# Thermophysical Properties in Combustion

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## *Information Sources - Journals*

- *Combustion and Flame*
- *Combustion Science and Technology*
- *Combustion Theory and Modeling*
- *Progress in Energy and Combustion Science*
- *Proceeding of the Combustion Institute*

## *Information Sources - Books*

- *Combustion – I. Glassman*
- *Principles of Combustion – K.K. Kuo*
- *Combustion Theory – F.A. Williams*
- *Droplets and Sprays – W.A. Sirignano*

## *Historical Perspective*

- Development of the Modern Society
  - Pre-Historical: Heating, Cooking, Metals
  - Steam Engine
  - Industrial Revolution
  - Otto Engine (1876) and Diesel Engine(1890)
  - II World War
    - Mobility ➢ Liquid Fuels
    - Gas Turbine
    - Rocket Engine
  - Oil Crisis (1973)

- Modern Engineering
  - Environment Aspects
  - Waste Treatment
  - Alternative Fuels
  - Cogeneration

## *Applications*

- Power Production
  - Thermoelectric Power Stations
  - Plane, Ship and Automobile Propulsion
  - Gas turbine Engines
  - Rocket Engines
- Process Industry
  - Steel, Glass, Ceramics, Cement
- Fire Prevention and Safety
- Heating – Industrial and Household
- Environment Aspects
  - Formation of Pollutants – $NO_x$, $SO_x$, CO
  - Formation of Particulates
  - Exhaust Gas Temperature Control

# *Motivation*

## • *Definitions*

- Oxidation
- Energy release
- Flame

## • *Background*

- Thermodynamics
- Chemical Kinetics
- Fluid Mechanics
- Heat and Mass Transfer
- Turbulence
- Materials Structure and Behavior

## • *Introduction*

- Thermodynamic Equilibrium
    - Mechanical + Thermal + Chemical
    - Thermodynamic properties + composition
- Nonequilibrium Thermodynamics – Rates of Change

## • *Thermodynamic Laws*

- *0th Law* – Temperature + Thermal Equilibrium
- *1th Law* – Internal Energy + Conservation of Energy
- *2th Law* – Entropy + Temperature Scale
- *3th Law* – Entropy Reference State and Value

## • Criteria for Equilibrium

**TABLE 1.1 General Equilibrium Criteria for Closed Thermodynamic System[15]**

| Variables Held Constant | Criteria for Thermodynamic Equilibrium in a Closed System |
|---|---|
| $p$ | $dH - T\,dS = dG + S\,dT = 0$ |
| $V$ | $dU - T\,dS = dA + S\,dT = 0$ |
| $T$ | $d(U - TS) + p\,dV = dG - V\,dp = dA + p\,dV = 0$ |
| $S$ | $dU + p\,dV = dH - V\,dp = 0$ |
| $p, T$ | $dG = 0$ |
| $V, T$ | $dA = 0$ |
| $p, S$ | $dH = 0$ |
| $V, S$ | $dU = 0$ |
| $S, U$ or $A, T$ | $dV = 0$ |
| $A, V$ or $G, p$ | $dT = 0$ |
| $U, V$ or $H, p$ | $dS = 0$ |
| $G, T$ or $H, S$ | $dp = 0$ |

## • Equilibrium Constants

$$K_p \equiv \frac{p_C^c\, p_D^d}{p_A^a\, p_B^b} \qquad \Delta G_0 = -R_u T \ln K_p$$

## • Specifying Amounts

- Mass Fraction - *Y*

$$Y_i = \frac{m_i}{\sum_{i=1}^{N} m_i}$$

- Mole Fraction - *X*

$$X_i = \frac{n_i}{\sum_{i=1}^{N} n_i}$$

- Fuel – Oxidant Ratio - F/O

$$F/O = \frac{m_{fuel}}{m_{oxidant}}$$

- Equivalence Ratio - ϖ

$$\phi = \frac{(F/O)}{(F/O)+(F/O)_{st}}$$

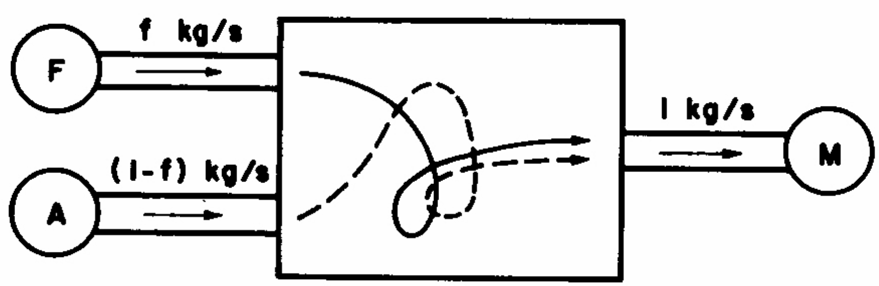
## • *Specifying Amounts*

- Mixture Fraction - $f$

$$f\xi_F + (1 - f)\xi_A = \xi_M$$

$$f = \frac{\xi_M - \xi_A}{\xi_F - \xi_A}$$

- $\nu$ - Extensive Property
- Sourceless
- $Y_{inert}$ ; $Y_F$ - $(F/O)_{st} Y_O$ ; ……
- Complete Reaction - $f_{st}$



## • *Standard Heat of Formation*

- Amount of energy (heat) absorbed or released when a substance is formed from its elements in the reference state (298.15K – 1 atm).
- Stable elements in the standard state have zero *heat of formation.*

## • *Heat of Reaction*

- Amount of energy (heat) absorbed or released during a reaction occurring at a given $T$ and $p$.

## • *Adiabatic Flame Temperature*

- Temperature of the products when the combustion process occurs adiabatically, with no work interactions, changes in kinetic or potential energy.
- Maximum temperature for the given conditions.

- ***Reaction Rates - Arrhenius***

$$aA + bB \xrightarrow{k_f} P$$

$$aA + bB \xleftarrow[k_b]{} P$$

$$\left.\frac{dC_A}{dt}\right|_f = -ak_f C_A^a C_B^b = -a\beta_f C_A^a C_B^b \, exp\left(-\frac{E_{a,f}}{R_u T}\right)$$

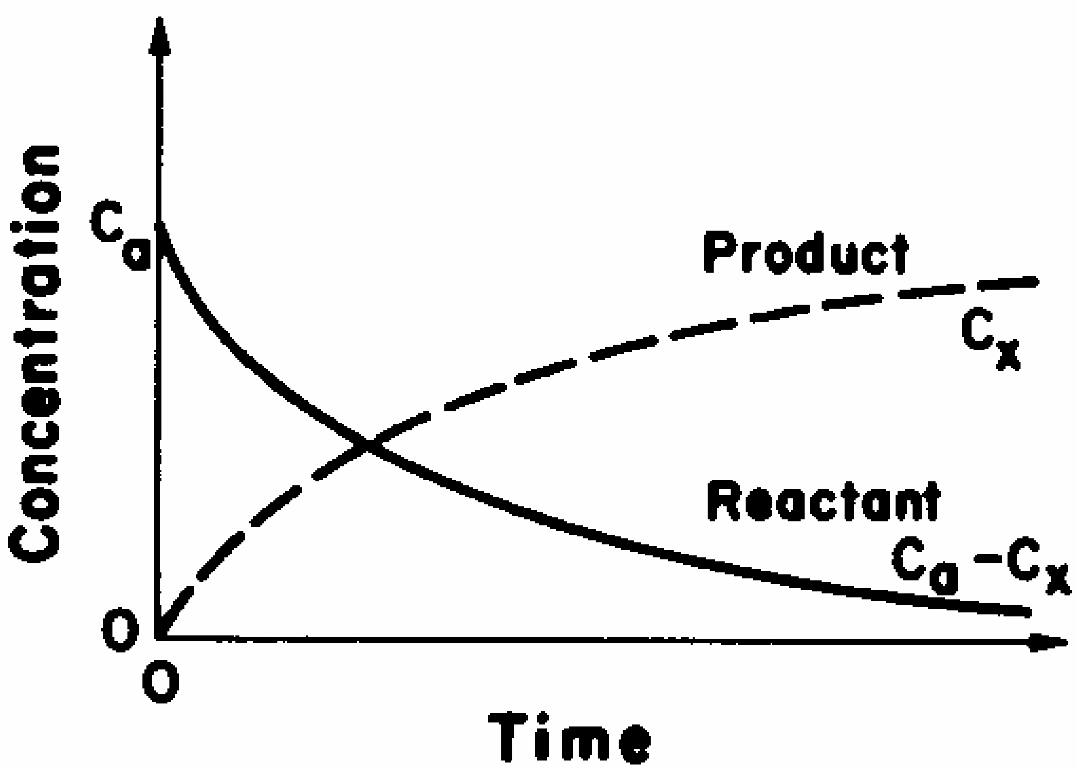
$$\left.\frac{dC_A}{dt}\right|_r = +ak_r C_A^a C_B^b = +a\beta_r C_A^a C_B^b \, exp\left(-\frac{E_{a,r}}{R_u T}\right)$$

- ***Equilibrium***

$$\left.\frac{dC_A}{dt}\right|_f = \left.\frac{dC_A}{dt}\right|_r$$

$$\left.\frac{dC_B}{dt}\right|_f = \left.\frac{dC_B}{dt}\right|_r$$

$$\left.\frac{dC_P}{dt}\right|_f = \left.\frac{dC_P}{dt}\right|_r$$



- **Continuity**

Rectangular coordinates $(x, y, z)$:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0 \qquad \text{(A)}$$

Cylindrical coordinates $(r, \theta, z)$:[a]

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0 \qquad \text{(B)}$$

Spherical coordinates $(r, \theta, \phi)$:[b]

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi) = 0 \qquad \text{(C)}$$

[a] $r \geq 0,\ 2\pi \geq \theta \geq 0.$

[b] $r \geq 0,\ 2\pi > \phi \geq 0,\ \pi \geq \theta \geq 0.$

- **Species**

$$\rho\left(\frac{\partial Y_i}{\partial t} + u_x\frac{\partial Y_i}{\partial x} + u_y\frac{\partial Y_i}{\partial y} + u_z\frac{\partial Y_i}{\partial z}\right)$$
$$+\frac{\partial}{\partial x}(\rho Y_i V_{ix}) + \frac{\partial}{\partial y}(\rho Y_i V_{iy}) + \frac{\partial}{\partial z}(\rho Y_i V_{iz}) = \omega_i$$

[where

$$V_{ix} = -\frac{\mathscr{D}}{Y_i}\frac{\partial Y_i}{\partial x}, \qquad V_{iy} = -\frac{\mathscr{D}}{Y_i}\frac{\partial Y_i}{\partial y}, \quad \text{and} \quad V_{iz} = -\frac{\mathscr{D}}{Y_i}\frac{\partial Y_i}{\partial z}$$

- **Momentum**

- **Energy**

$$\rho C_p\left(\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right)-\left(\frac{\partial p}{\partial t}+u_x\frac{\partial p}{\partial x}+u_y\frac{\partial p}{\partial y}+u_z\frac{\partial p}{\partial z}\right)$$

$$=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)-\sum_{i=1}^{N}\omega_i\Delta h^{\circ}_{f,i}$$

$$-\left[\frac{\partial}{\partial x}\left(\rho T\sum_{i=1}^{N}C_{pi}Y_iV_{ix}\right)+\frac{\partial}{\partial y}\left(\rho T\sum_{i=1}^{N}C_{pi}Y_iV_{iy}\right)+\frac{\partial}{\partial z}\left(\rho T\sum_{i=1}^{N}C_{pi}Y_iV_{iz}\right)\right]$$

$$+\mu\left\{2\left[\left(\frac{\partial u_x}{\partial x}\right)^2+\left(\frac{\partial u_y}{\partial y}\right)^2+\left(\frac{\partial u_z}{\partial z}\right)^2\right]+\left[\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial y}\right]^2\right.$$

$$\left.+\left[\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right]^2+\left[\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right]^2-\frac{2}{3}\left[\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right]^2\right\}$$

$$+\rho\sum_{i=1}^{N}Y_i(f_{ix}V_{ix}+f_{iy}V_{iy}+f_{iz}V_{iz}) \qquad \text{(A)}$$

## • Source Term

$$\omega_i = W_i \sum_{k=1}^{M} ( \nu''_{i,k} - \nu'_{i,k} ) B_k T^{\alpha} \exp\left[ -\frac{E_{ak}}{R_u T} \right] \prod_{j=1}^{N} \left( \frac{X_j p}{R_u T} \right)^{\nu'_{i,k}}$$

- Coupling
- *M – Total number of chemical reactions*
- *N – Total number of chemical species*

## • Equation of State

# Conservation Equations

- ***Unknowns***

$$Y_1, Y_2, Y_3, \ldots\ldots., Y_N, \rho, T, p, u, v, w$$

- ***Equations***

- 1 overall mass continuity equation
- 3 momentum equations
- 1 energy equation
- $N - 1$ species equation
- 1 equation of state
- 1 equation relating all $Y_i$

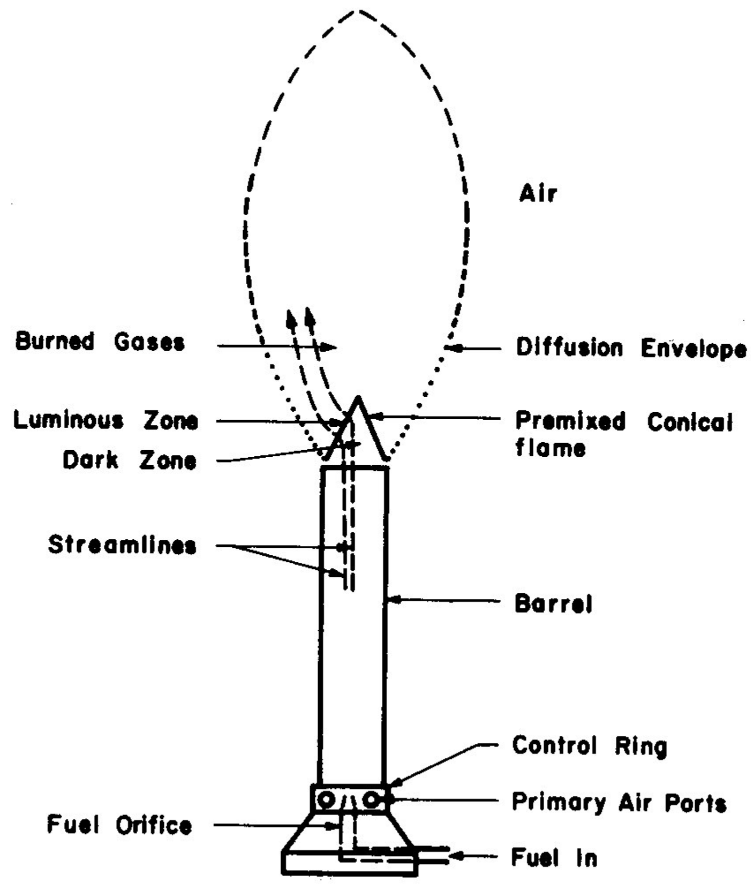
- ***Premixed x Diffusion Flames***

## • *Deflagration Waves*

- Subsonic
- Propagated by Energy Release from Chemical Reaction

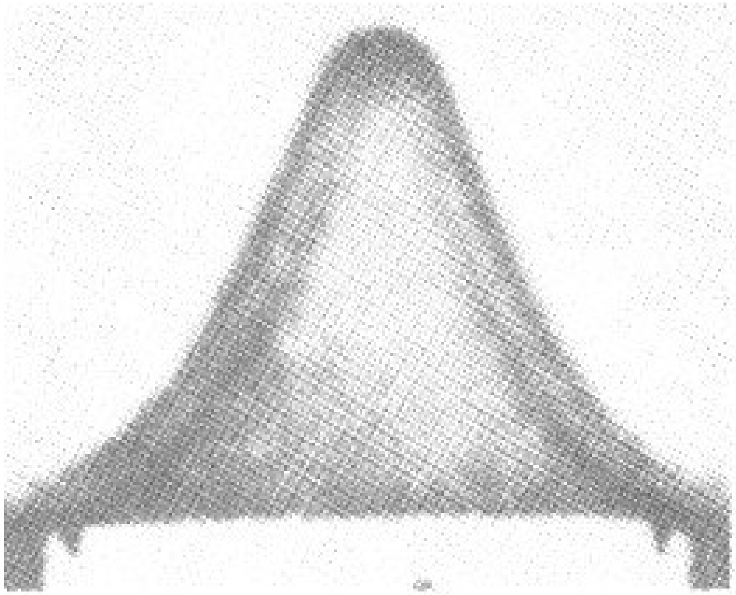
## • *Experiment*

- Bunsen, 1855
- Gaseous fuel
- Entrainment of Primary Air
- Anchored Flame



## Experiment (cont.)

- Luminous Conical Region
  - 1mm thick
  - Color – Fuel/Air Ratio
- Reaction and Heat release
- Nonstationary
- Nonplanar
- Find Laminar Flame Speed

