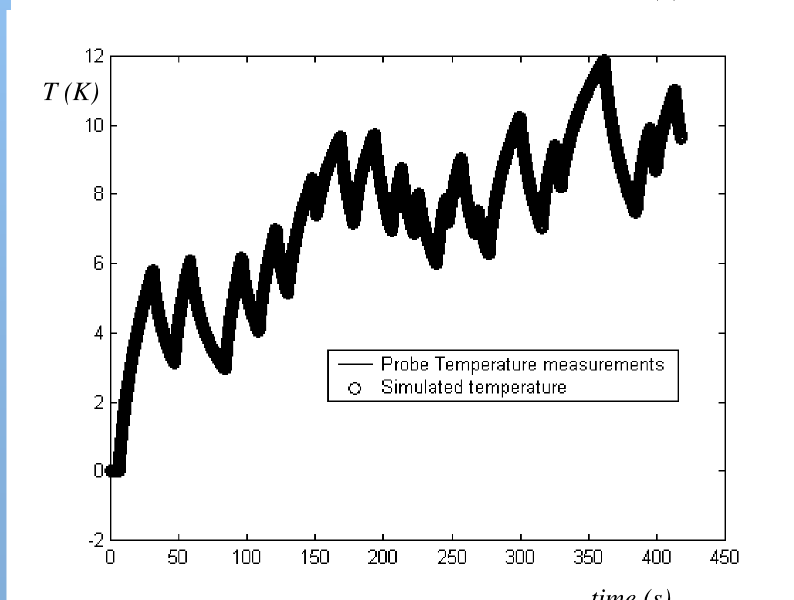
Reduced sensitivity coefficients (modulus)



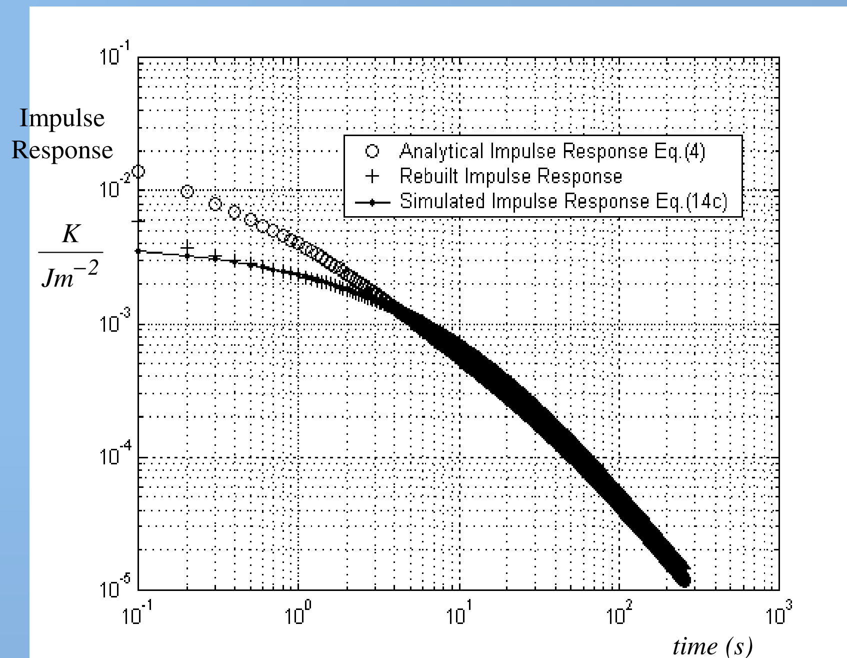
Independence of sens. coef.

Pseudo-random heating





Experimental and fitted temperature



Simulated and recovered I.R.

Pseudo-random heating

Linear regression with *n+1* successive measurements

$$D^{1/2}\mathbf{Y}_n = \mathbf{H}_n \boldsymbol{\beta} + \mathbf{E}_n$$

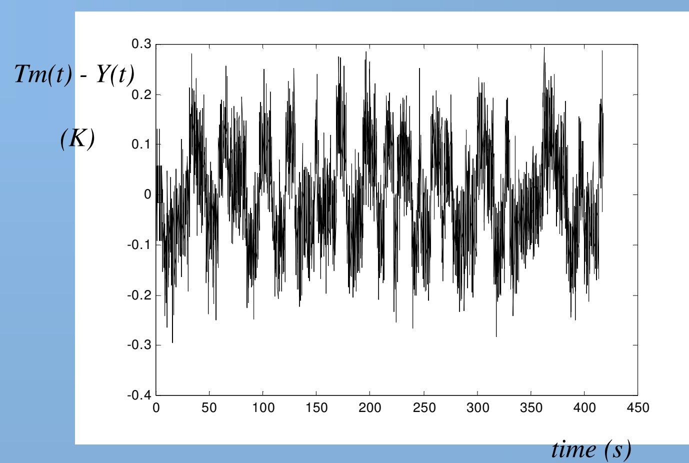
Fractional derivatives of $T_m(t)$ and $q(t)$

OLS Estimator

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{H}_n^t \mathbf{H}_n\right)^{-1} \mathbf{H}_n^t D^{1/2} \mathbf{Y}_n$$

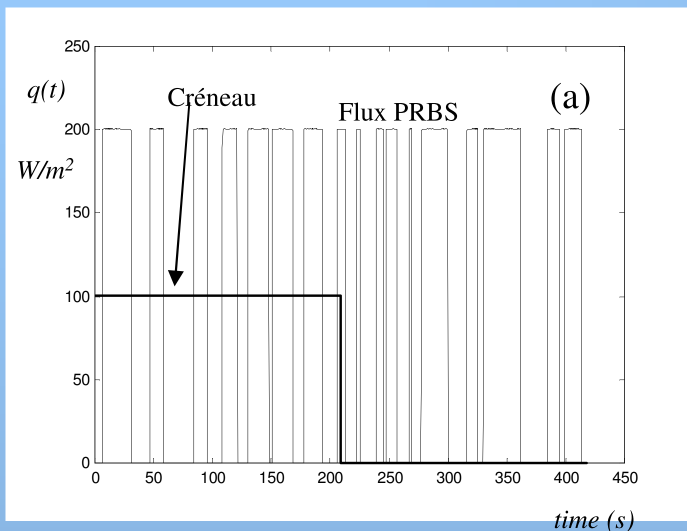
Sample	Thermal Properties Reference Data	Estimation Results	Relative Error (%)
Calcium Silicate Skamol Super 1100-E	$E = 123.4$ Wm^{-2}K^{-1}s$^{1/2}$ $k = 0.074$ Wm^{-1}K^{-1}	$E = 110.8$ Wm^{-2}K^{-1}s$^{1/2}$ $k = 0.074$ Wm^{-1}K^{-1}	-10 0.4
Agar Agar Gel 3 gr. / l.	$E = 1597$ Wm^{-2}K^{-1}s$^{1/2}$ $k = 0.613$ Wm^{-1}K^{-1}	$E = 1405$ Wm^{-2}K^{-1}s$^{1/2}$ $k = 0.606$ Wm^{-1}K^{-1}	-12 -1.1
Extruded Polystyrene Owens Thermofoam	$E = 43.87$ Wm^{-2}K^{-1}s$^{1/2}$ $k = 0.025$ Wm^{-1}K^{-1}	$E = 44.93$ Wm^{-2}K^{-1}s$^{1/2}$ $k = 0.026$ Wm^{-1}K^{-1}	0.2 6.4