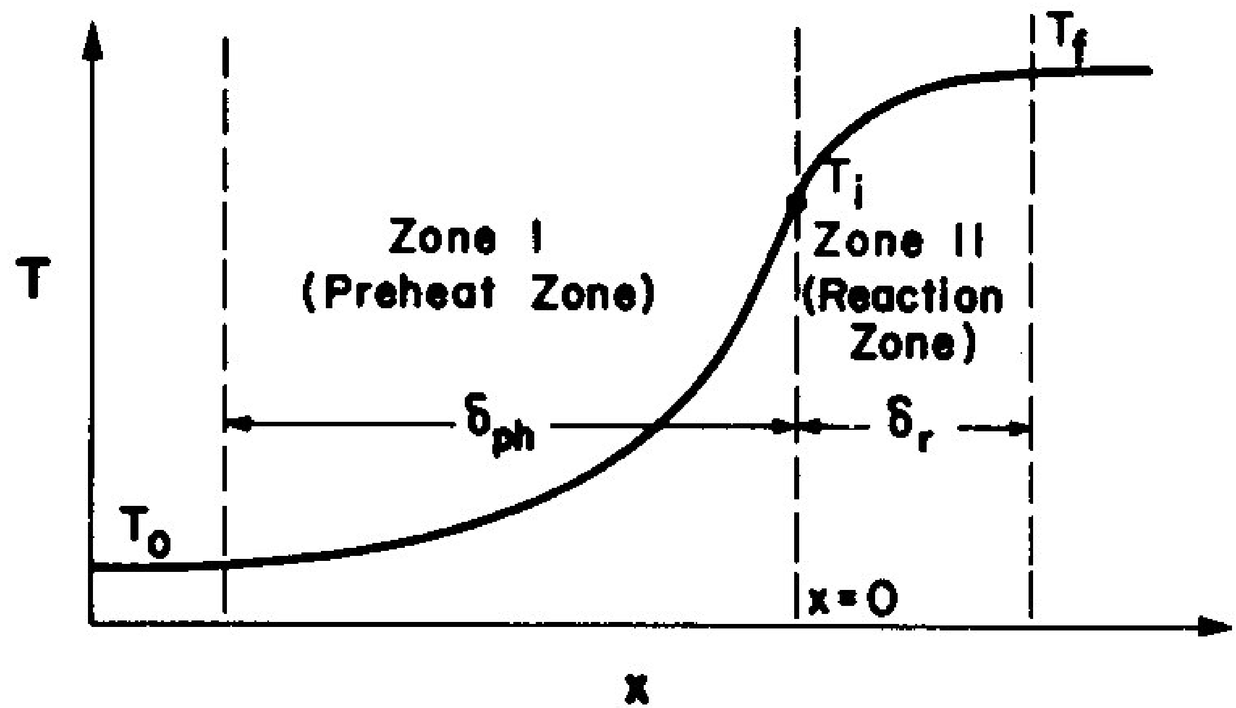


**Inner and Outer Cones**

- ***Thermal Theory***

$$S_L = C_1\sqrt{\alpha \cdot RR}$$

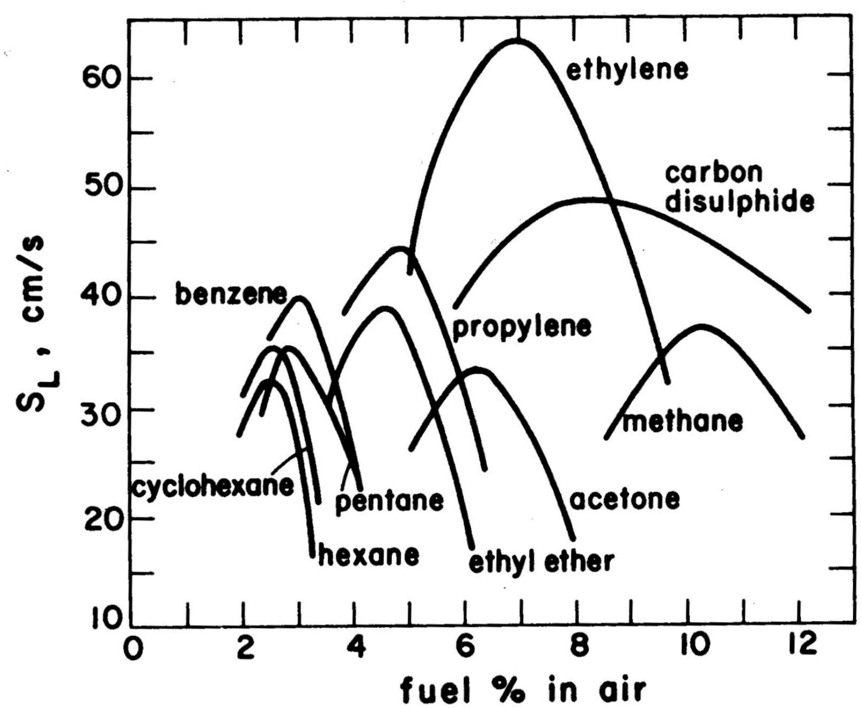
  - Mallard and Le Chatelier



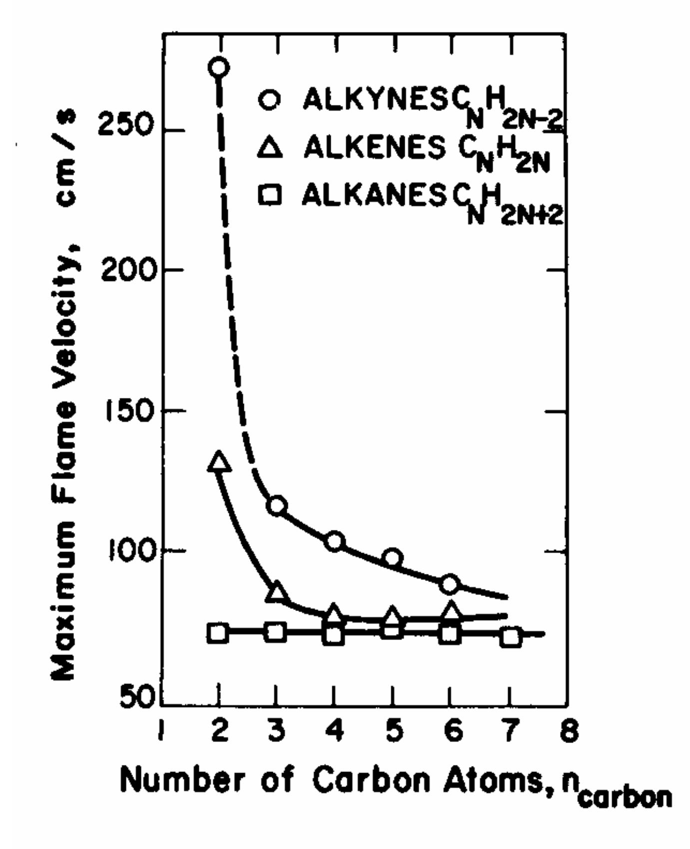

***Typical Temperature Variation***

# • *Effect of Chemical Properties*

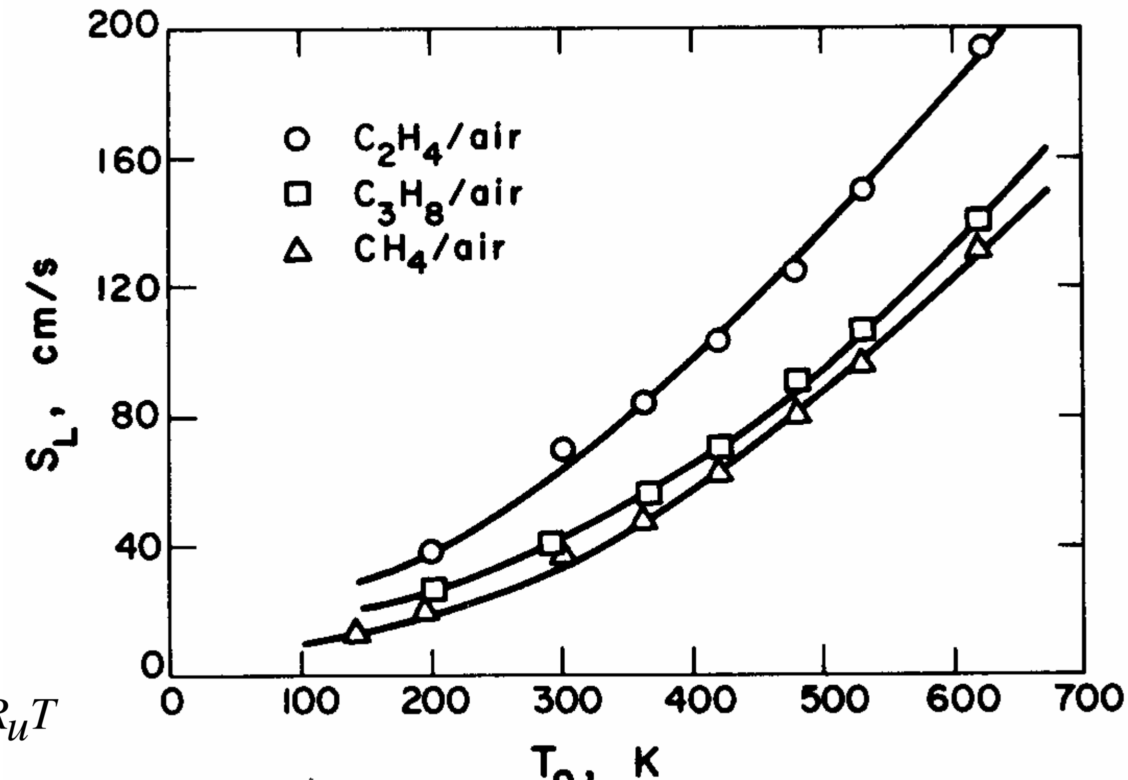
## • *Mixture Ratio*




## • *Fuel Molecular Structure*



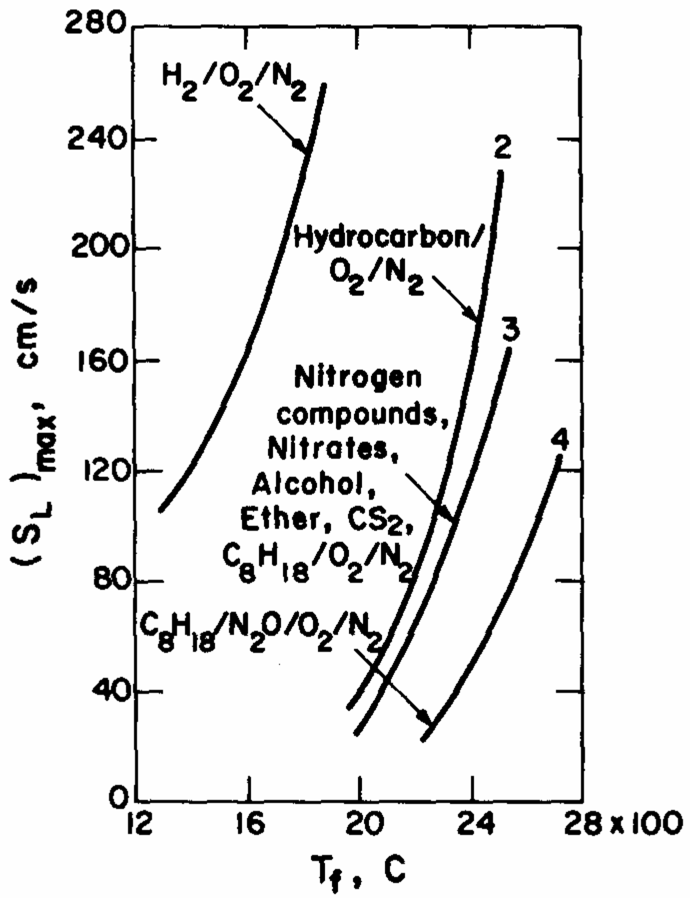
- ***Effect of Physical Properties***
  - ***Pressure –*** $S_L = C_2 \cdot p^{n-2}$
  - ***Initial Temperature***



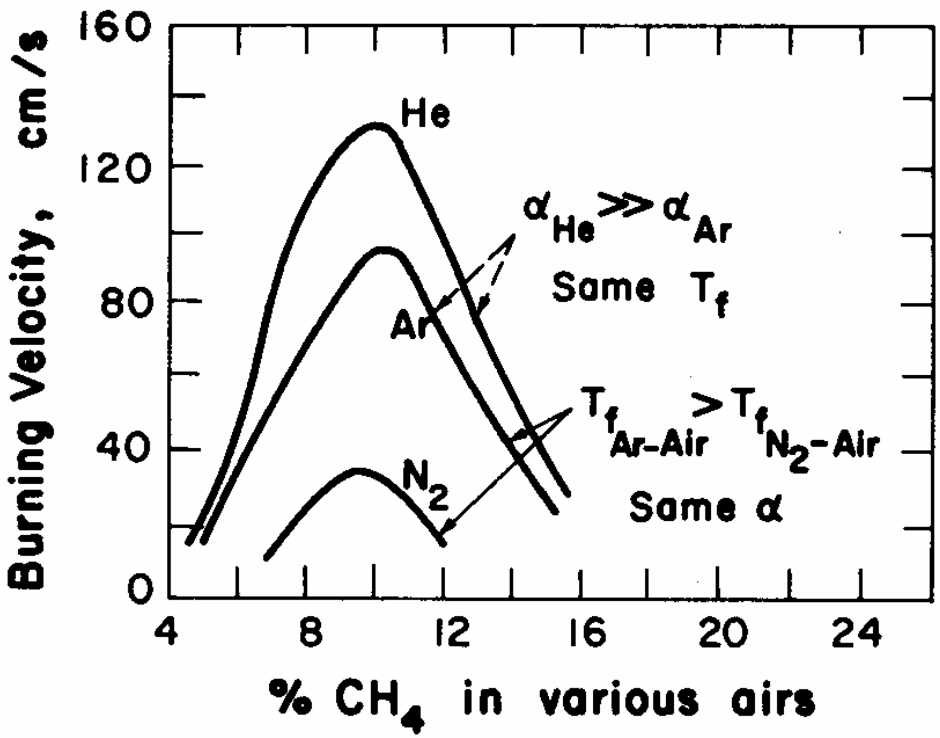

$$S_L = C_3 \cdot e^{-E/2R_uT}$$

# • Effect of Physical Properties

## • Flame Temperature

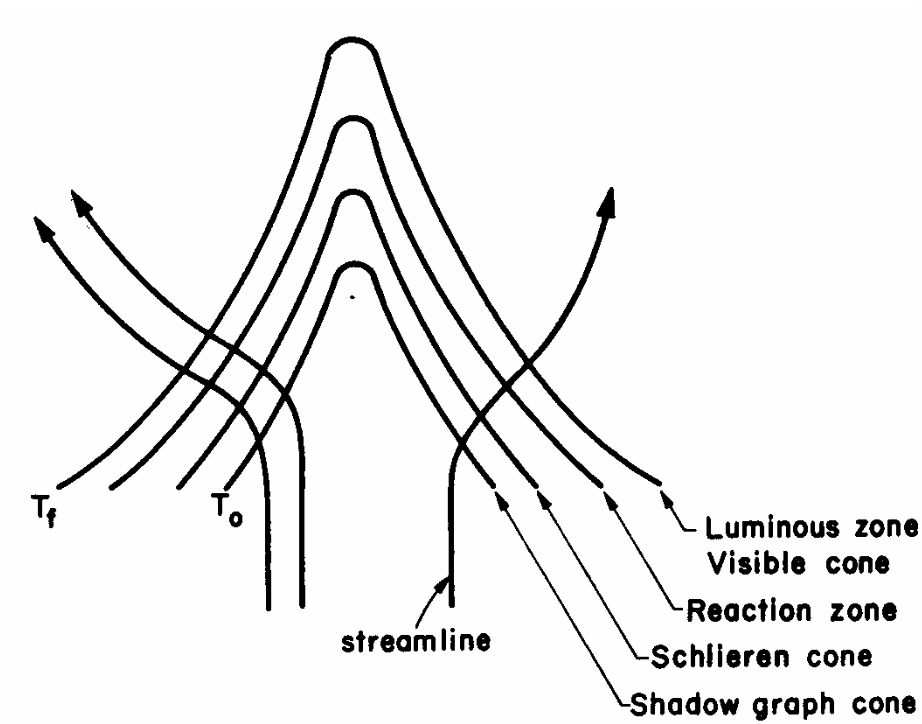


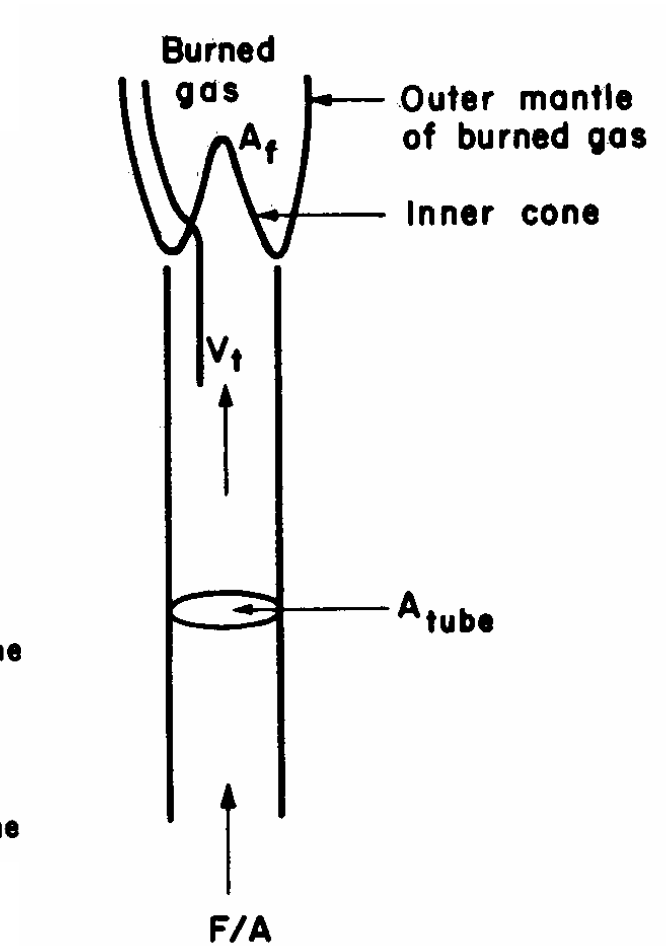

## • $\alpha$ and $C_p$



- ***$S_L$ - Experimental Methods***

  - ***Bunsen-burner***

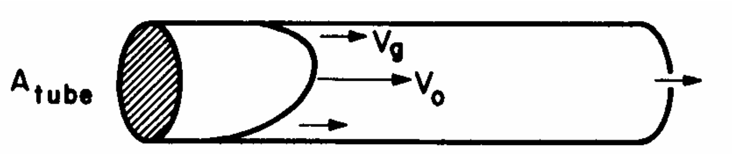



$$S_L = V_t \cdot A_t / A_f$$

# • $S_L$ - Experimental Methods (cont.)

## • Transparent Tube




$$S_L = (V_o - V_g)A_{tube} / A_f$$

## • Constant-Volume Bomb

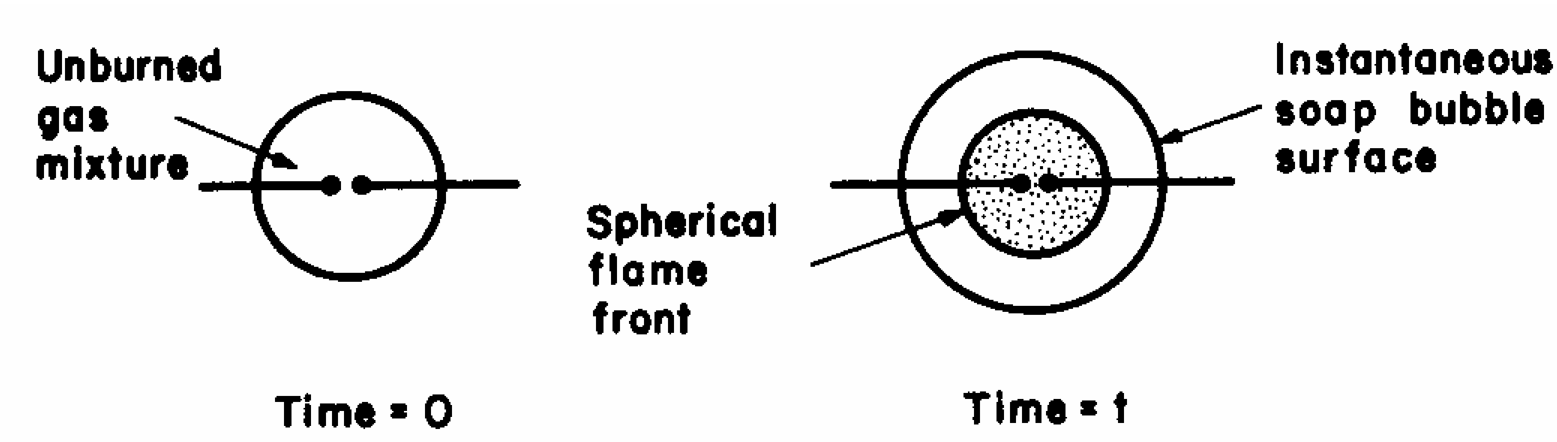
- **Constant-Volume**
- **Temperature and pressure rise**
- **Flame accelerates**