Experimental results

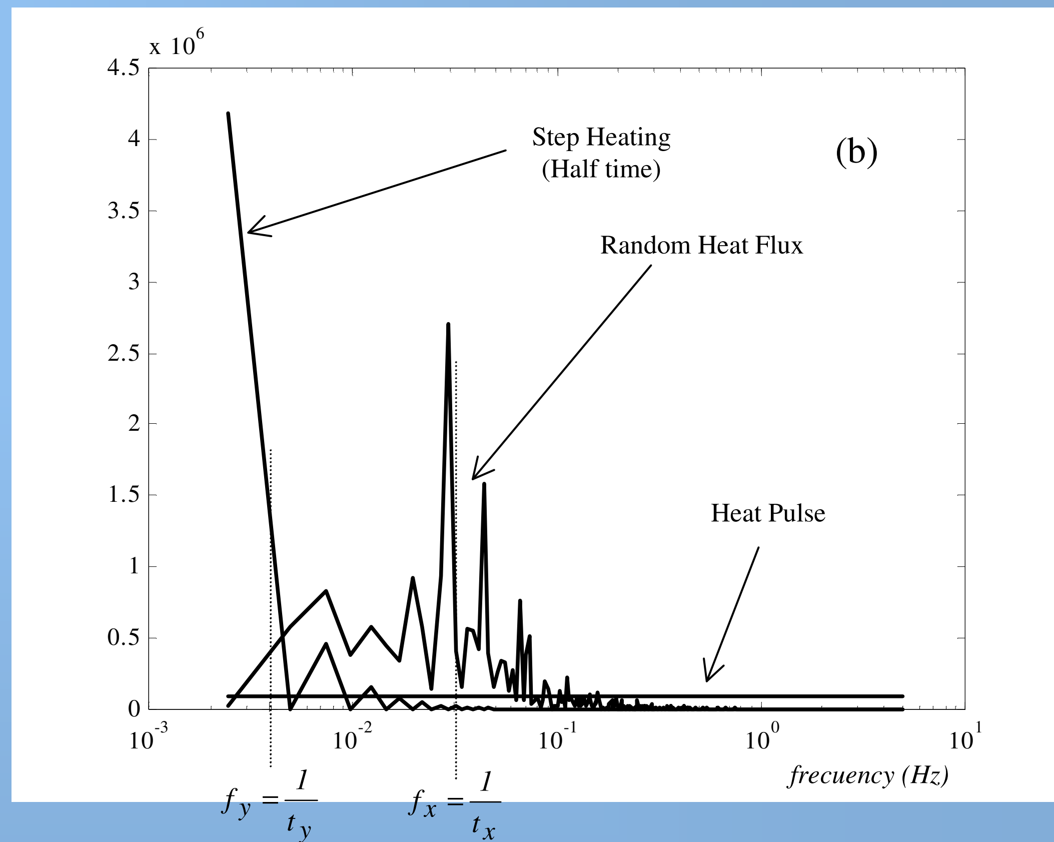


Residual function

Pseudo-random heating



PRBS heating

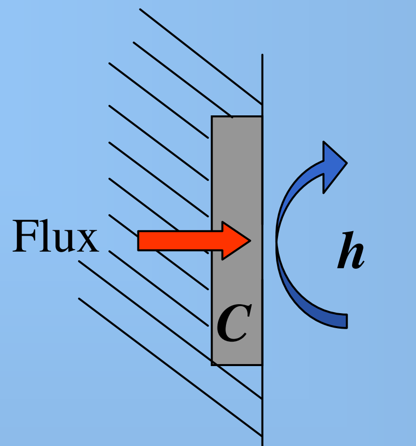


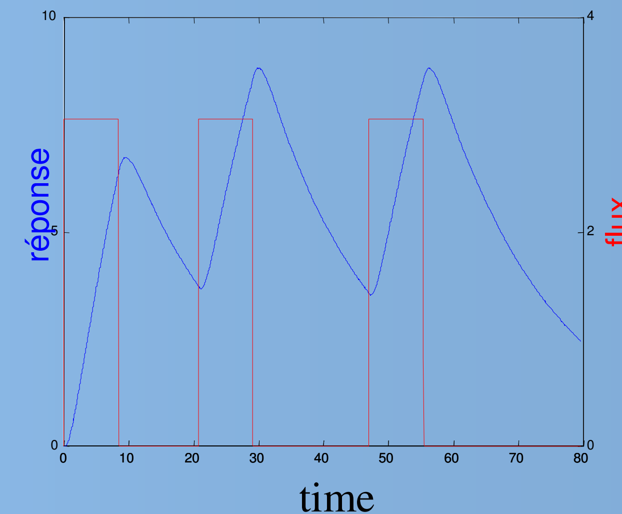
Power spectral density

Pseudo-random heating

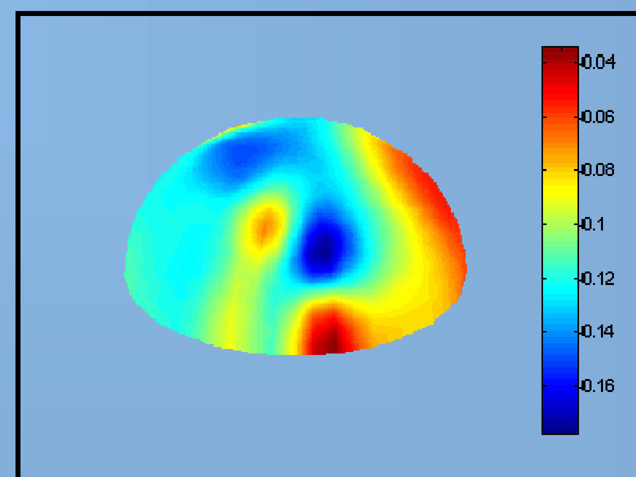
Convective coefficients mapping

Thermal characterization of cyclist casque





$$C\frac{dT}{dt} = \varphi(t) - hT$$

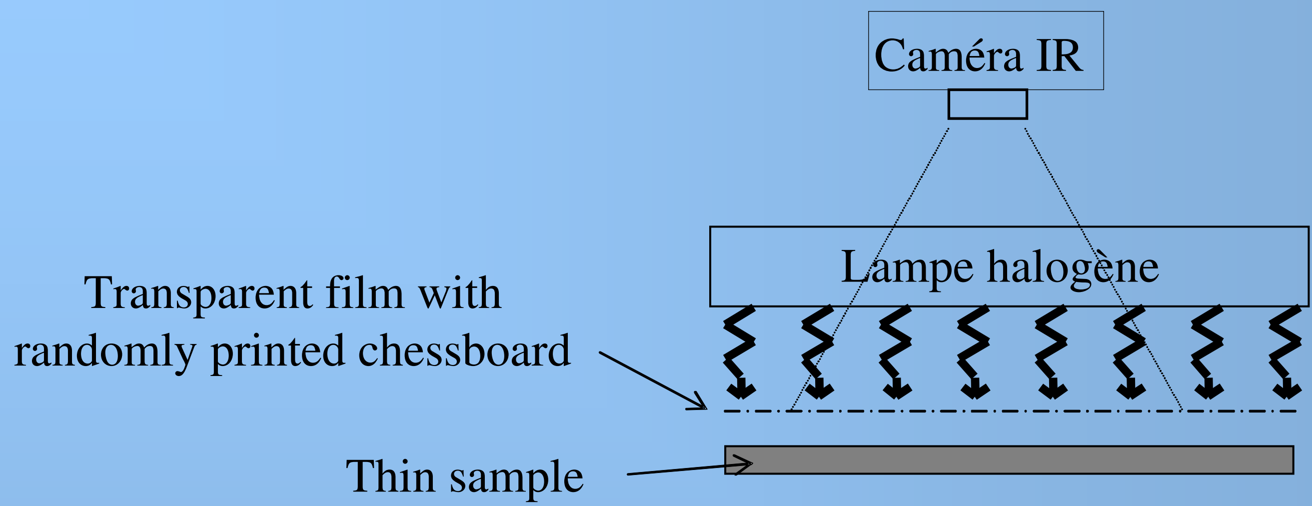


Characteristic frequencies are estimated : $\dfrac{h}{C}$

The Thermal Quadrupole Formalism.
Application to the estimation of thermophysical properties by random heating

1. The Thermal Quadrupole Formalism
2. Estimation of thermophysical properties by random heating
3. Thermal diffusivity mapping from spatial random heating
4. Conclusion

Thermal diffusivity mapping from spatial random heating



$$\rho c(x,y)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k(x,y)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(x,y)\frac{\partial T}{\partial y}\right)-\frac{2h}{e}(T-T_\infty)$$