$$S_L = \frac{dr}{dt} - \frac{R^3 - r^3}{3p\gamma_u r^2}$$

## • *$S_L$ - Experimental Methods (cont.)*

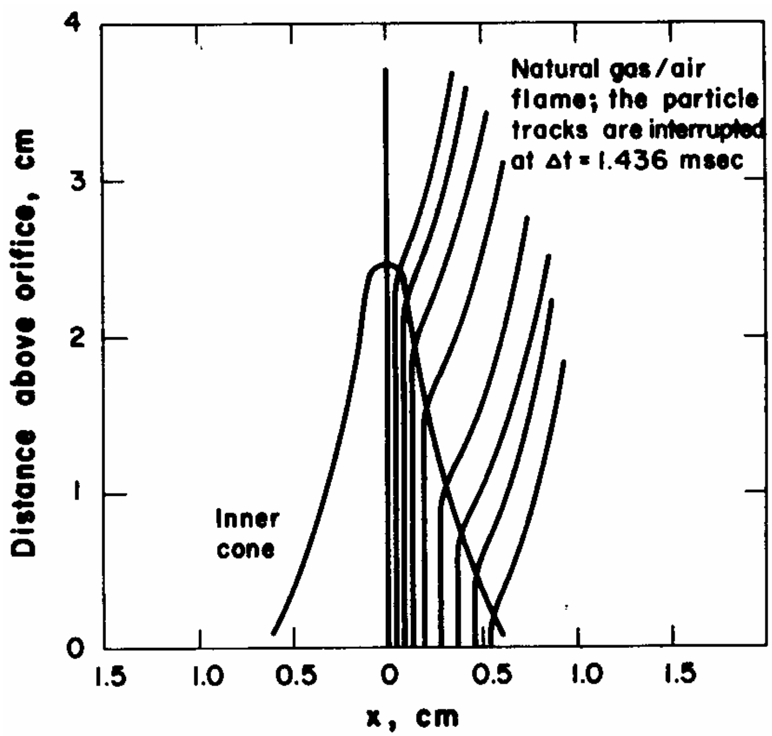
### • *Soap-Bubble*

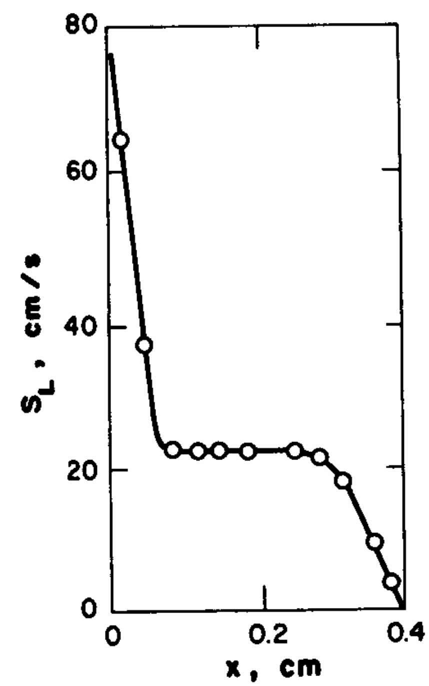
- **Constant Pressure**




$$S_L = u_r \frac{\rho_b}{\rho_u} = u_r \frac{T_b}{T_u}$$

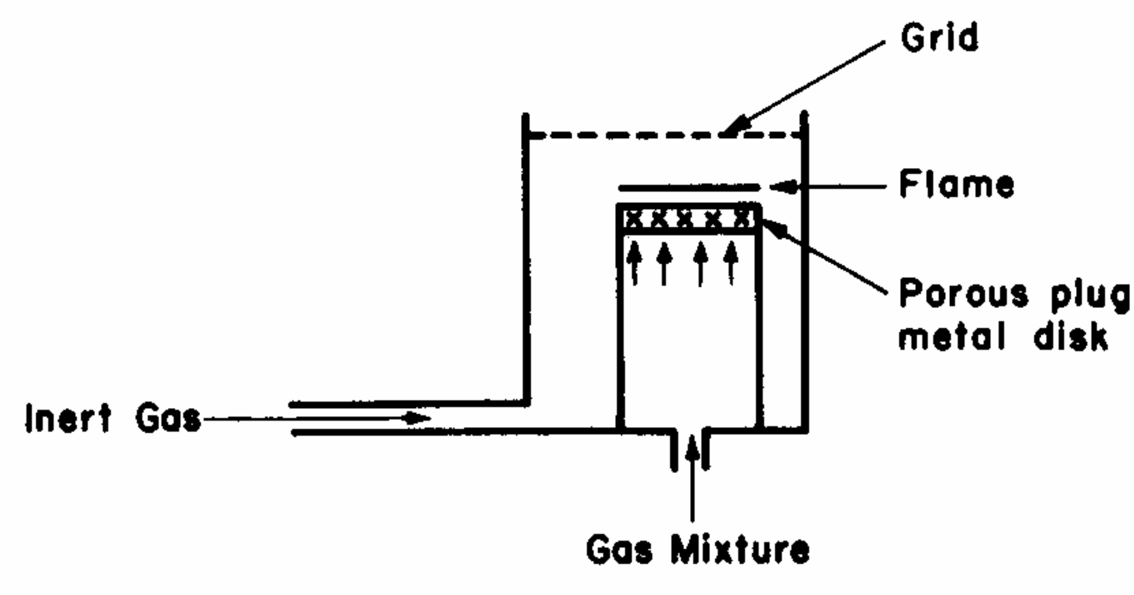
- ***$S_L$ - Experimental Methods (cont.)***
  - ***Particle Tracking***

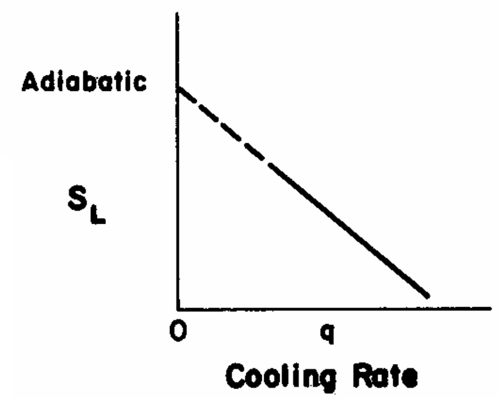


- ***$S_L$ - Experimental Methods (cont.)***

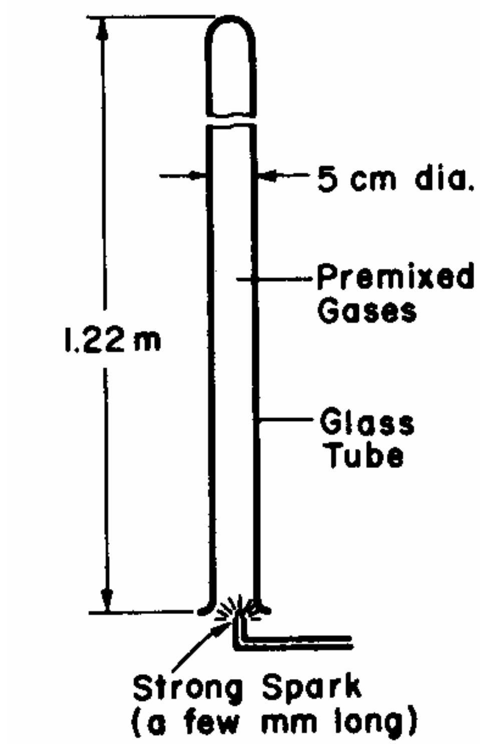
  - ***Flat Burner***

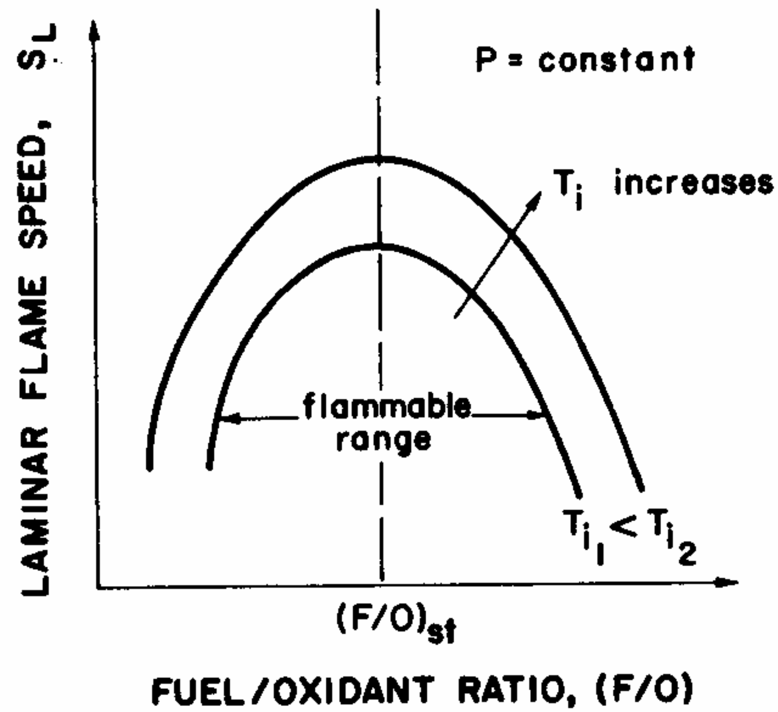


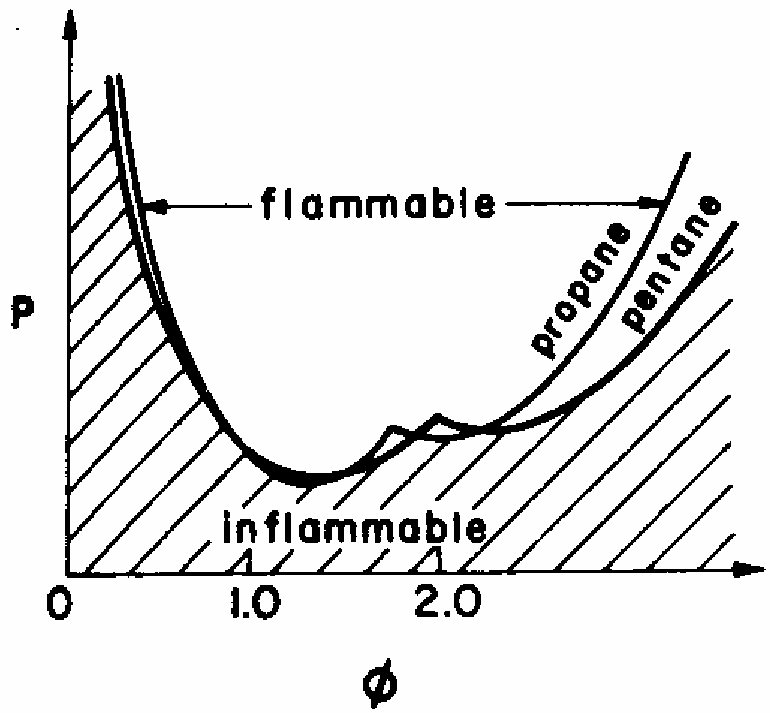
- ***Flame Quenching***

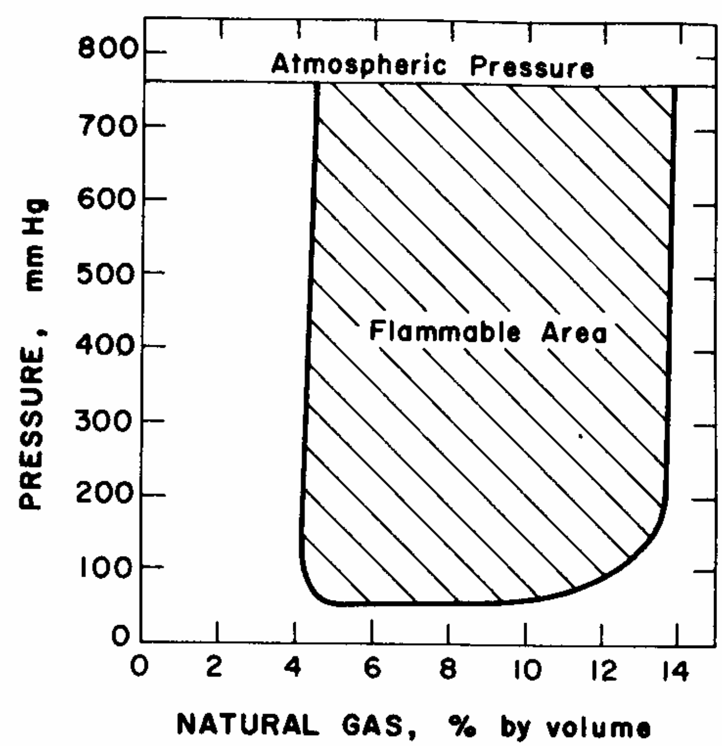
- ***Flammability Limits***



## • *Flammability Limits*



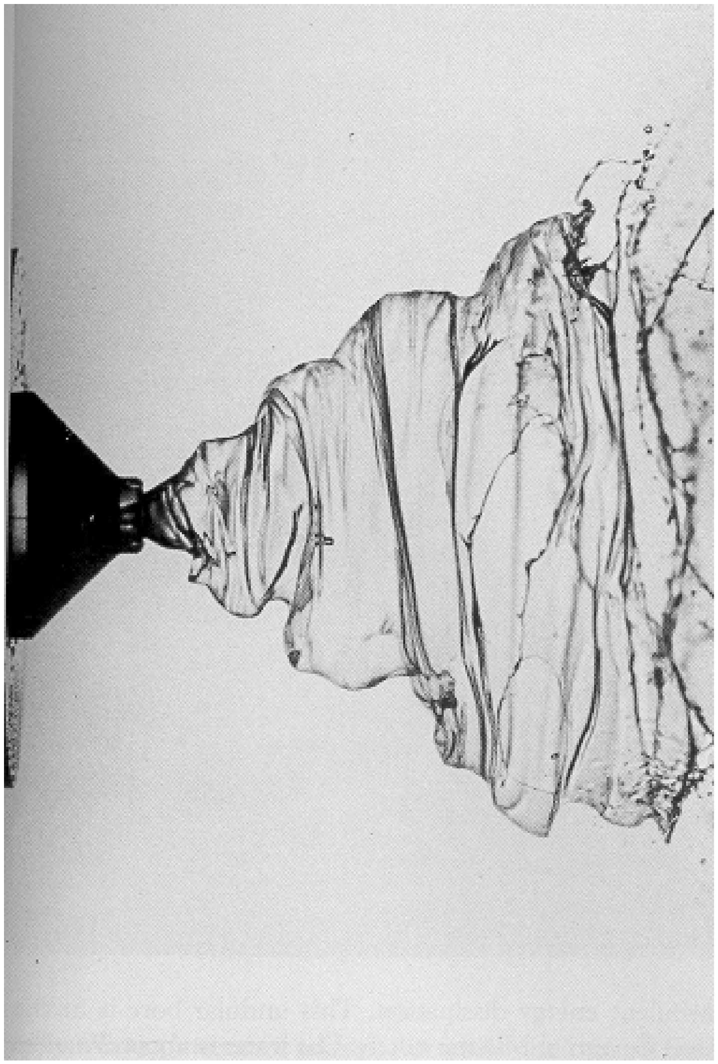



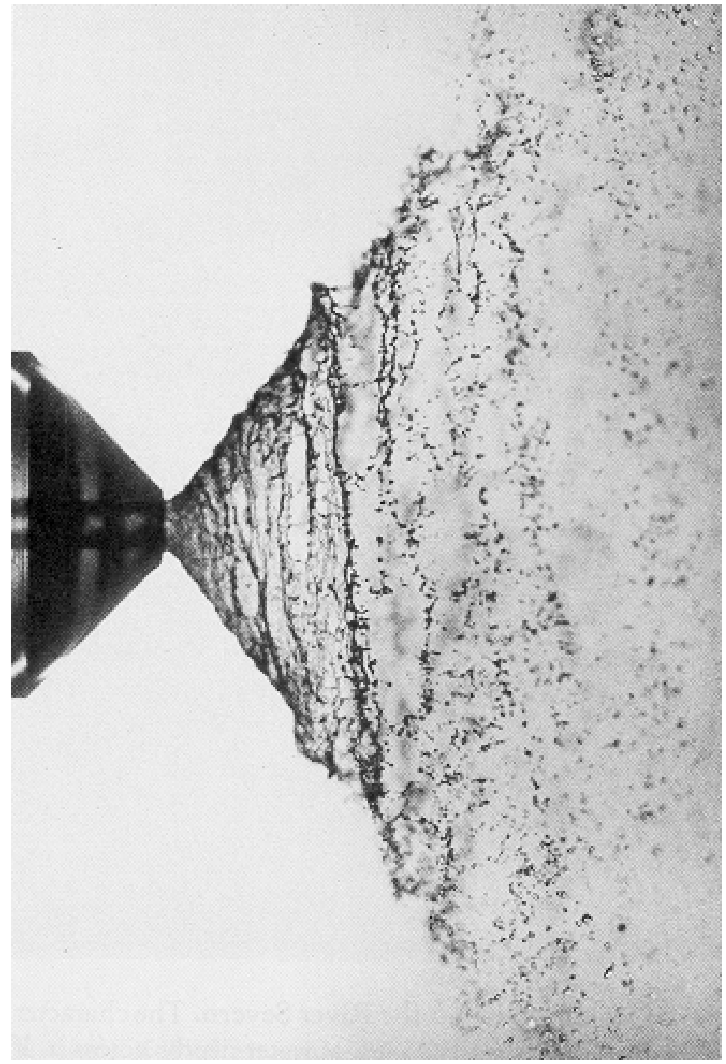
- ***Flammability Limits***

**TABLE 5.5 Flammability Limits of Certain Fuel–Air Mixtures**

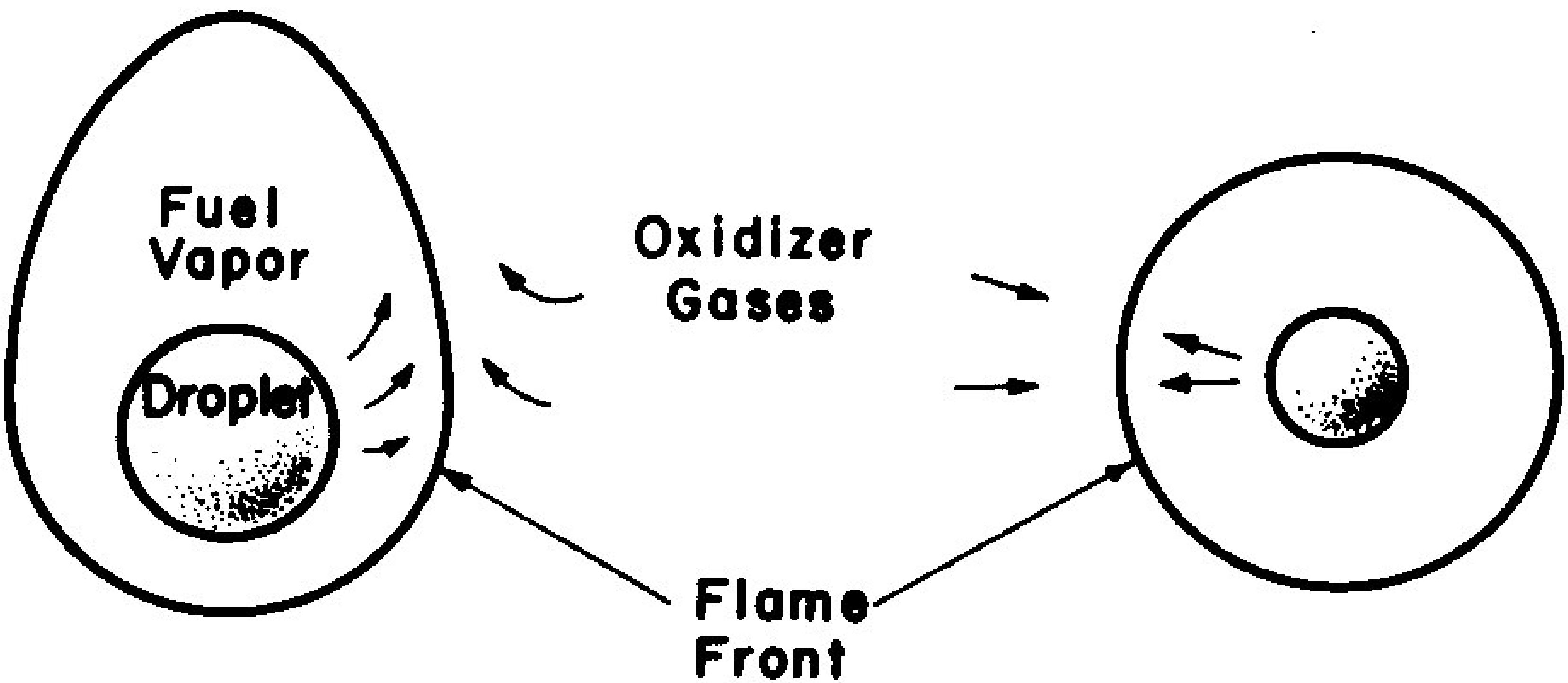
| | Lower (Lean) Limits, Vol. (%) in Air | Upper (Rich) Limits, Vol. (%) in Air | Stoichiometric, Vol. (%) in Air |
|---|---|---|---|
| Methane, $CH_4$ | 5 | 15 | 9.47 |
| Heptane, $C_7H_{16}$ | 1 | 6 | 1.87 |
| Hydrogen, $H_2$ | 4 | 74.2 | 29.2 |
| Carbon monoxide, CO | 12.5 | 74.2 | 29.5 |
| Acetylene | 2.5 | 80 | 7.7 |
| Ethane, $C_2H_4$ | 2.82 | 15.34 | 5.64 |
| Ethylene oxide, $C_2H_4O$ | 3.0 | 100 | 7.72 |
| Propane, $C_3H_8$ | 2.05 | 11.38 | 4.02 |
| Methanol ($g$), $CH_3OH$ | 5.88 | 49.94 | 12.24 |