Discretization

$$\mathbf{T}^{\mathbf{t}+\Delta t}-\mathbf{T}^{\mathbf{t}}=\mathbf{A}.*\Delta\mathbf{T}^{\mathbf{t}}+\mathbf{C}^{-\mathbf{1}}.*\delta_{\mathbf{x}}\mathbf{K}.*\delta_{\mathbf{x}}\mathbf{T}^{\mathbf{t}}+\mathbf{C}^{-\mathbf{1}}.*\delta_{y}\mathbf{K}.*\delta_{y}\mathbf{T}^{\mathbf{t}}-\mathbf{H}.*(\mathbf{T}^{\mathbf{t}}-T_\infty)$$

Thermal diffusivity mapping from spatial random heating

$$\hat{\mathbf{T}}' - \hat{\mathbf{T}} = \begin{bmatrix} \hat{\mathbf{T}}^{\mathbf{t}_0+\Delta\mathbf{t}} - \hat{\mathbf{T}}^{\mathbf{t}_0} \\ . \\ \hat{\mathbf{T}}^{\mathbf{t}+\Delta\mathbf{t}} - \hat{\mathbf{T}}^{\mathbf{t}} \\ . \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} \Delta\hat{\mathbf{T}}^{\mathbf{t}_0} & \delta_x \hat{\mathbf{T}}^{\mathbf{t}_0} & \delta_y \hat{\mathbf{T}}^{\mathbf{t}_0} & \hat{\mathbf{T}}^{\mathbf{t}_0} - T_\infty \\ . & . & . & . \\ \Delta\hat{\mathbf{T}}^{\mathbf{t}} & \delta_x \hat{\mathbf{T}}^{\mathbf{t}} & \delta_y \hat{\mathbf{T}}^{\mathbf{t}} & \hat{\mathbf{T}}^{\mathbf{t}} - T_\infty \\ . & . & . & . \end{bmatrix}$$

$$\hat{\beta} = \left(\mathbf{X}^t \mathbf{X}\right)^{-1} \mathbf{X}^t \left(\hat{\mathbf{T}}' - \hat{\mathbf{T}}\right)$$

Point by point estimation

$\mathbf{X}^t \mathbf{X}$ = 4x4 matrix

$$\beta_{ij} = \begin{bmatrix} a_{ij} \\ \dfrac{\delta_x k_{ij}}{(\rho c)_{ij}} \\ \dfrac{\delta_y k_{ij}}{(\rho c)_{ij}} \\ H_{ij} \end{bmatrix}$$

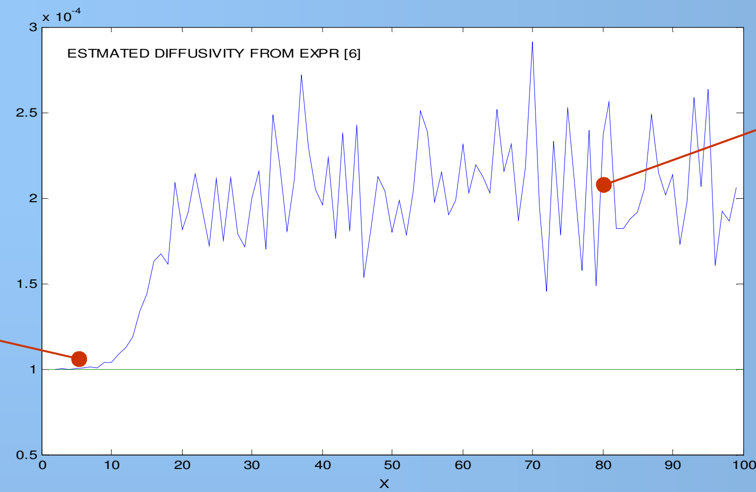
Sequential implementation of the sums
(Recursive estrimation)

Simplified model

$$\hat{\beta} \equiv \mathbf{A} \Rightarrow \hat{\mathbf{A}} = \frac{\sum_{i=0}^{n} \Delta\hat{\mathbf{T}}^{t_i} .* \left(\hat{\mathbf{T}}^{t_i+\Delta t} - \hat{\mathbf{T}}^{t_i}\right)}{\sum_{i=0}^{n} \left(\Delta\hat{\mathbf{T}}^{t_i}\right)^2}$$