# *Liquid Sprays*

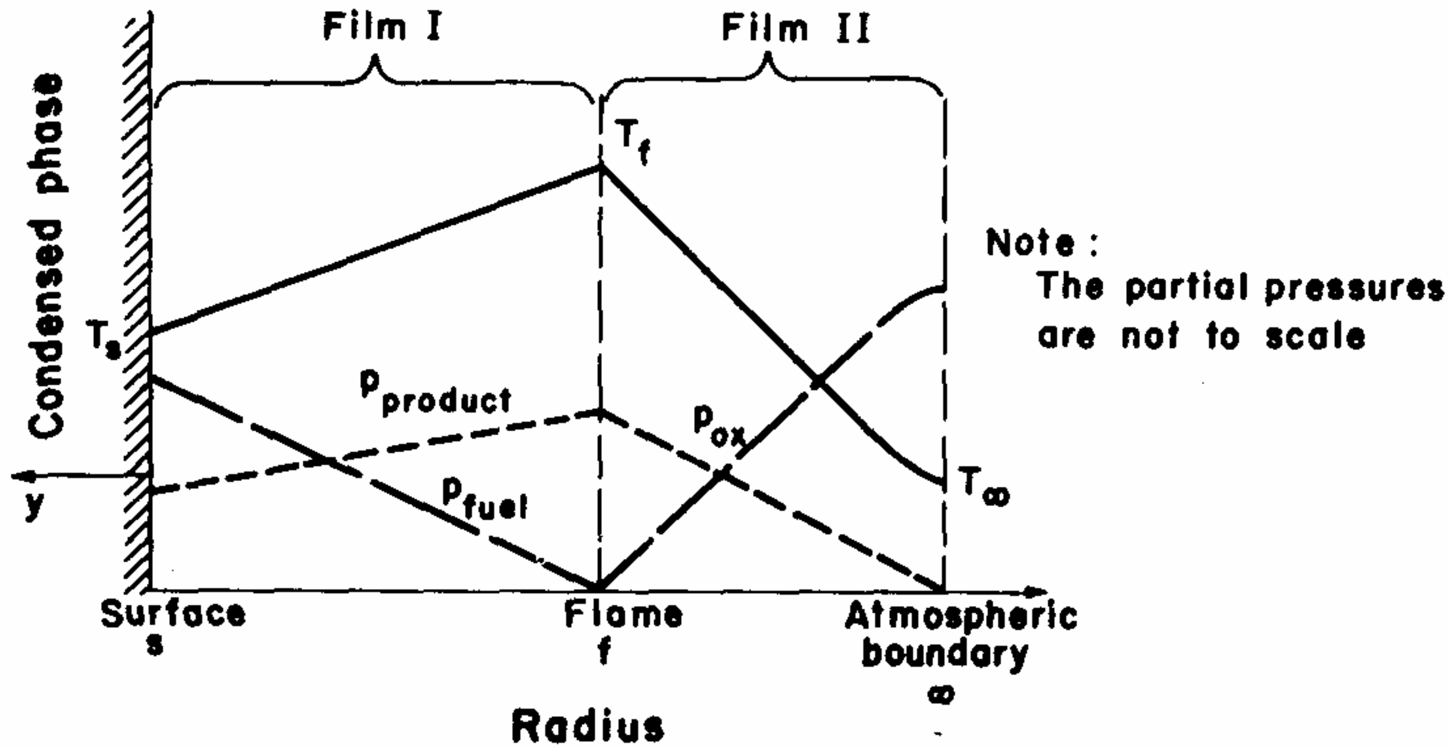




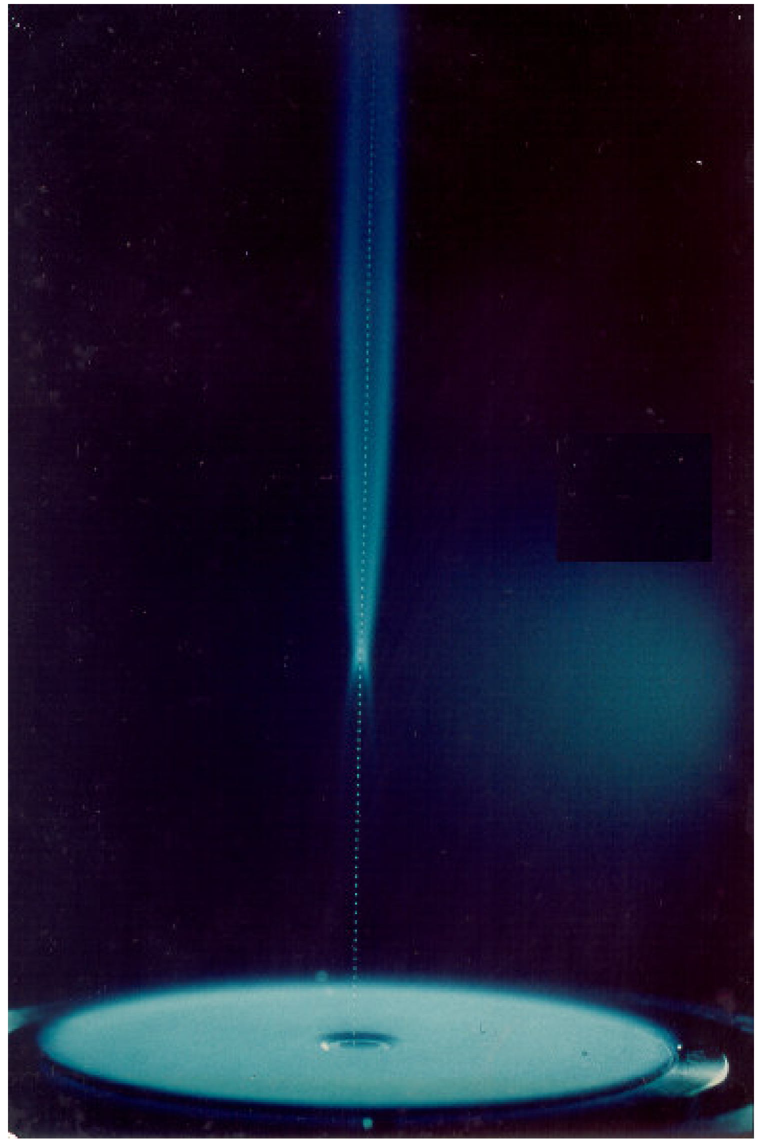
# *Isolated Droplets*



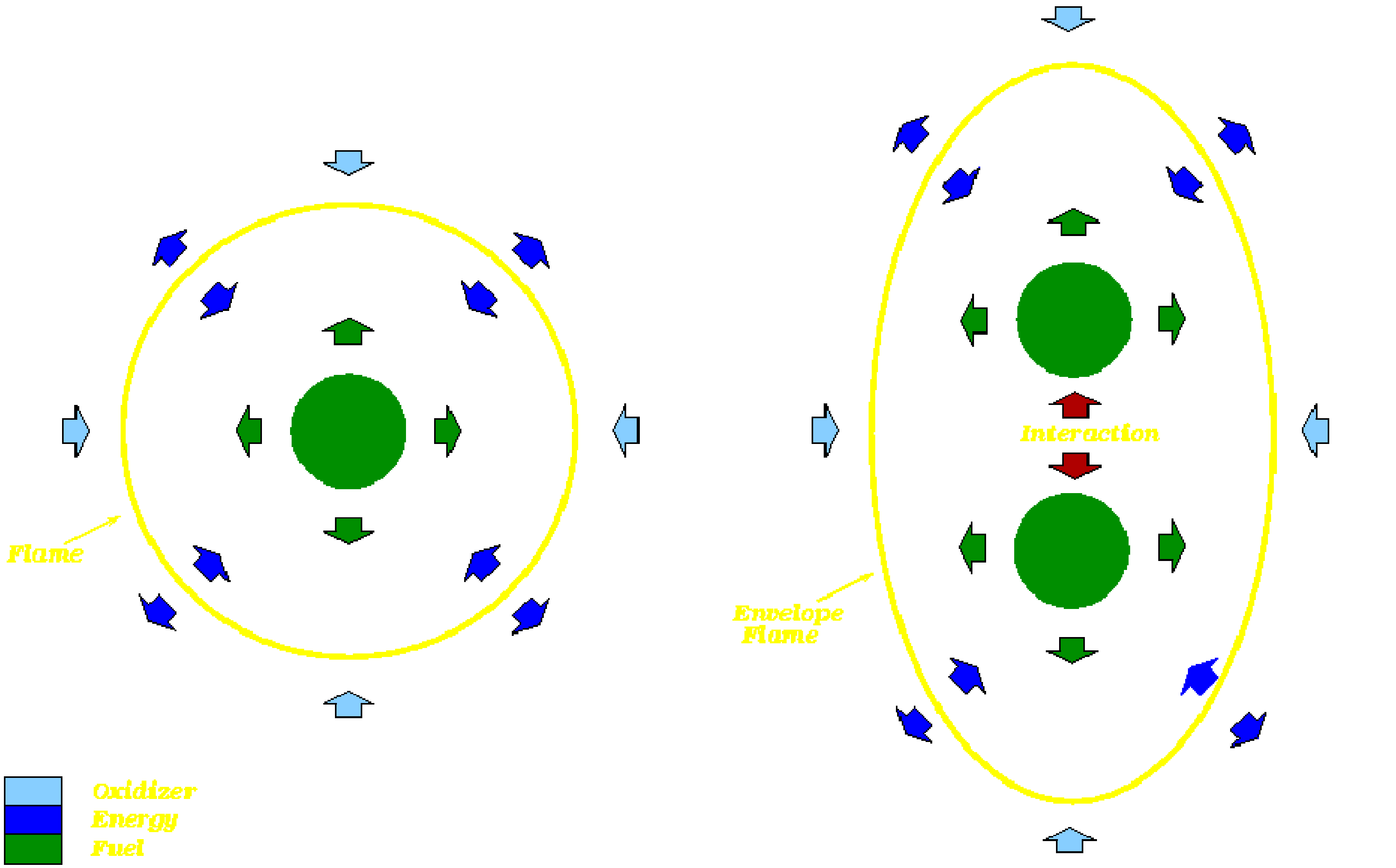
# *Isolated Droplets*



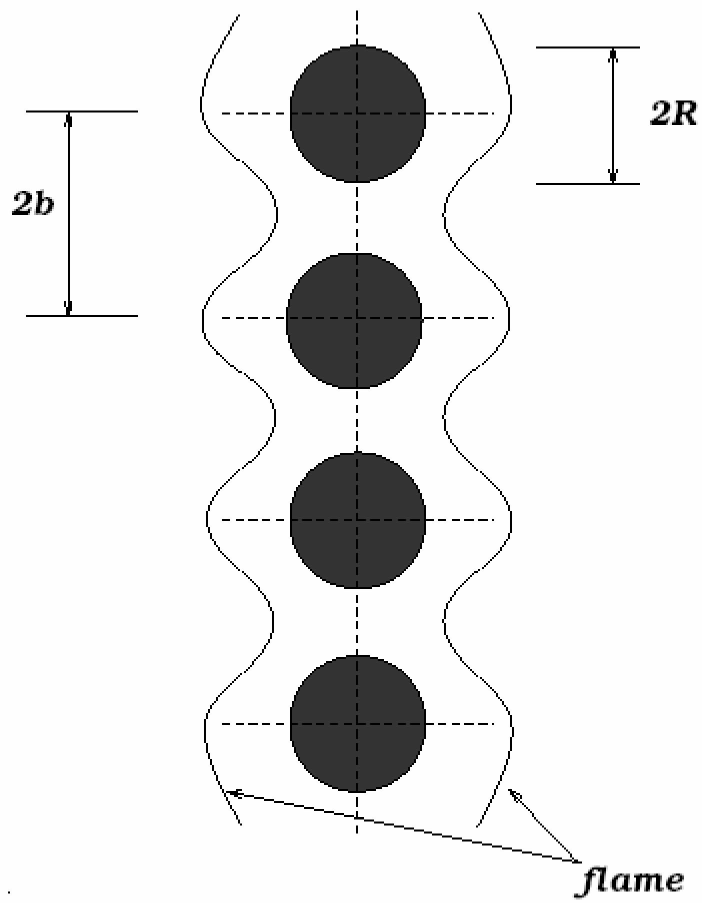
# *Droplet Stream*



# Interaction Phenomenon

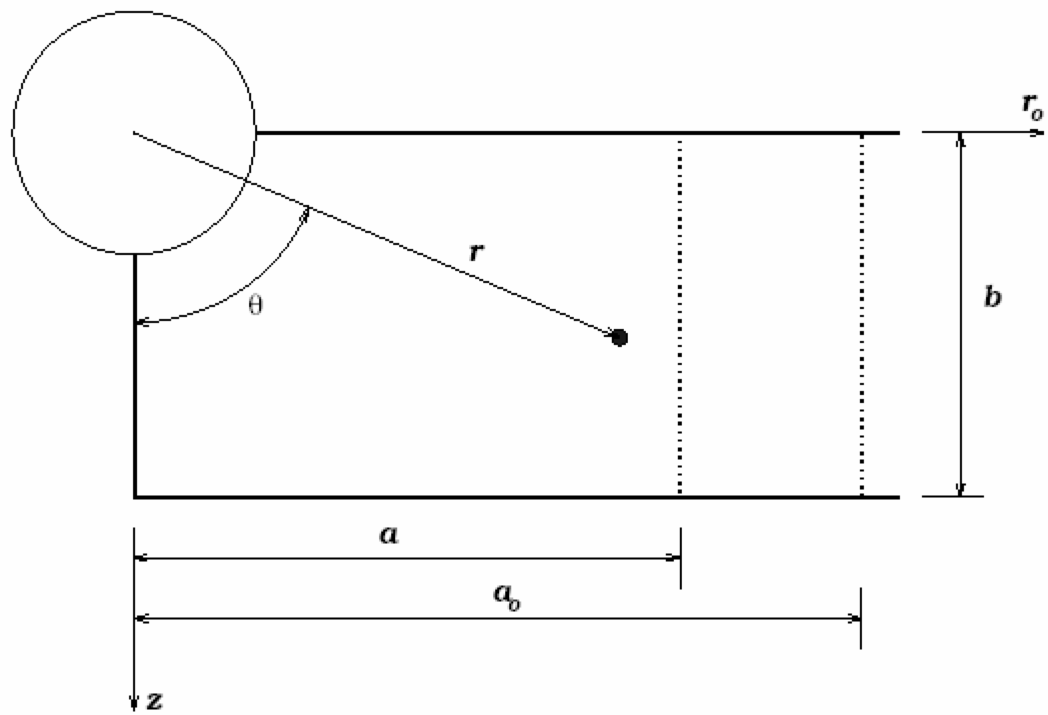


# Stream of Droplets




Center portion of a stream of droplets and principal dimensions.

# Physical Domain




Physical domain and principal dimensions.

## Governing Equations

*Droplet-Stream Combustion*

$$\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial \Phi^*}{\partial r^*}\right)+\frac{1}{r^{*2}sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial \Phi^*}{\partial \theta}\right)=0$$

$$u_r^*\frac{\partial T^*}{\partial r^*}+\frac{u_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta}=\frac{1}{Ja}\left(\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial T^*}{\partial r^*}\right)+\frac{1}{r^{*2}sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial T^*}{\partial \theta}\right)\right)+\dot{q}^*$$

$$u_r^*\frac{\partial Y_f^*}{\partial r^*}+\frac{u_\theta^*}{r^*}\frac{\partial Y_f^*}{\partial \theta}=\frac{1}{JaLe}\left(\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial Y_f^*}{\partial r^*}\right)+\frac{1}{r^{*2}sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial Y_f^*}{\partial \theta}\right)\right)+\dot{w}_f^*$$

$$u_r^*\frac{\partial Y_o^*}{\partial r^*}+\frac{u_\theta^*}{r^*}\frac{\partial Y_o^*}{\partial \theta}=\frac{1}{JaLe}\left(\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial Y_o^*}{\partial r^*}\right)+\frac{1}{r^{*2}sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial Y_o^*}{\partial \theta}\right)\right)+\dot{w}_o^*$$

# Boundary Conditions



$$\frac{\partial \Phi^*}{\partial r^*} = V_s^*(\theta) \quad T^* = T_s^* \quad Y_f^* = Y_{f,s}^* \quad Y_o^* = Y_{o,s}^* \quad ; r^* = 1 \quad 0 < \theta < \frac{\pi}{2}$$

$$\Phi^* \to 0 \quad T^* \to 1 \quad Y_f^* \to Y_{f,\infty}^* \quad Y_o^* \to Y_{o,\infty}^* \quad ; r^* \to \infty$$

$$\frac{\partial \Phi^*}{\partial \theta} = 0 \quad \frac{\partial T^*}{\partial \theta} = 0 \quad \frac{\partial Y_f^*}{\partial \theta} = 0 \quad \frac{\partial Y_o^*}{\partial \theta} = 0 \quad ; \begin{cases} \theta = 0 & 1 < r^* < b^* \\ \theta = \frac{\pi}{2} & 1 < r^* < \infty \end{cases}$$

$$-\frac{\partial \Phi^*}{\partial \theta} sin\theta + \frac{\partial \Phi^*}{\partial r^*} b^* = 0 \qquad -\frac{\partial T^*}{\partial \theta} sin\theta + \frac{\partial T^*}{\partial r^*} b^* = 0$$

$$-\frac{\partial Y_f^*}{\partial \theta} sin\theta + \frac{\partial Y_f^*}{\partial r^*} b^* = 0 \qquad -\frac{\partial Y_o^*}{\partial \theta} sin\theta + \frac{\partial Y_o^*}{\partial r^*} b^* = 0 \quad ; \begin{cases} r^* cos\theta = b^* \\ 0 < \theta < \theta_c \end{cases}$$

# Boundary Conditions

*Droplet-Stream Combustion*

Coupling condition

$$\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1} = V_s^* = \left.\frac{\partial \Phi^*}{\partial r^*}\right|_{r^*=1}$$

$$Y_{f,s}^* V_s^* = \frac{1}{Ja\ Le}\left.\frac{\partial Y_f^*}{\partial r^*}\right|_{r^*=1}$$

$$Y_{o,s}^* V_s^* = \frac{1}{Ja\ Le}\left.\frac{\partial Y_o^*}{\partial r^*}\right|_{r^*=1}$$

nondimentional variables

$$r^* = \frac{r}{R} \qquad \Phi^* = \frac{\Phi}{V_{ref} R} \qquad T^* = \frac{T - T_{sat}}{T_\infty - T_{sat}}$$

$$Y_f^* = 1 - Y_f \qquad Y_o^* = Y_o$$

# Coupling Functions

*Droplet-Stream Combustion*

$$\beta_{f,o} = \frac{Y_f^* + Y_o^* \nu}{Y_{f,s}^* + Y_{o,s}^* \nu}$$

$$\beta_{f,t} = \frac{Y_f^* - JaIT^*}{Y_{f,s}^* - I}$$

$$\beta_{o,t} = \frac{Y_o^* \nu + JaIT^*}{Y_{o,s}^* \nu + I}$$

Nondimensional Parameters

$$Ja = \frac{c_p(T_\infty - T_{sat})}{h_{fg}} = \frac{V_{ref} R}{\alpha} \qquad Le = \frac{D}{\alpha}$$

$$I = \frac{h_{fg}}{h_r}$$

# Coupling Function Equation



$$u_r^* \frac{\partial \beta_i}{\partial r^*} + \frac{u_\theta^*}{r^*} \frac{\partial \beta_i}{\partial \theta} = \frac{1}{Ja} \left( \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} \left( r^{*2} \frac{\partial \beta_i}{\partial r^*} \right) + \frac{1}{r^{*2} sin\theta} \frac{\partial}{\partial \theta} \left( sin\theta \frac{\partial \beta_i}{\partial \theta} \right) \right)$$

Boundary conditions

$$\beta_i = \beta_{i;s} \quad ; r^* = 1 \quad 0 < \theta < \frac{\pi}{2}$$

$$\beta_i \rightarrow \beta_{i;\infty} \quad ; r^* \rightarrow \infty$$

$$\frac{\partial \beta_i}{\partial \theta} = 0 \qquad \theta = 0 \quad 1 < r^* < b^*$$

$$\frac{\partial \beta_i}{\partial \theta} = 0 \quad ; \theta = \frac{\pi}{2} \quad 1 < r^* < \infty$$

$$-\frac{\partial \beta_i}{\partial \theta} sin\theta + \frac{\partial \beta_i}{\partial r} b = 0 \quad ; r^* cos\theta = b \quad 0 < \theta < \theta_c$$

$$i \equiv (f,o),\ (f,t)\ or\ (o,t)$$

# Coupling Function Equation

***Droplet-Stream Combustion***

Coupling Condition

$$\frac{1}{Ja}\frac{\partial \beta_i}{\partial r^*}\bigg|_{r^*=1} = V_s^*$$

Linear Relation

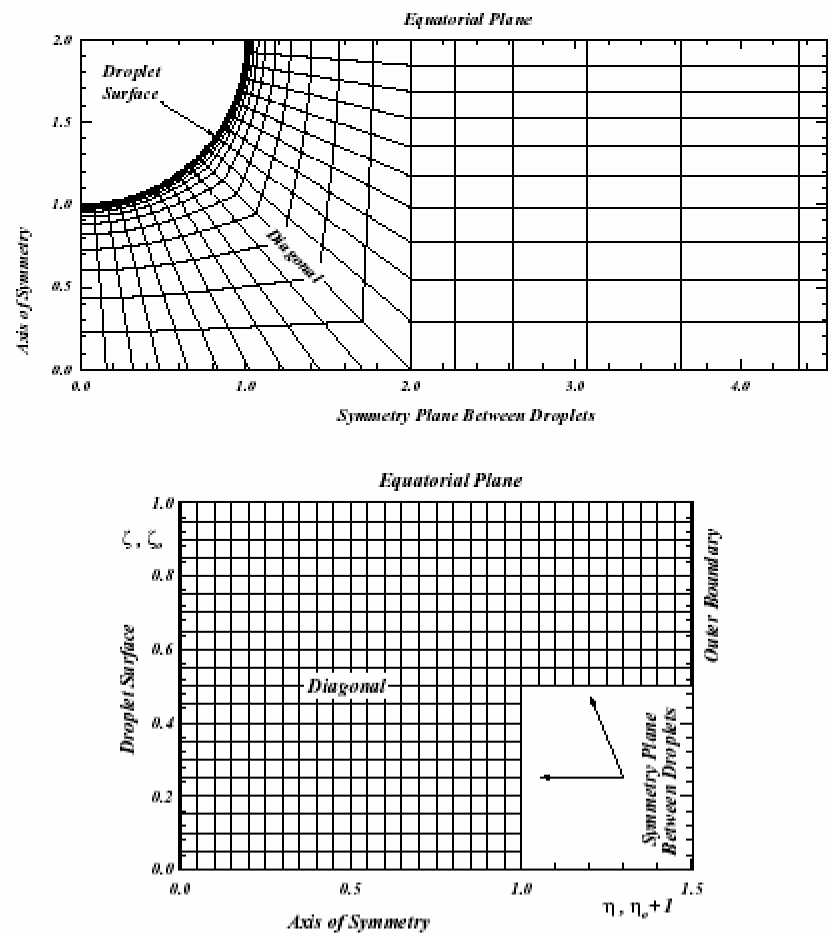
$$\beta_j = \beta_{f,o} + (\beta_{j;\infty} - \beta_{f,o;\infty})$$

$j \equiv (f,t)$ or $(o,t)$

Flame Position

$$\beta_{f,o} = \frac{1}{Y_{f,s}^*}$$

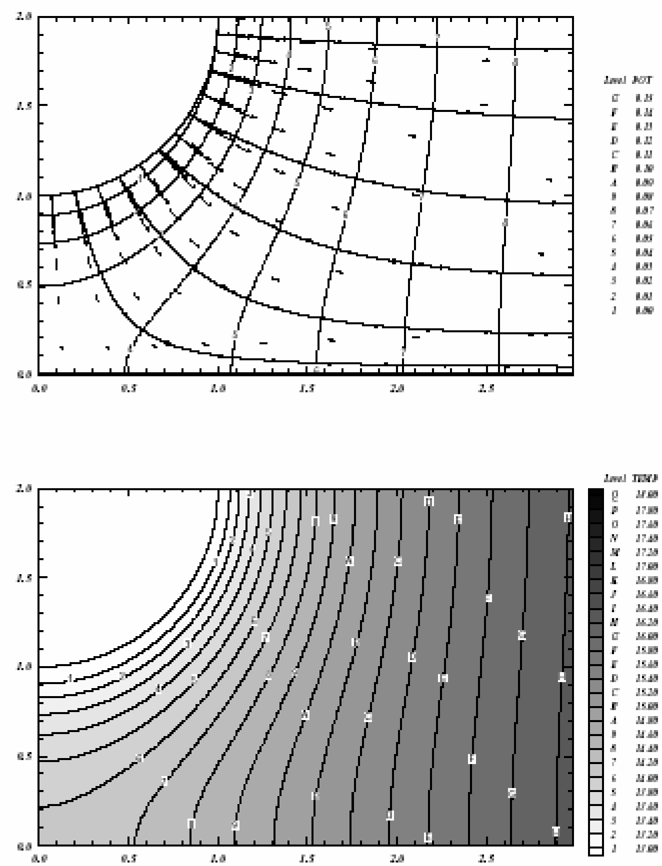
# Grid




Computational 21x21 grids in the physical plane (above) and in the transformed plane (below).

# Results - Stream of Porous-Spheres

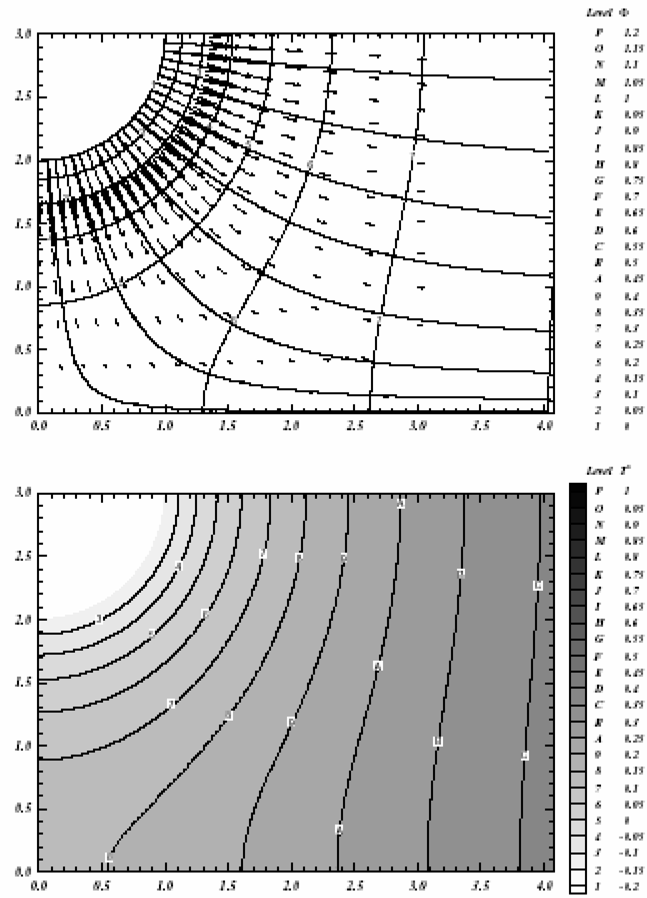
*Droplet-Stream Combustion*



Equip. lines, streamlines, velocity vectors (above) and temperature regions – $b^* = 2$ ( $Ja = 2$ - 161x161 points)

# Results - Stream of Droplets

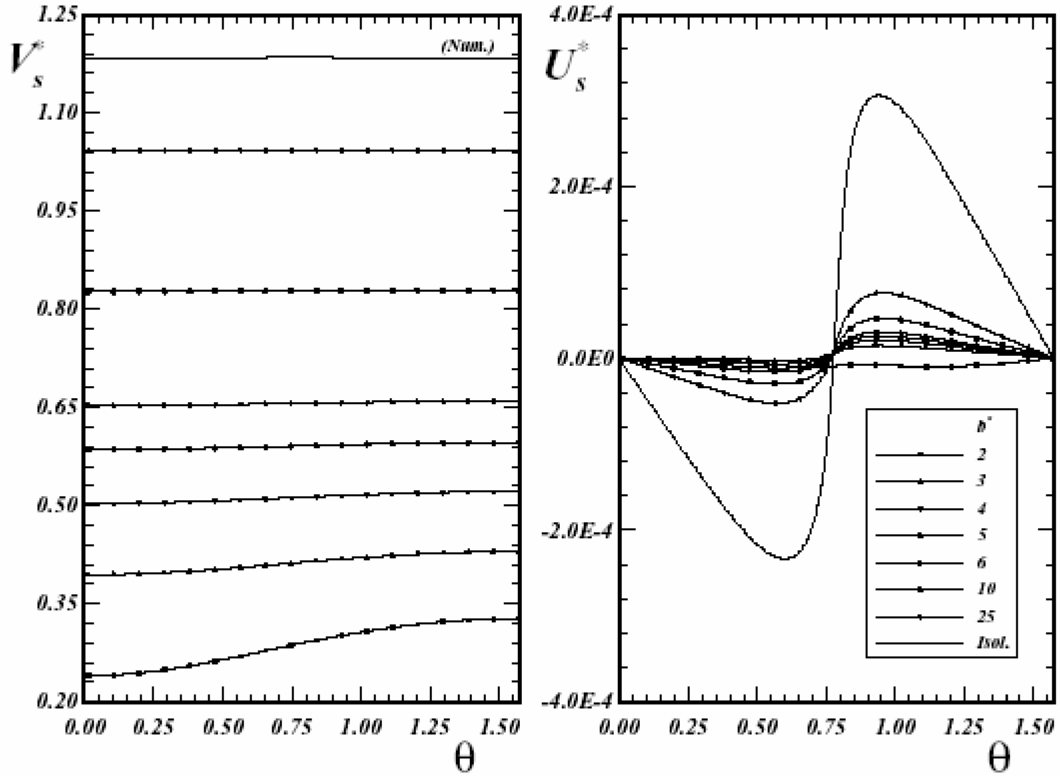
*Droplet-Stream Combustion*



Equip. lines, streamlines, velocity vectors (above) and temperature regions – $b^* = 3$)

# Results - Droplet Surface Velocity

*Droplet-Stream Combustion*



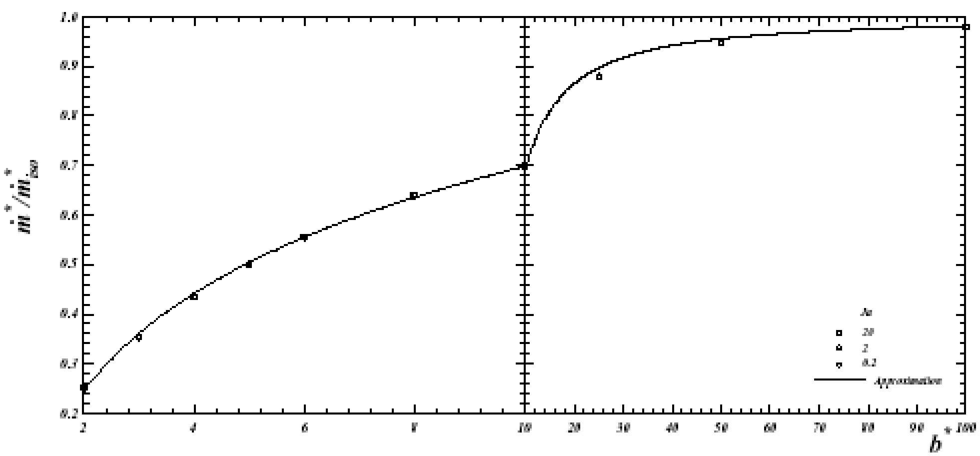

Droplet surface velocity: normal (right) and tangential (left). ($Ja = 2$)

# Mass Vaporization Rate



$$\dot{m}^* = \frac{\dot{m}}{4\pi\rho R^2 V_{ref}} = \int_0^{\pi/2} V_s^*(\theta) sin(\theta) d\theta$$

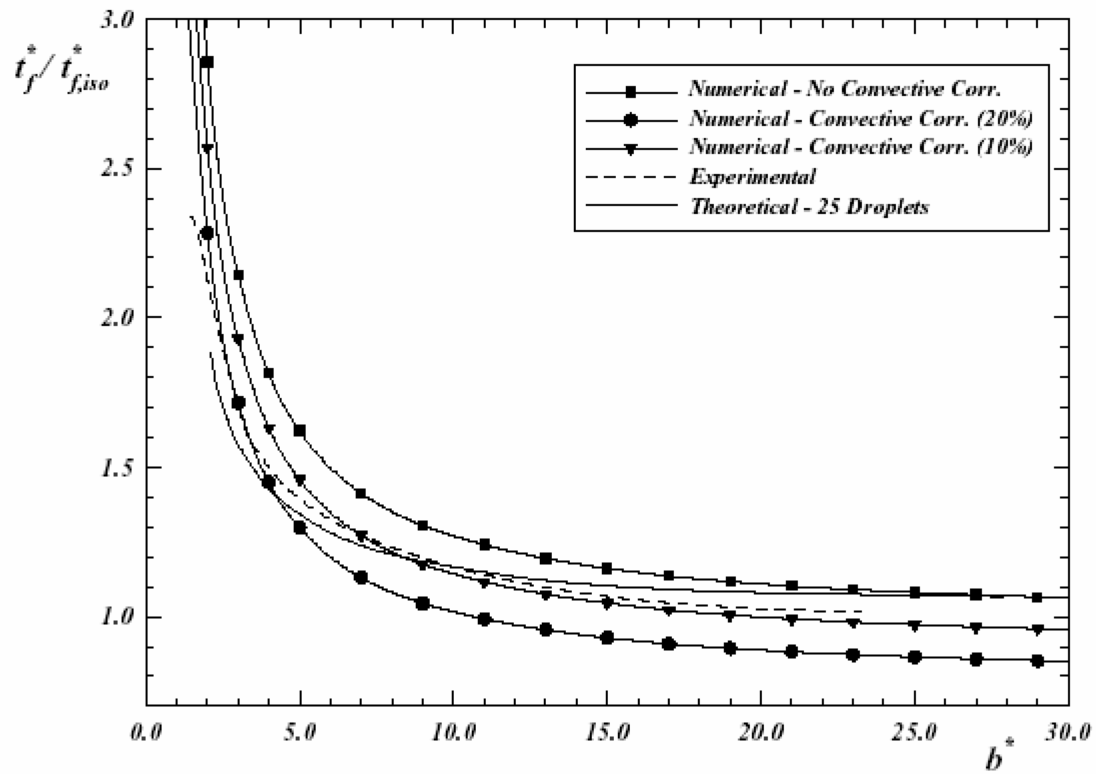
$$\frac{\dot{m}^*}{\dot{m}^*_{iso}} = 1 - g_0 \left(b^*\right)^{-g_1} \qquad g_0 = 0.9972 \quad g_1 = 1.1845$$




Ratio of the Nondimensional Droplet Vaporization Rate in a Stream to an Isolated Droplet.

# Droplet Lifetime

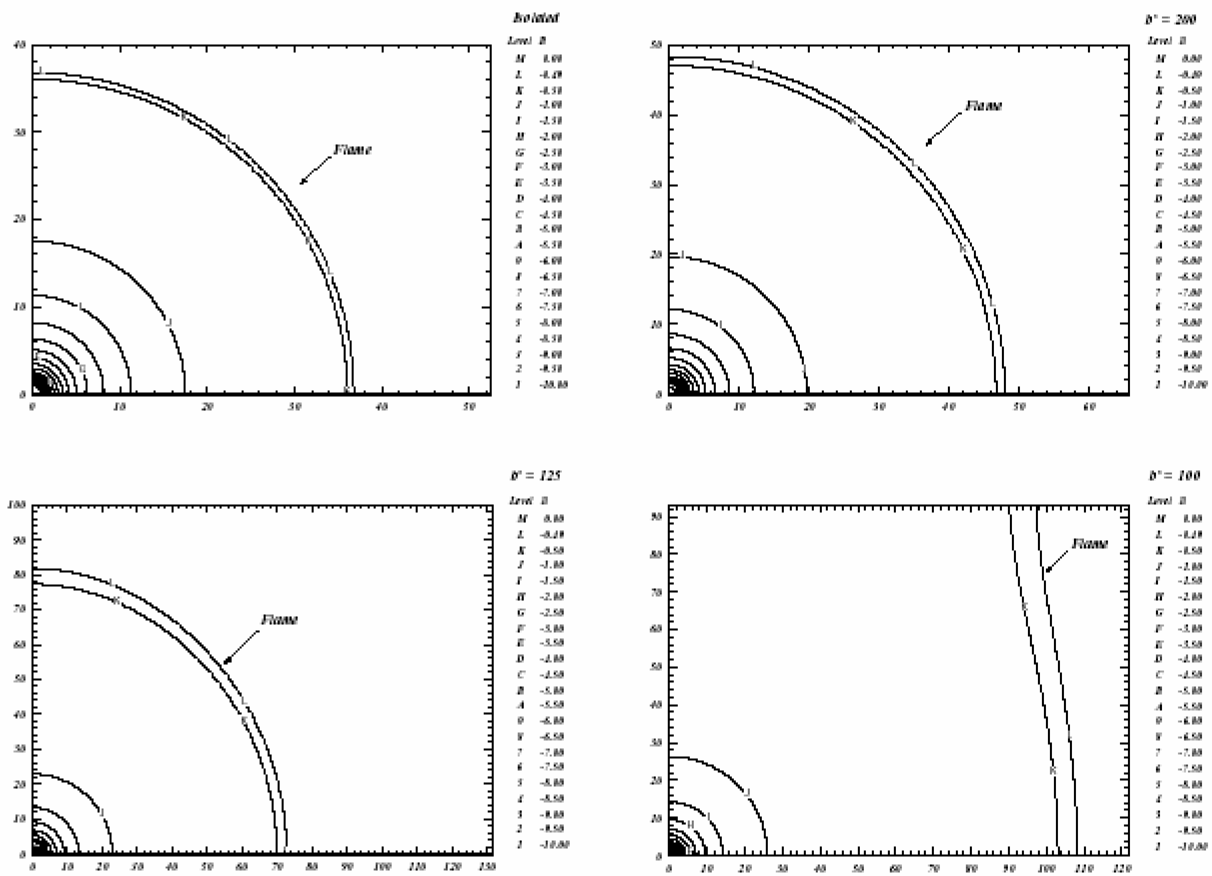
*Droplet Stream Combustion*




Ratio of the Nondimensional Droplet Vaporization Rate in a Stream to an Isolated Droplet.

# Flame Interference

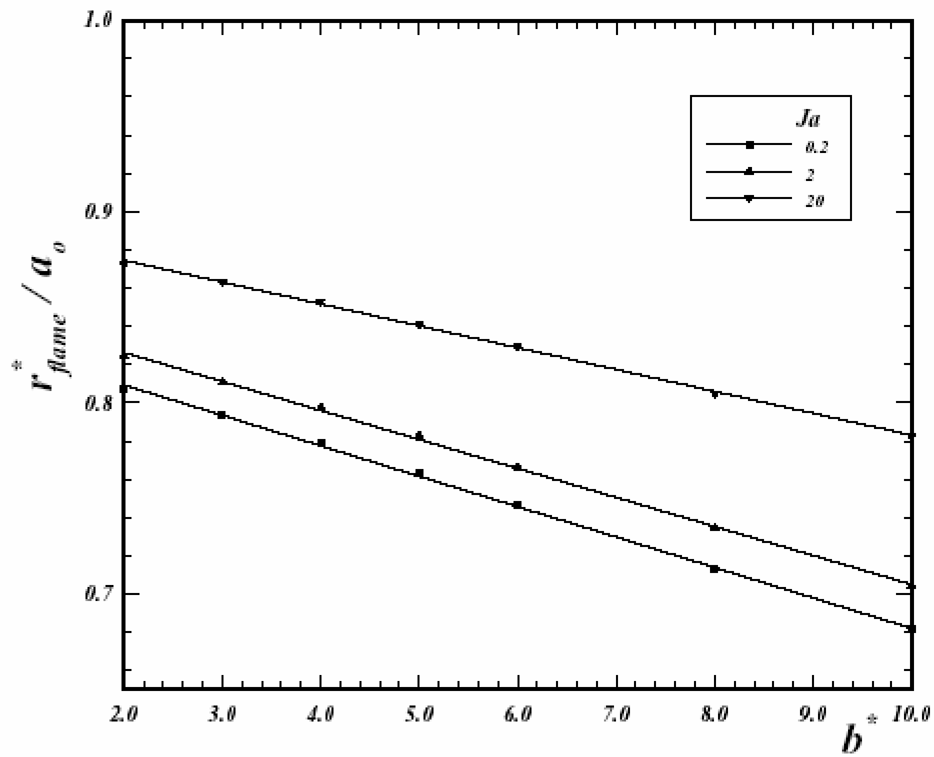
## Droplet Stream Combustion




Flame distortion and position

# Truncation Distance Influence






Ratio of the Cylindrical Flame-Stream distance to the truncating radius

# Truncation Distance Influence



| $b^*$ | 2 | | 10 | | 25 | |
|---|---|---|---|---|---|---|
| | $\dot{m}^*/\dot{m}^*_{iso}$ | $r^*_{flame}/a^*_o$ | $\dot{m}^*/\dot{m}^*_{iso}$ | $r^*_{flame}/a^*_o$ | $\dot{m}^*/\dot{m}^*_{iso}$ | $r^*_{flame}/a^*_o$ |
| $a^*_o$ | | | | | | |
| 800 | 0.2787 | 0.8375 | 0.7344 | 0.7150 | 0.9021 | 0.5075 |
| 1600 | 0.2543 | 0.8244 | 0.6988 | 0.7044 | 0.8798 | 0.2211 |
| 3200 | 0.2327 | 0.7192 | 0.6672 | 0.6144 | 0.8592 | 0.4897 |

Mass vaporization ratio and relative flame position for different physical domain truncating distances.