Thermal diffusivity mapping from spatial random heating

Correct estimation in the perturbated region



Important bias in the unperturbated region

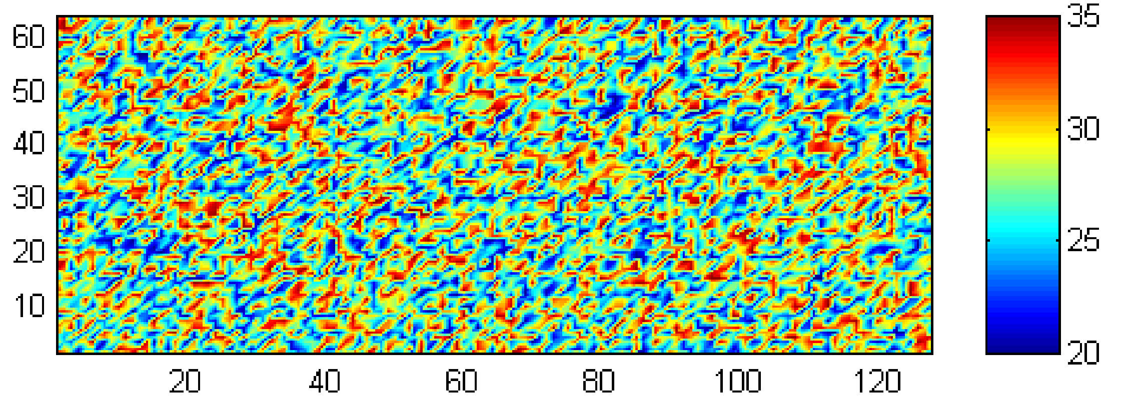
Homogeneous plate with local heating

Periodic heating ⇒ Periodic bias in the unperturbated points

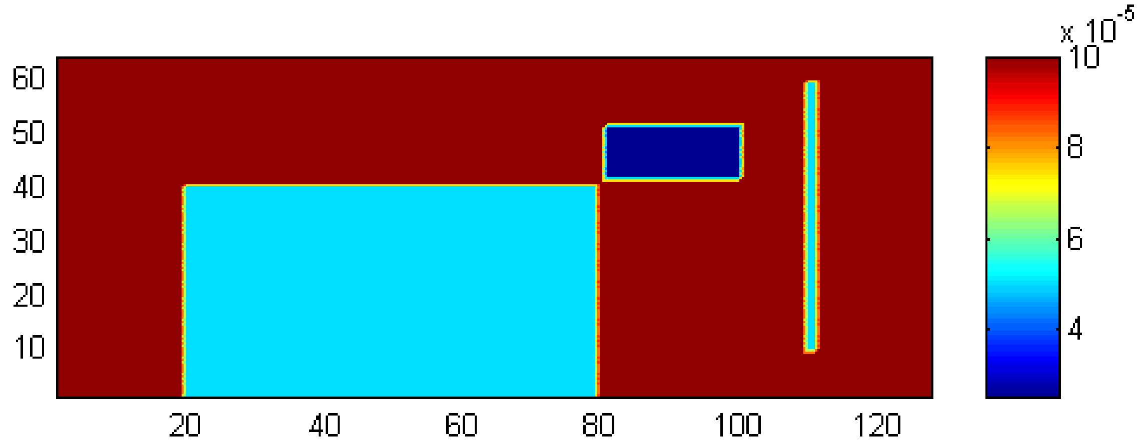
$$\hat{\mathbf{A}} = \frac{\sum_{i=0}^{n} \mathbf{\Delta}\hat{\mathbf{T}}^{t_i} .* \left(\hat{\mathbf{T}}^{t_i+\Delta t} - \hat{\mathbf{T}}^{t_i}\right)}{\sum_{i=0}^{n} \left(\mathbf{\Delta}\hat{\mathbf{T}}^{t_i}\right)^2}$$