## Governing Equations

***Velocity Field***

$$\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial \Phi^*}{\partial r^*}\right)+\frac{1}{r^{*2}sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial \Phi^*}{\partial \theta}\right)=0$$

## Coupling Functions

***Droplet-Stream Combustion***

$$\beta_{f,o}=\frac{Y_f^*+Y_o^*\nu}{Y_{f,s}^*+Y_{o,s}^*\nu} \qquad \beta_{f,t}=\frac{Y_f^*-JaIT^*}{Y_{f,s}^*-I} \qquad \beta_{o,t}=\frac{Y_o^*\nu+JaIT^*}{Y_{o,s}^*\nu+I}$$

Nondimensional Parameters

$$t^*=\frac{V_{ref}\cdot t}{a+a_o}$$

# Governing Equations

*Coupling Function*

$$\frac{1}{a^* + a_0^*}\frac{\partial \beta_i}{\partial t^*} + u_r^*\frac{\partial \beta_i}{\partial r^*} + \frac{u_\theta^*}{r^*}\frac{\partial \beta_i}{\partial \theta} =$$

$$= \frac{1}{Ja}\left(\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial \beta_i}{\partial r^*}\right) + \frac{1}{r^{*2} sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial \beta_i}{\partial \theta}\right)\right)$$

Boundary conditions

$$\beta_i = \beta_{i;s} \quad ; \ r^* = 1 \quad 0 < \theta < \frac{\pi}{2}$$

$$\beta_i \rightarrow \beta_{i;\infty} \quad ; \ r^* \rightarrow \infty$$

$$\frac{\partial \beta_i}{\partial \theta} = 0 \qquad \theta = 0 \quad 1 < r^* < b^*$$

$$\frac{\partial \beta_i}{\partial \theta} = 0 \quad ; \ \theta = \frac{\pi}{2} \quad 1 < r^* < \infty$$

$$-\frac{\partial \beta_i}{\partial \theta} sin\theta + \frac{\partial \beta_i}{\partial r} b = 0 \quad ; \ r^* cos\theta = b \quad 0 < \theta < \theta_c$$

$$i \equiv (f,o),\ (f,t)\ or\ (o,t)$$

# Coupling Function Equation

*Droplet-Stream Combustion*

Coupling Condition

$$\frac{1}{Ja}\frac{\partial \beta_i}{\partial r^*}\bigg|_{r^*=1} = V_s^*$$

Linear Relation

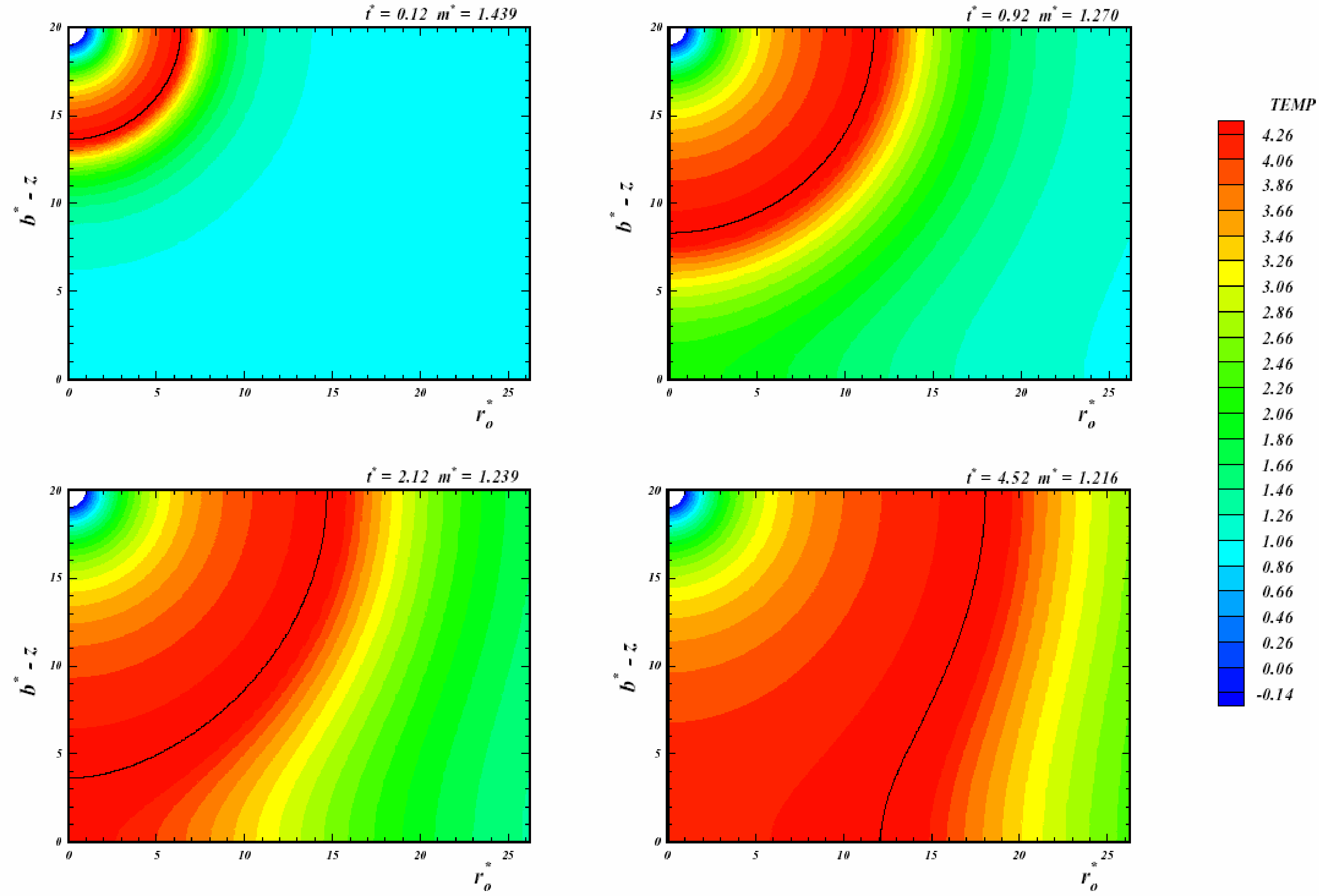
$$\beta_j = \beta_{f,o} + (\beta_{j;\infty} - \beta_{f,o;\infty})$$

$j \equiv (f, t) \text{ or } (o, t)$

Flame Position

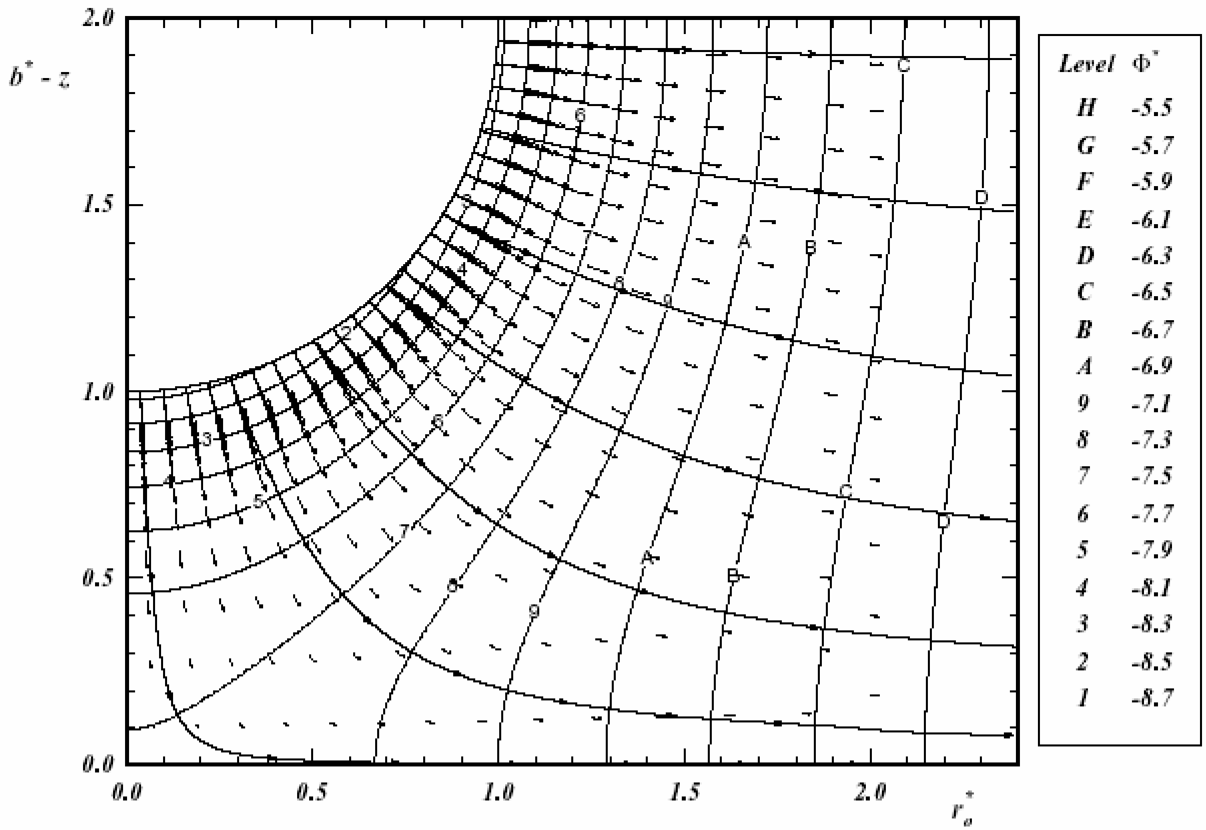
$$\beta_{f,o} = \frac{1}{Y_{f,s}^*}$$

# Droplet Mass Vaporization



# Velocity Field

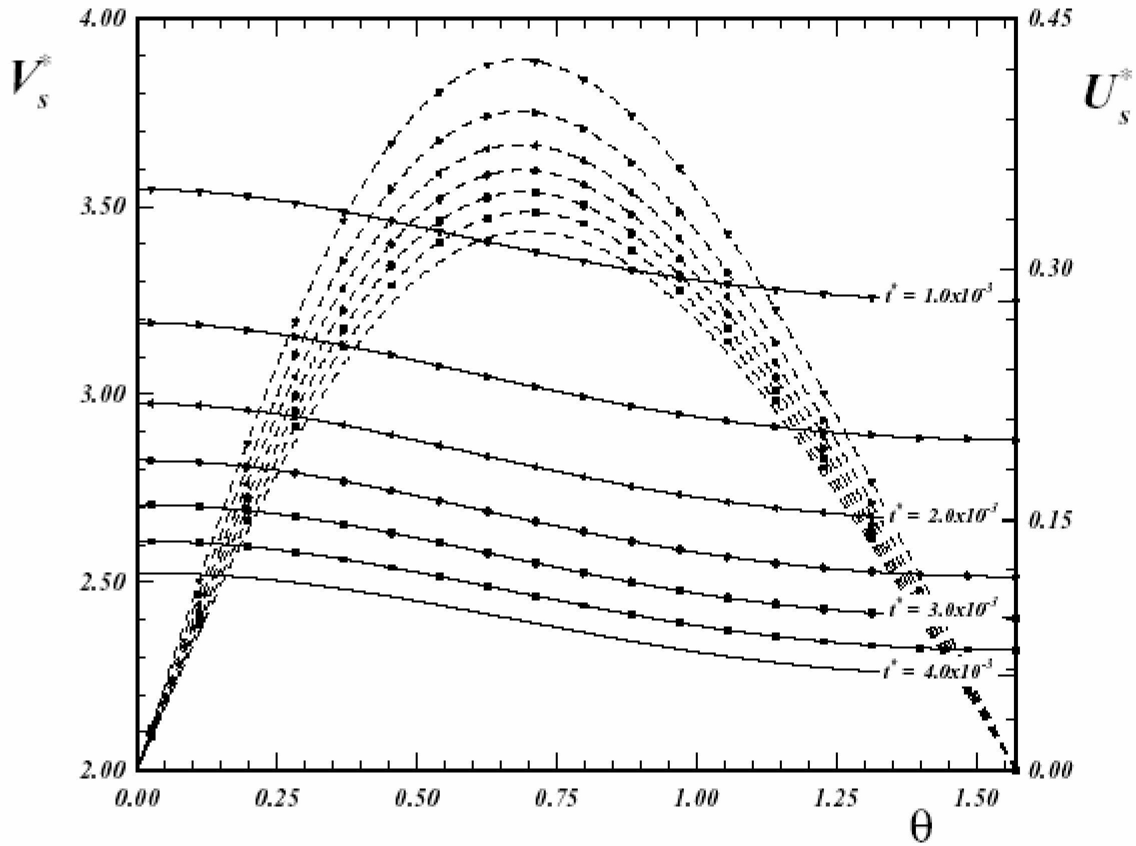
## *Droplet-Stream Combustion*




Equip. lines, streamlines, velocity vectors for a burning droplet – $b^* = 2$ ($Ja = 2$)

# Droplet Surface Velocity

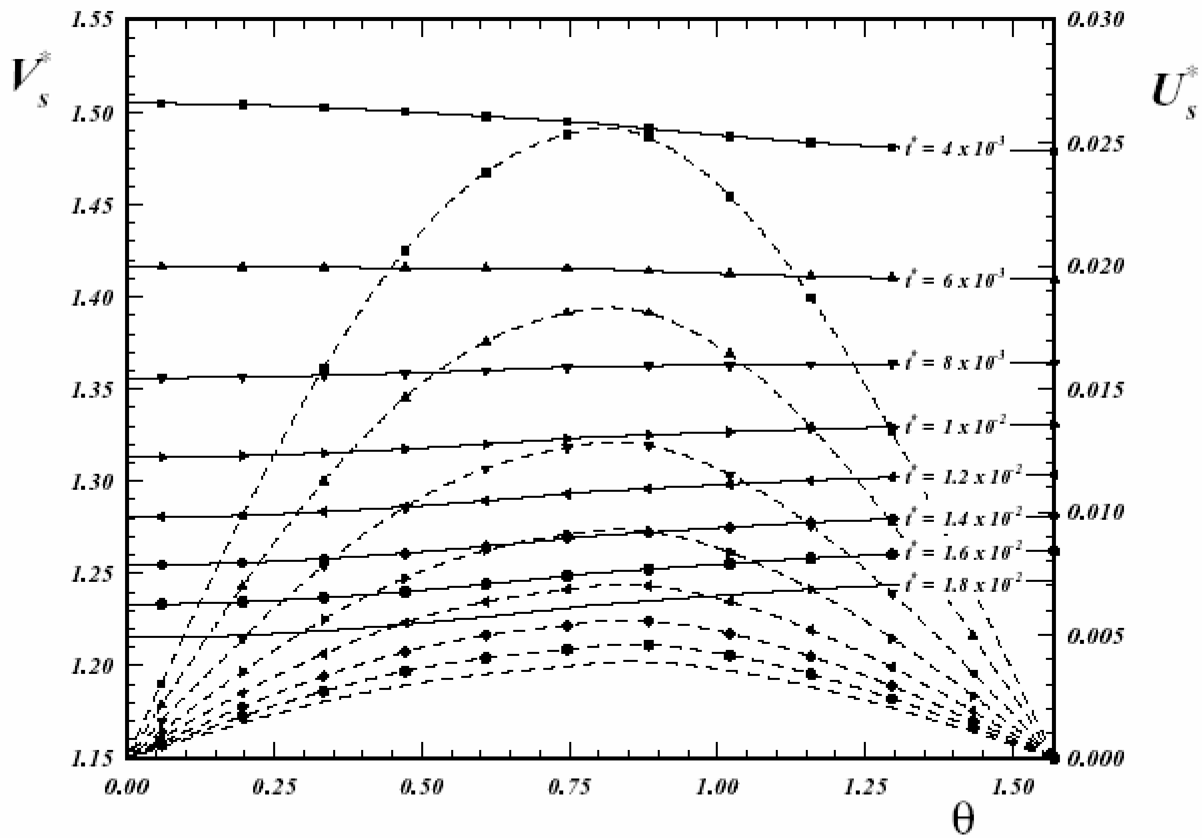
*Droplet-Stream Combustion*




Droplet surface velocity: normal (solid) and tangential (dashed) - $b^* = 2$.

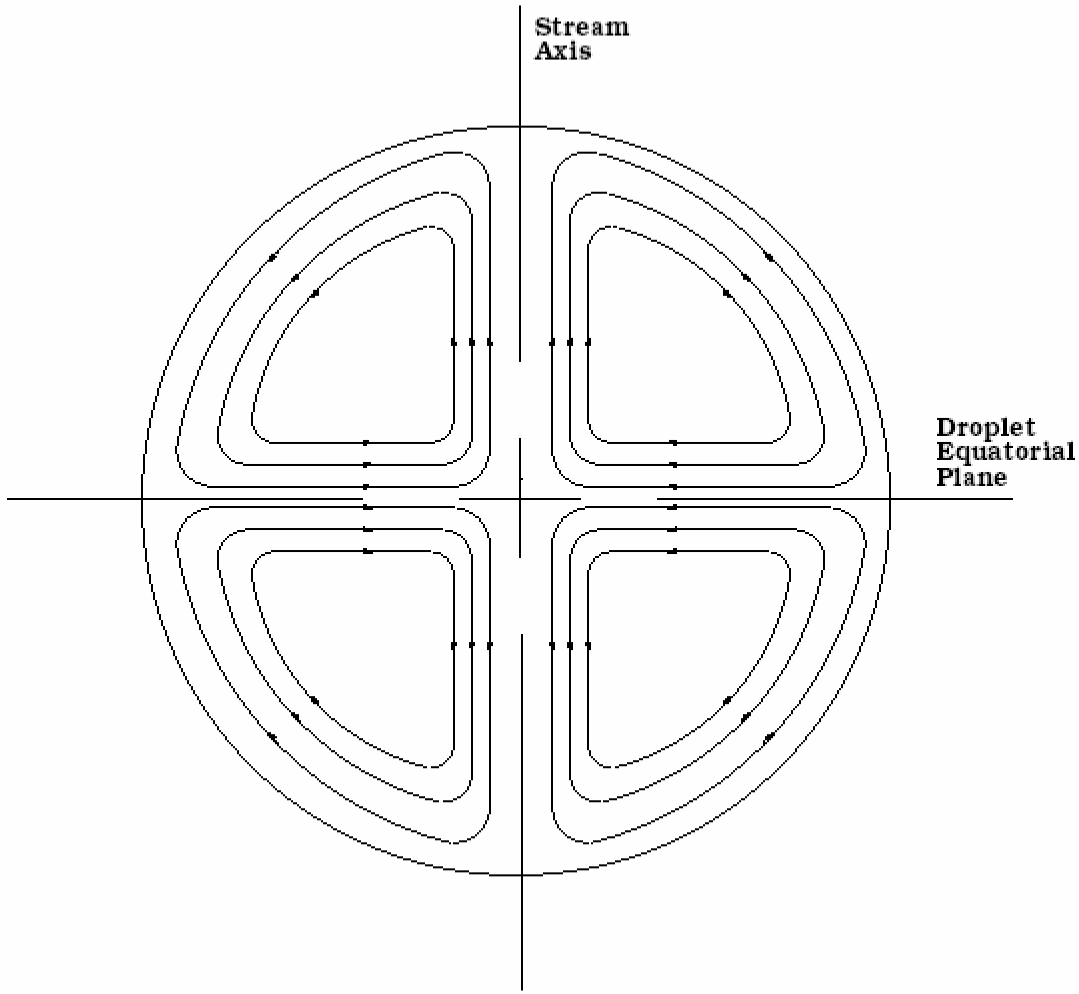
# Droplet Surface Velocity

*Droplet-Stream Combustion*

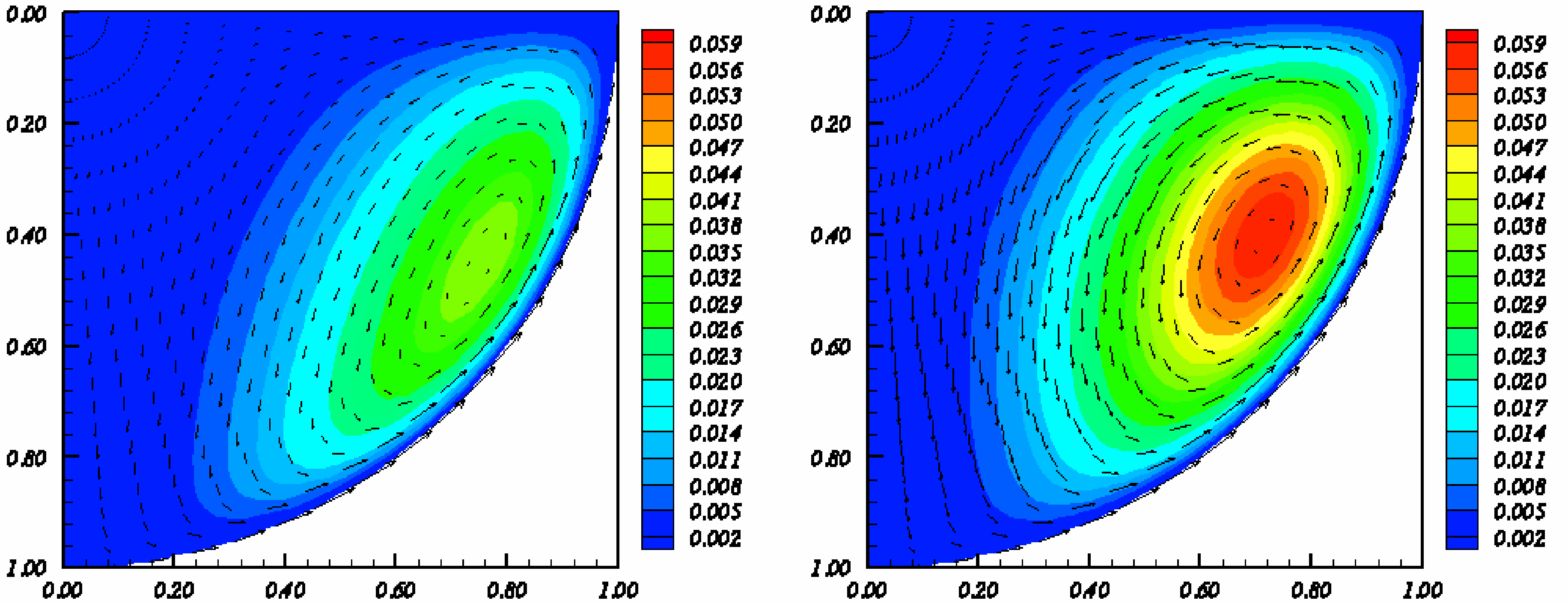



Droplet surface velocity: normal (solid) and tangential (dashed) - $b^* = 4$.

# Circulation Pattern

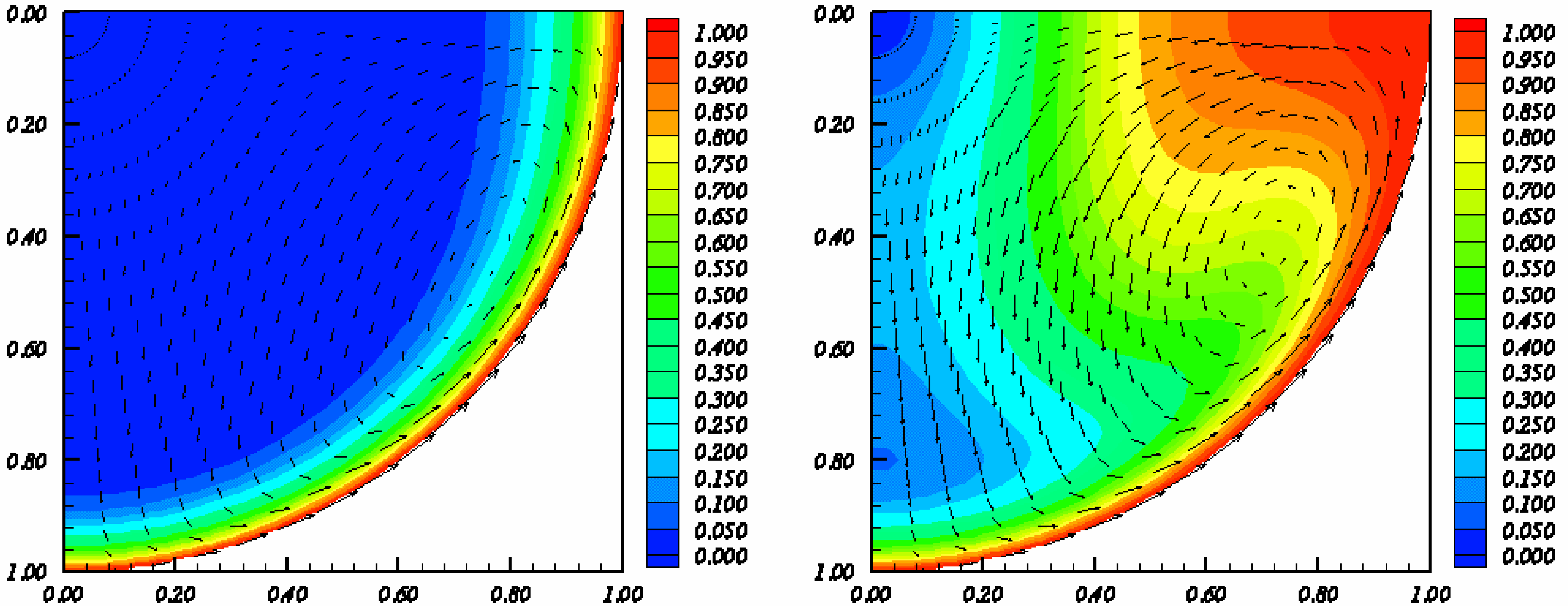


## Results




*Stream Function Re=100, Pr=1 for t = 0.05 (left) and t = 0.95 (right).*

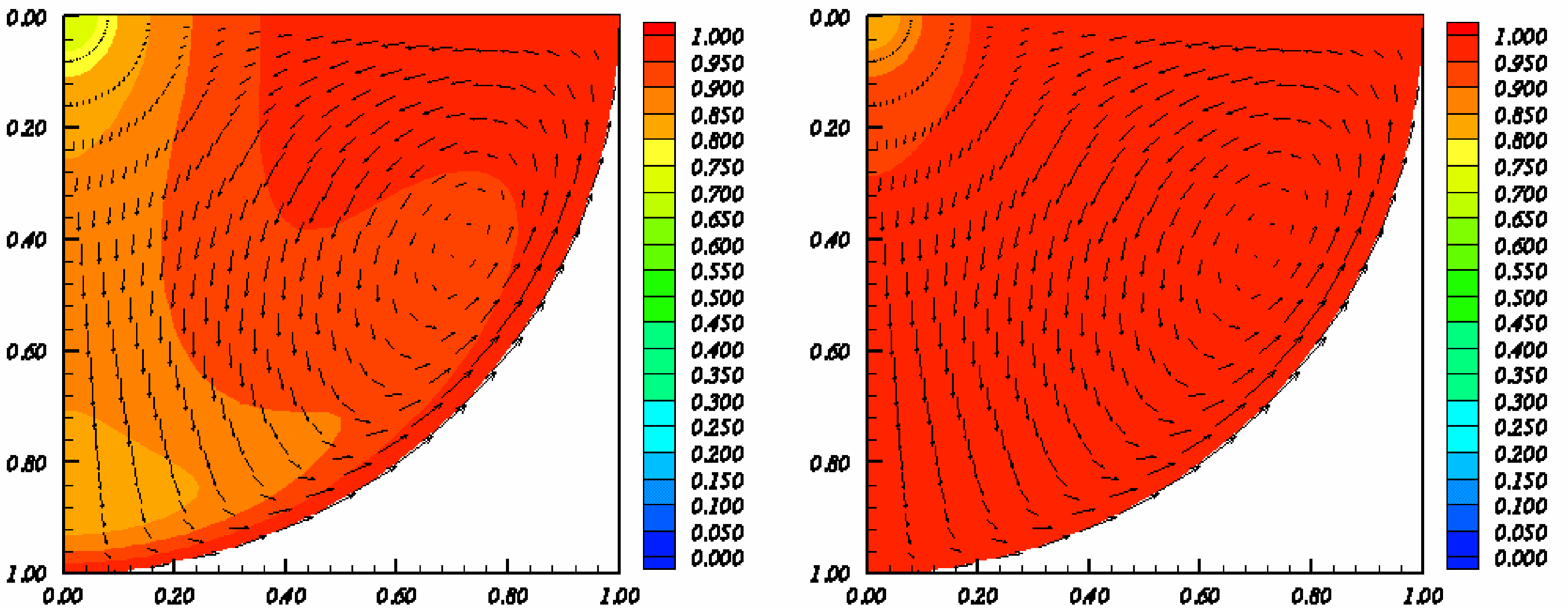
# Results




*Temperature field and the velocity vectors for t = 0.05 (left) and t = 0.30 (right).*

*Re = 100, Pr = 1*

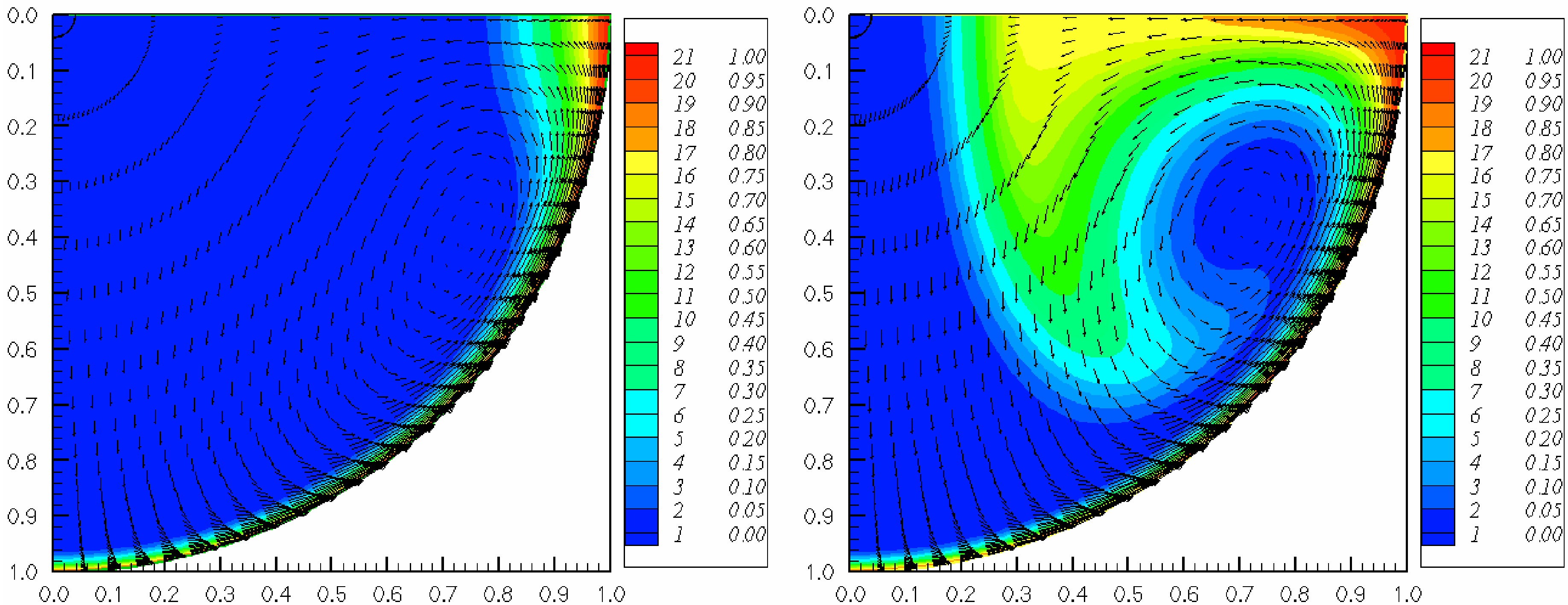
# Results




*Temperature field and the velocity vectors for t = 0.6 (left) and t = 0.9 (right).*

*Re = 100, Pr = 1*

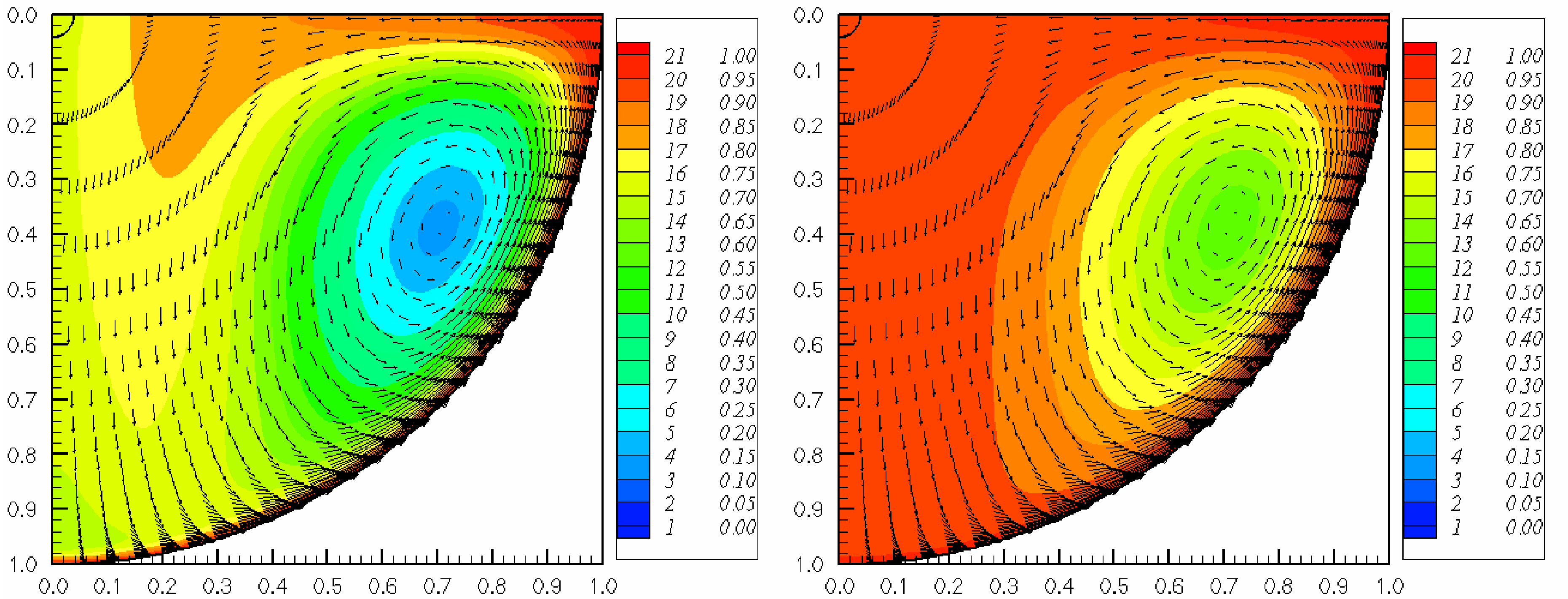
# Results




*Temperature field and the velocity vectors for t = 1.0 (left) and t = 4.0 (right).*

*Re = 100, Pr = 10*

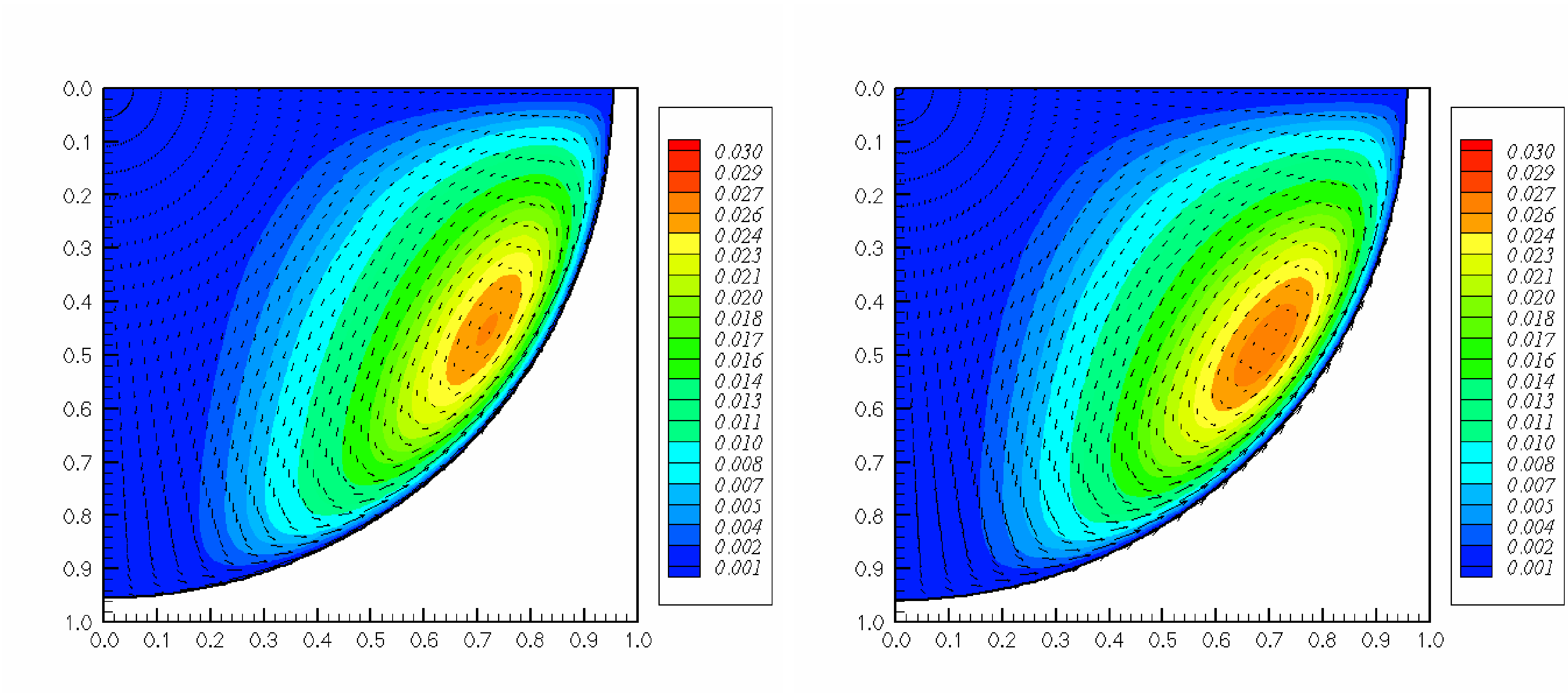
# Results




*Temperature field and the velocity vectors for t = 10.0 (left) and t = 25.0 (right).*