Thermal diffusivity mapping from spatial random heating

Initial randomly distributed temperature field



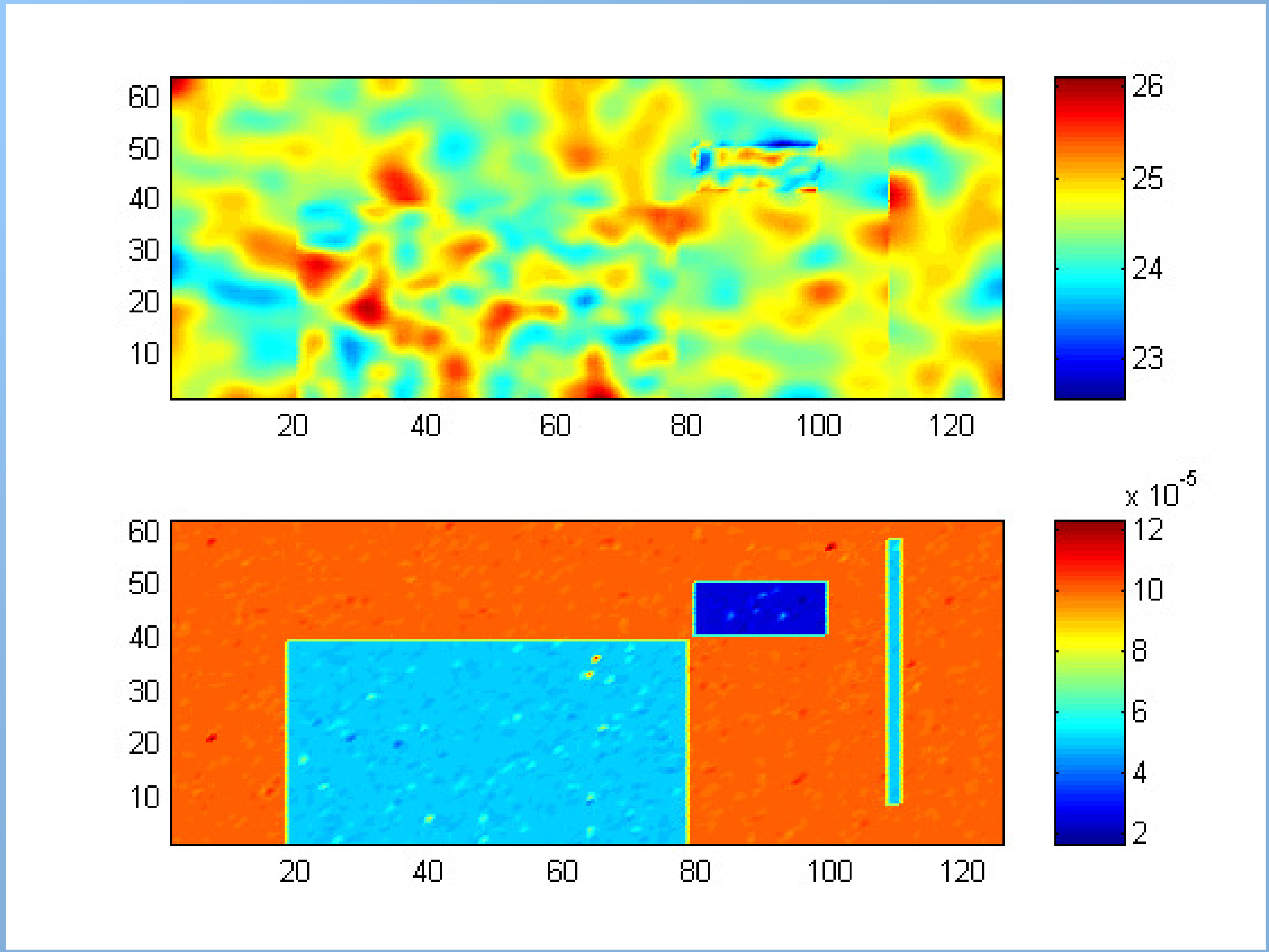
Sample



Thermal diffusivity mapping from spatial random heating

Final temperature field

Estimated thermal diffusivity field



The Thermal Quadrupole Formalism.
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4. Conclusions

- Limitation of numerical performance of hybrid quadrupoles

- Complex Geometry

- Both time and spatial random heating

- Thermal tomography

Lost quadrupole

A última pergunta ?

Cadê a melhor receita da Caipirinha ?