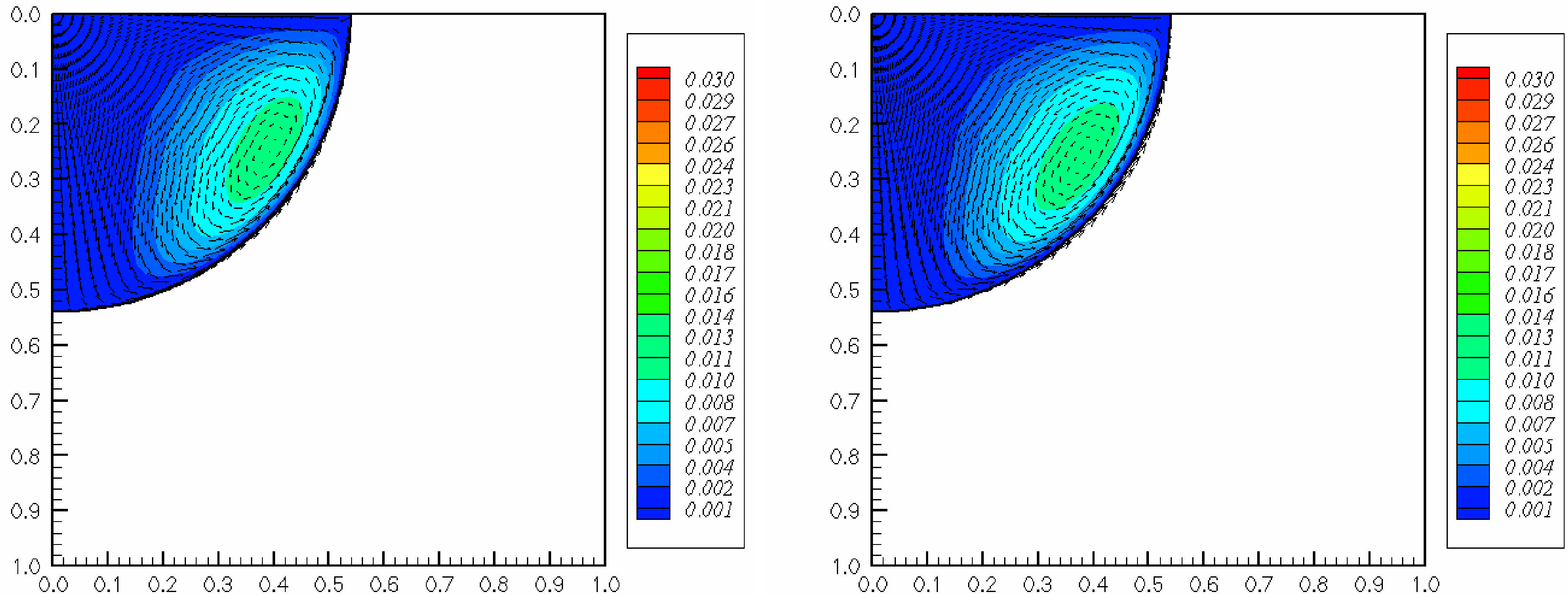
*Re = 100, Pr = 10*

# Results



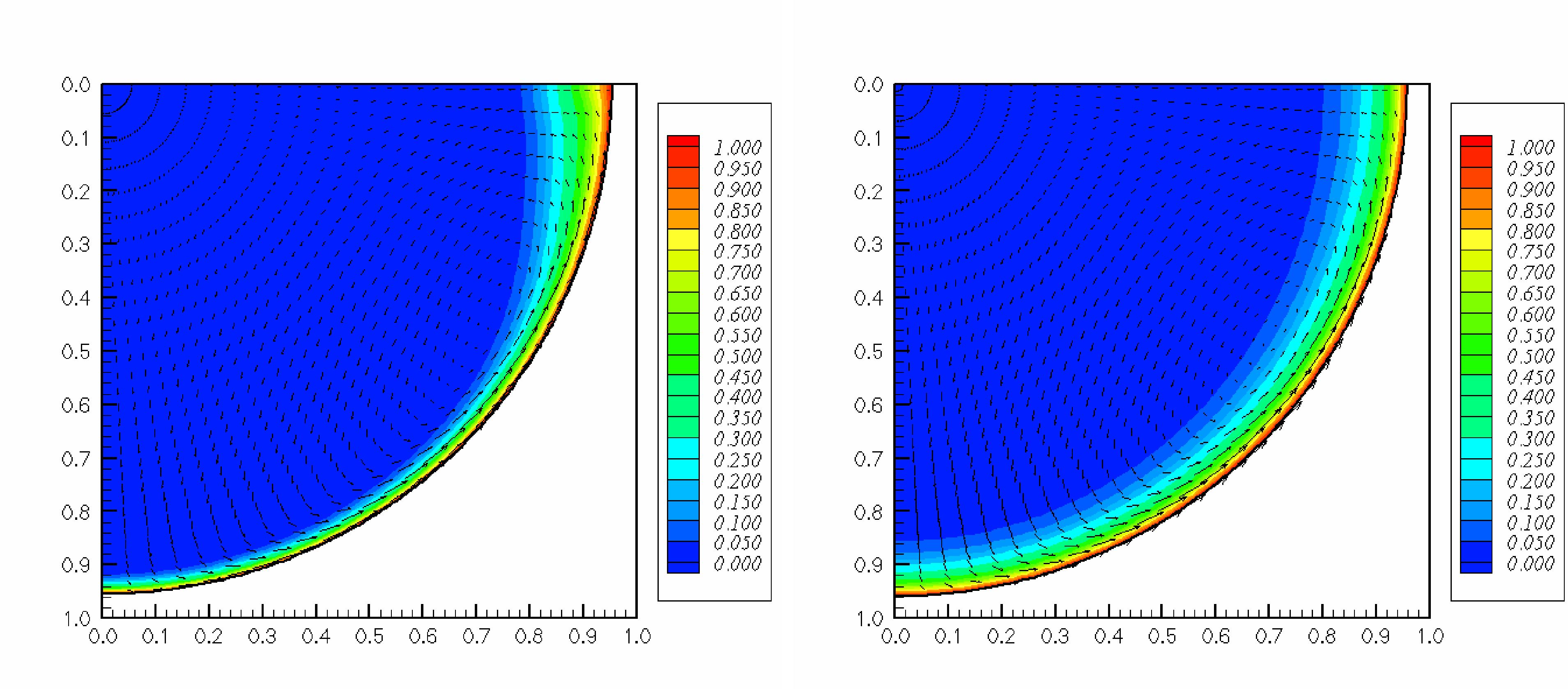

*Stream Function Re=100 (right) and Re=10 (left) for Pr=10, t = 1.0.*

# Results



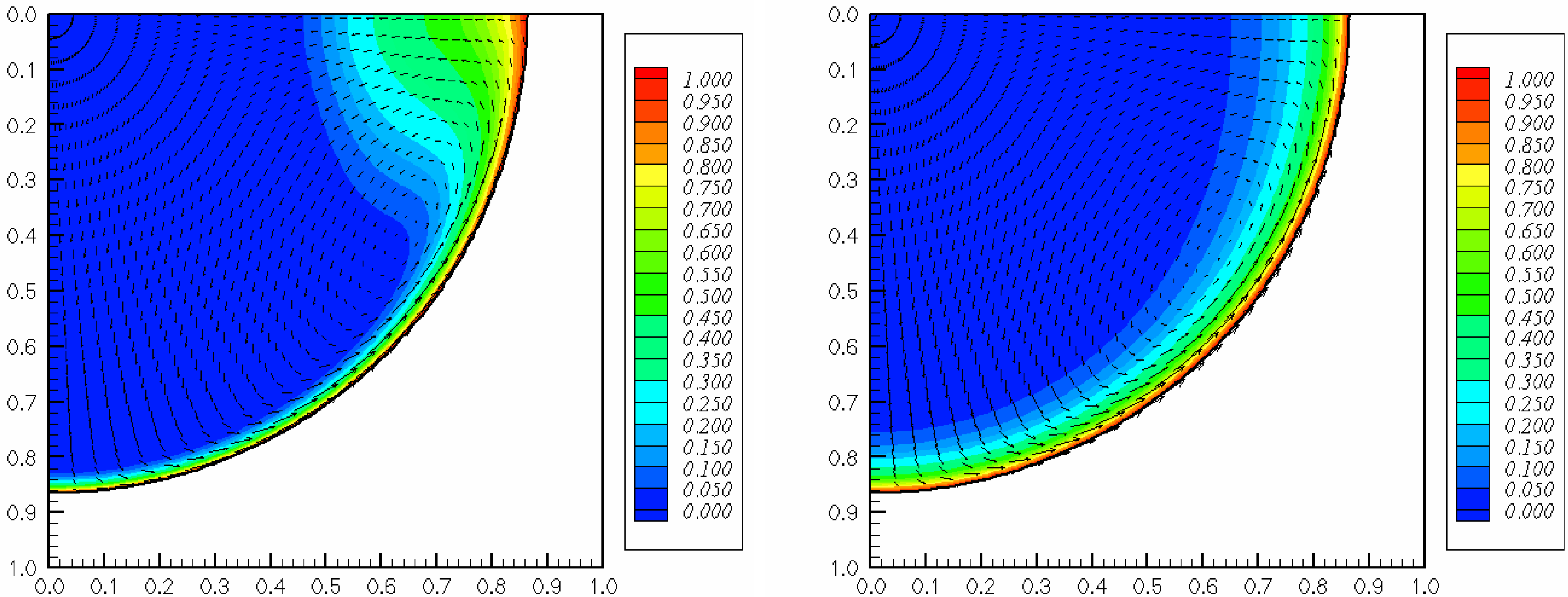

*Stream Function Re=100 (right) and Re=10 (left) for Pr=10, t = 7.0.*

# Results



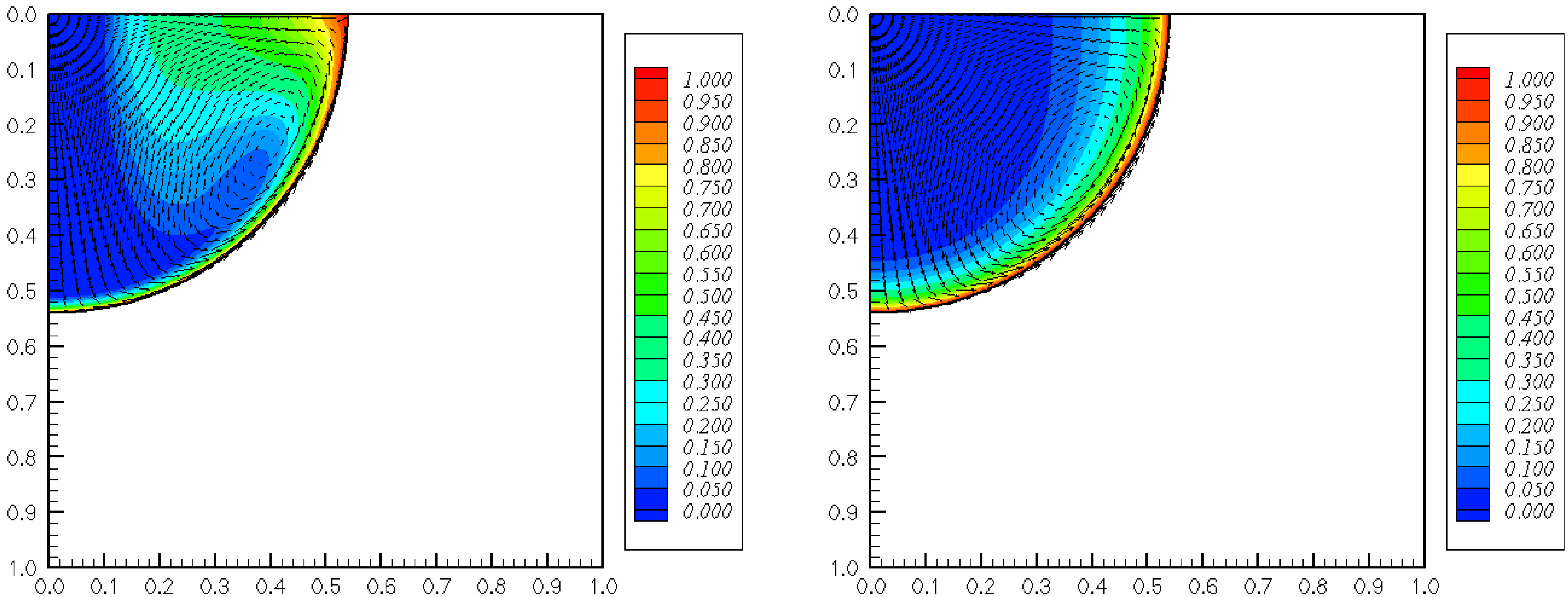

*Temperature field and the velocity vectors for Re=100 (right) and Re=10 (left)*
*Pr=10, t = 1.0*

# ***Results***




*Temperature field and the velocity vectors for Re=100 (right) and Re=10 (left)*
*Pr=10, t = 3.0*

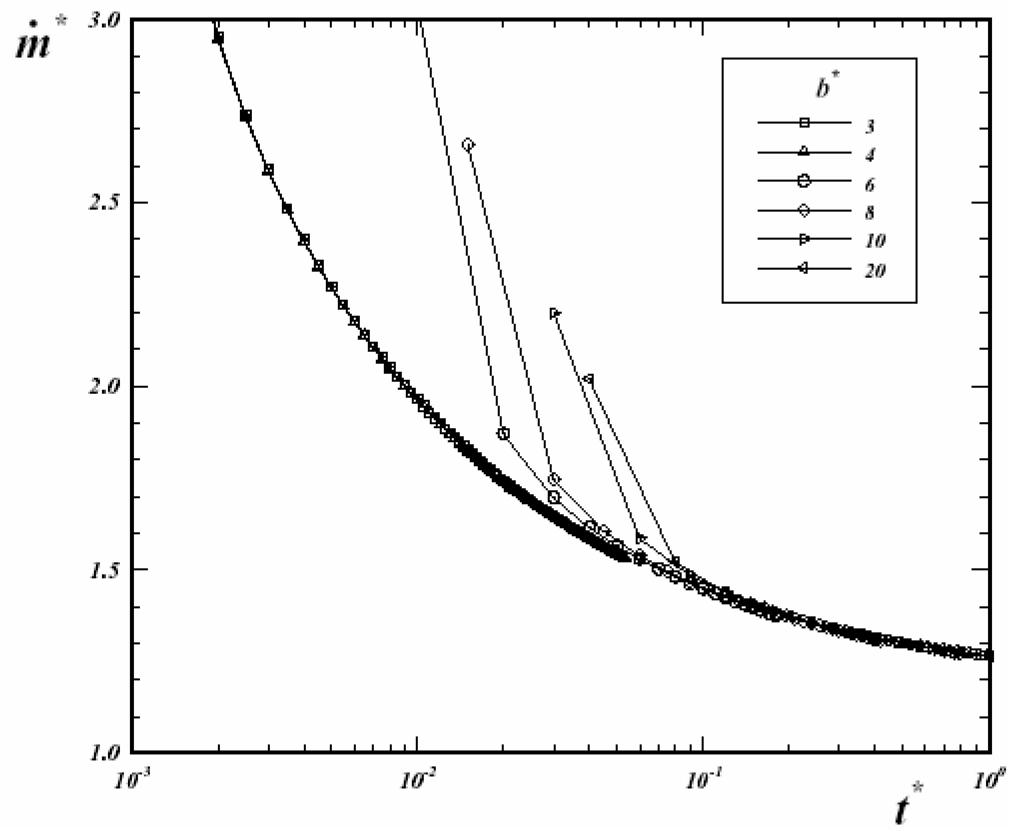
# Results




*Temperature field and the velocity vectors for Re=100 (right) and Re=10 (left)*
*Pr=10, t = 7.0*

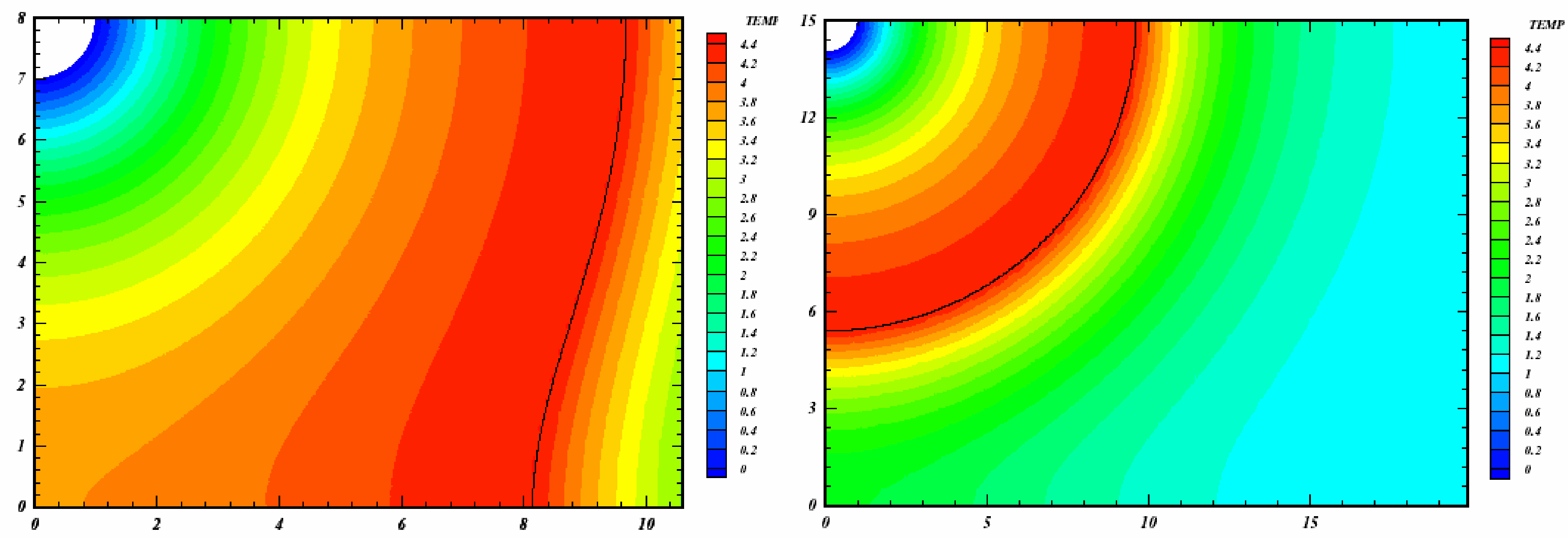
# Mass Vaporization Rate

*Droplet Stream Combustion*



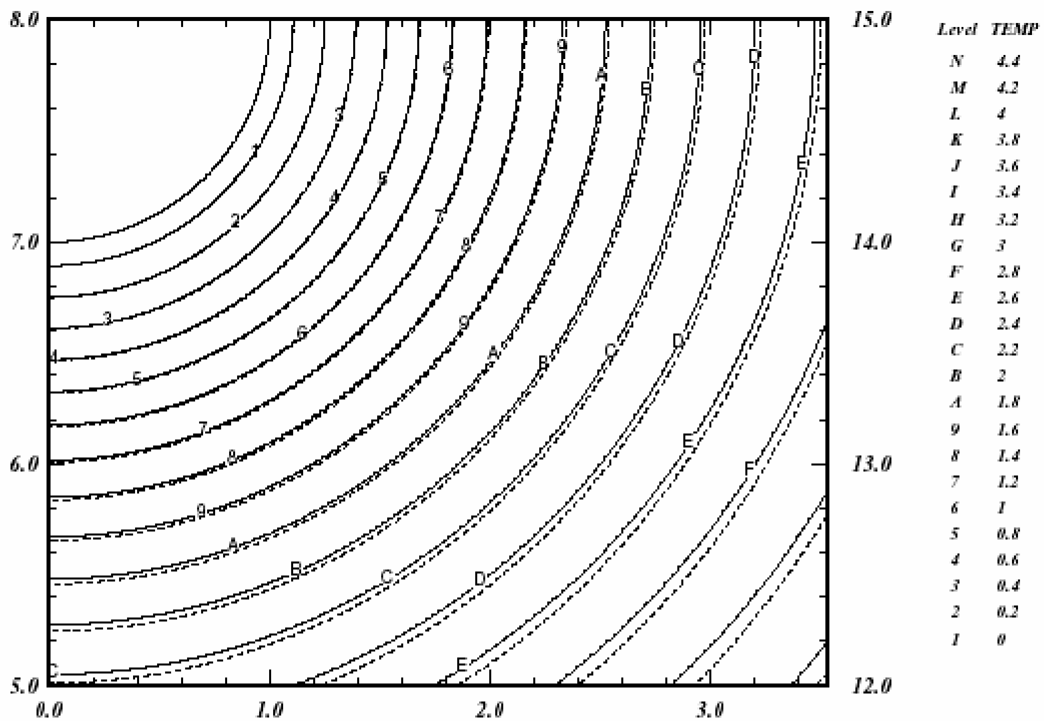

Droplet mass vaporization rate for different interdroplet spacings

# *Droplet Mass Vaporization*



# Mass Vaporization Rate

## Droplet Stream Combustion




Temperature profiles and flame position for for $b^* = 8$ (dashed) and $b^* = 15$ (solid) – $t^* = 0.42$.

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- *Edson Gomes Moreira Filho*
- *Erick Galante*
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## Sponsors

***CNPq, CAPES, FAPERJ, FINEP, IME, PETROBRAS, UFRJ***

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